$\boxed{\text{Lecture 2}}$

Key example of how we will use Hoeffding inequality:

Suppose we develop a q. algorithm that outputs 1 w/ probability $\frac{1+s}{2}$ & -1 w/ probability $\frac{1-s}{2}$ where $s \in [-1, 1]$

How to estimate s?

Suppose Z is a R.V. corresponding to the above, then

$$\mathbb{E}[Z] = (+1)\,\frac{1+s}{2} + (-1)\,\frac{1-s}{2}$$

$$= s$$

So an algorithm for estimating s consists of taking n i.i.d. samples of Z $(Z_1, \ldots, Z_n)$, & calculating the sample mean

$$\overline{Z}^n = \frac{1}{n} \sum_{i=1}^{n} Z_i$$

Hoeffding then guarantees that

$$\Pr\{ |\overline{Z}^n - s| \le \varepsilon \} \ge 1 - \delta$$

for $\quad n \ge \frac{2}{\varepsilon^2} \ln\left(\frac{2}{\delta}\right)$

b/c Z takes values $+1$ or -1

other basic concepts from probability theory

- can have a joint distribution $P_{XY}(x,y)$ for two RVs $X+Y$.

- Marginal distribution for X (when Y is not accessible) given by

$$P_X(x) = \sum_{y \in \mathcal{Y}} P_{XY}(x,y)$$

similar for Y:

$$P_Y(y) = \sum_{x \in \mathcal{X}} P_{XY}(x,y)$$

- conditional prob. distribution: prob. to observe x given that y has been observed

$$P_{X|Y}(x|y) = \frac{P_{XY}(x,y)}{P_Y(y)}$$

Another useful inequality from probability theory is the union bound

- Given events $E_1, \ldots, E_\ell$

$$\Pr\{E_1 \cup \cdots \cup E_\ell\} \leq \sum_{i=1}^{\ell} \Pr[E_i]$$

Typically flip this around & use it to bound the probability that an intersection of events does not occur:

$$1 - \Pr\{E_1 \cap \cdots \cap E_\ell\}$$
$$= \Pr\{(E_1 \cap \cdots \cap E_\ell)^c\}$$
$$= \Pr\{E_1^c \cup \cdots \cup E_\ell^c\}$$
$$\leq \sum_{i=1}^{\ell} \Pr\{E_i^c\}$$

"as long as each individual event has small failure prob., then overall failure prob. is small"

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Review of linear algebra basics