

Lecture 9

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Quantum teleportation

- possibly the most remarkable
QIP protocol

- combine classical communication
+ entanglement to realise
q. communication

- neither can generate
q. comm. on their own,
but they can when combined

Before presenting teleportation protocol,
let us recall the Bell states

$$|\Phi_{zx}\rangle_{AB} = (Z_A^z X_A^x \otimes I_B) |\Phi\rangle_{AB} \text{ so that}$$

$$|\Phi_{0,0}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

$$|\Phi_{1,0}\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle)$$

②

$$|\Phi_{0,1}\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$$

$$|\Phi_{1,1}\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)$$

can abbreviate as

$$|\Phi_{z,x}\rangle = \frac{1}{\sqrt{2}} (|0x\rangle + (-1)^z |1\bar{x}\rangle)$$

where $\bar{x} = 1 \oplus x$

we also have the identities

$$|00\rangle = \frac{1}{\sqrt{2}} (|\Phi_{00}\rangle + |\Phi_{10}\rangle)$$

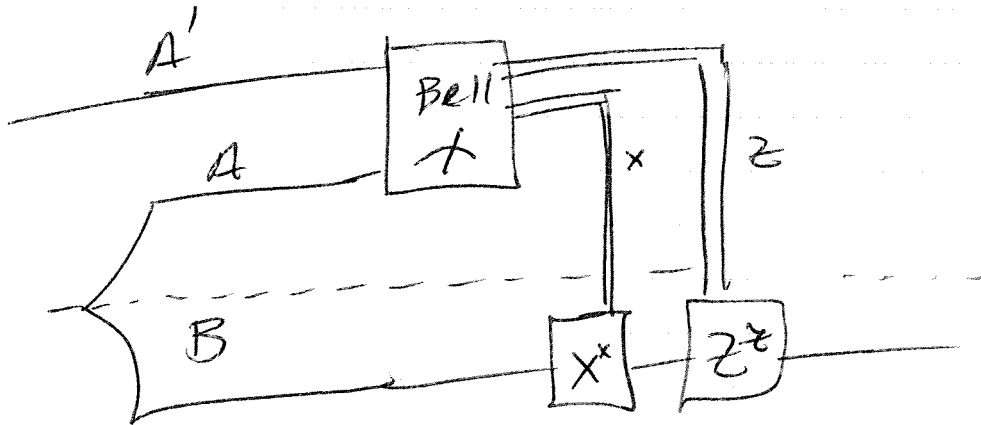
$$|01\rangle = \frac{1}{\sqrt{2}} (|\Phi_{01}\rangle + |\Phi_{11}\rangle)$$

$$|10\rangle = \frac{1}{\sqrt{2}} (|\Phi_{01}\rangle - |\Phi_{11}\rangle)$$

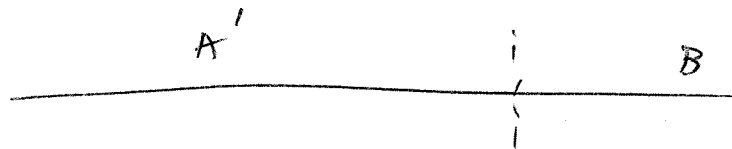
$$|11\rangle = \frac{1}{\sqrt{2}} (|\Phi_{00}\rangle - |\Phi_{10}\rangle)$$

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Diagram for teleportation:



realizes an ideal qubit channel



Why does it work? (use state vector formalism)

$$|\psi\rangle_{A'} = \alpha|0\rangle_{A'} + \beta|1\rangle_{A'}$$

"state to be teleported"

$|\Phi\rangle_{AB}$ initial entangled state

(4)

mutual global state vector is

$$|\psi\rangle_{A'} \otimes |\Phi\rangle_{AB} =$$

$$\frac{1}{\sqrt{2}} \left(\alpha |000\rangle_{A'AB} + \alpha |011\rangle_{A'AB} \right. \\ \left. + \beta |100\rangle_{A'AB} + \beta |111\rangle_{A'AB} \right)$$

Now use previous identities to write as

$$\frac{1}{\sqrt{2}} \left(\alpha \left[\frac{1}{\sqrt{2}} (|\Phi_{00}\rangle + |\Phi_{10}\rangle) \right] |0\rangle_B \right. \\ \left. + \alpha \left[\frac{1}{\sqrt{2}} (|\Phi_{01}\rangle + |\Phi_{11}\rangle) \right] |1\rangle_B \right. \\ \left. + \beta \left[\frac{1}{\sqrt{2}} (|\Phi_{01}\rangle - |\Phi_{11}\rangle) \right] |0\rangle_B \right. \\ \left. + \beta \left[\frac{1}{\sqrt{2}} (|\Phi_{00}\rangle - |\Phi_{10}\rangle) \right] |1\rangle_B \right)$$

(5)

$$= \frac{1}{2} \left(\begin{aligned} &|\Phi_{00}\rangle_{A'A} \otimes (\alpha|0\rangle_B + \beta|1\rangle_B) \\ &+ |\Phi_{10}\rangle_{A'A} \otimes (\alpha|0\rangle_B - \beta|1\rangle_B) \\ &+ |\Phi_{01}\rangle_{A'A} \otimes (\alpha|1\rangle_B + \beta|0\rangle_B) \\ &+ |\Phi_{11}\rangle_{A'A} \otimes (\alpha|1\rangle_B - \beta|0\rangle_B) \end{aligned} \right)$$

$$= \frac{1}{2} \left(\begin{aligned} &|\Phi_{00}\rangle_{A'A} \otimes |\psi\rangle_B + \\ &|\Phi_{10}\rangle_{A'A} \otimes Z_B |\psi\rangle_B + \\ &|\Phi_{01}\rangle_{A'A} \otimes X_B |\psi\rangle_B + \\ &|\Phi_{11}\rangle_{A'A} \otimes X_B Z_B |\psi\rangle_B \end{aligned} \right)$$

After Alice measures $A'A$ in Bell basis, state becomes

$$|\Phi_{00}\rangle_{A'A} \otimes |\psi\rangle_B, \dots, |\Phi_{11}\rangle_{A'A} \otimes X_B Z_B |\psi\rangle_B$$

(b)

w/ equal probability $1/4$.

- Alice communicates measurement outcomes x & z using

two classical bit channels

- Bob then performs X^x & followed by Z^z on system B.

final state is $|+\rangle_B$, so that teleportation is complete.

remarks: Before receiving classical bits, reduced state of Bob is maximally mixed.

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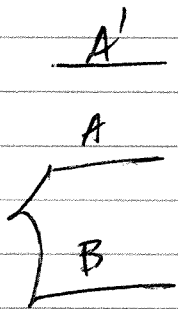
If an eavesdropper gains access to classical bits, they contain no information about q_i state.

There is no instantaneous teleportation: classical bits are needed to reconstruct state.

No violation of no-cloning theorem. state is "disembodied" & reconstructed.

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protocol can be generalized to
qudits, consider diagram:



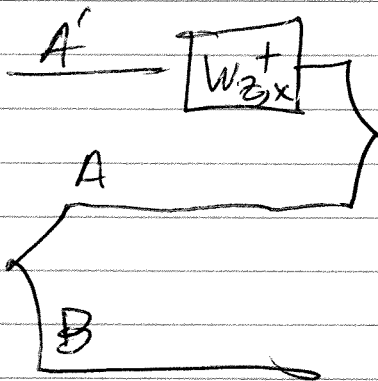
Measure in Bell basis

$$\{ |\Phi_{z,x}\rangle \langle \Phi_{z,x}| \}_{z,x}$$

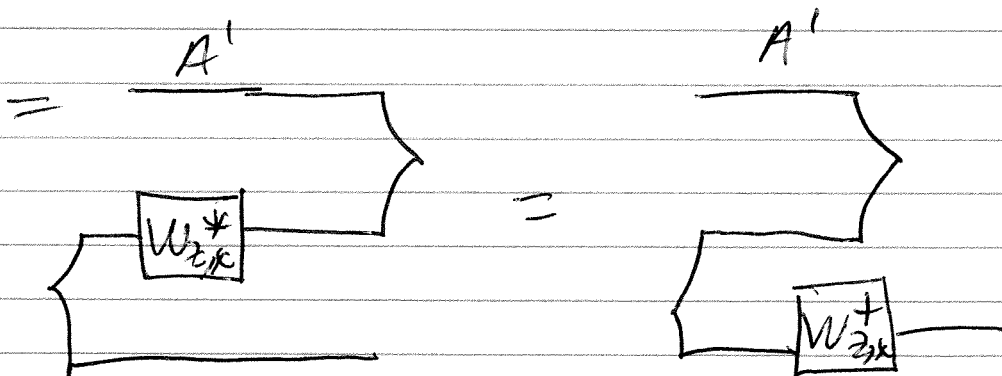
where

$$|\Phi_{z,x}\rangle = (W_{z,x} \otimes I) |\Phi\rangle$$

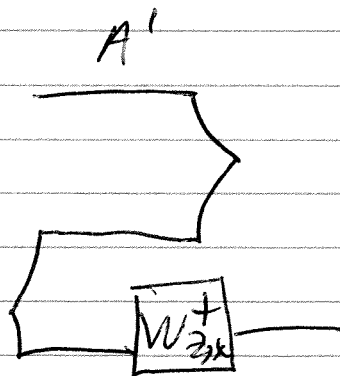
Diagram becomes



use transpose
trick



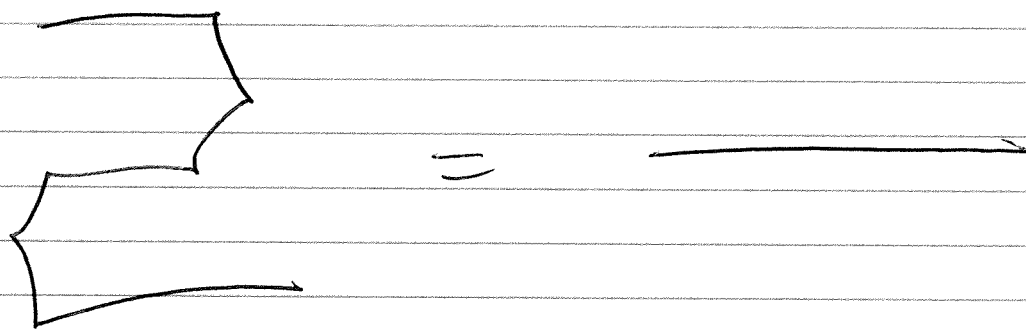
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then Bob applies $W_{z,x}$

to undo $W_{z,x}^\dagger$.



can analyze mathematically