Lecture 13

Quantum teleportation

- possibly the most remarkable QIP protocol
- combine classical communication + entanglement to realize q. communication
- neither can generate q. comm. on their own, but they can when combined

Before presenting teleportation protocol, let us recall the Bell states

$$|\Phi_{z,x}\rangle_{AB} = \left(Z_A^z X_A^x \otimes I_B\right) |\Phi\rangle_{AB} \quad \text{so that}$$

$$|\Phi_{0,0}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$
$$|\Phi_{1,0}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$|\Phi_{0,1}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

$$|\Phi_{1,1}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

can abbreviate as

$$|\Phi_{z,x}\rangle = \frac{1}{\sqrt{2}}\left(|0\,x\rangle + (-1)^z |1\,\bar{x}\rangle\right)$$

where $\bar{x} = 1 \oplus x$

we also have the identities

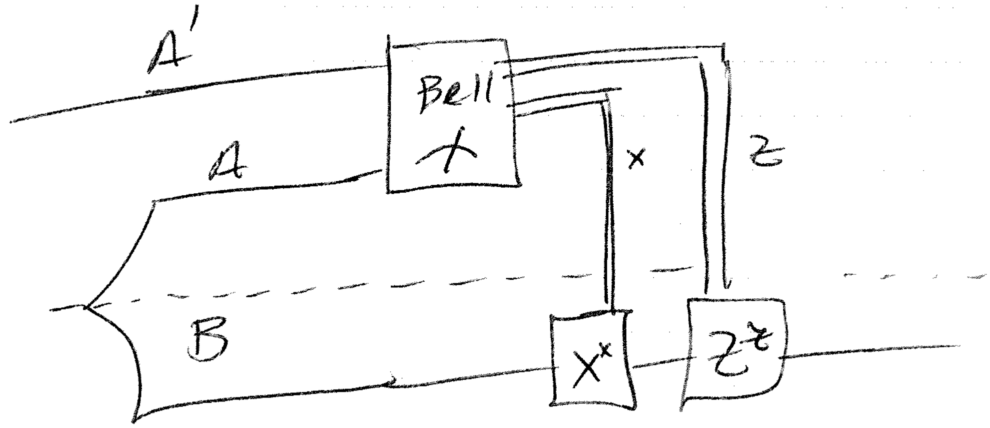
$$|00\rangle = \frac{1}{\sqrt{2}}\left(|\Phi_{00}\rangle + |\Phi_{10}\rangle\right)$$

$$|01\rangle = \frac{1}{\sqrt{2}}\left(|\Phi_{01}\rangle + |\Phi_{11}\rangle\right)$$

$$|10\rangle = \frac{1}{\sqrt{2}}\left(|\Phi_{01}\rangle - |\Phi_{11}\rangle\right)$$

$$|11\rangle = \frac{1}{\sqrt{2}}\left(|\Phi_{00}\rangle - |\Phi_{10}\rangle\right)$$

Diagram for teleportation:



realizes an ideal qubit channel

why does it work? (use state vector formalism)

$|\psi\rangle_{A'} = \alpha|0\rangle_{A'} + \beta|1\rangle_{A'}$ "state to be teleported"

$|\Phi\rangle_{AB}$ initial entangled state

Total global state vector is

$$|\psi\rangle_{A'} \otimes |\Phi\rangle_{AB} =$$

$$\frac{1}{\sqrt{2}}\Big(\alpha|000\rangle_{A'AB} + \alpha|011\rangle_{A'AB} + \beta|100\rangle_{A'AB} + \beta|111\rangle_{A'AB}\Big)$$

Now use previous identities to write as

$$\frac{1}{\sqrt{2}}\Big(\alpha\Big[\frac{1}{\sqrt{2}}\big(|\Phi_{00}\rangle + |\Phi_{10}\rangle\big)\Big]|0\rangle_B$$
$$+\alpha\Big[\frac{1}{\sqrt{2}}\big(|\Phi_{01}\rangle + |\Phi_{11}\rangle\big)\Big]|1\rangle_B$$
$$+\beta\Big[\frac{1}{\sqrt{2}}\big(|\Phi_{01}\rangle - |\Phi_{11}\rangle\big)\Big]|0\rangle_B$$
$$+\beta\Big[\frac{1}{\sqrt{2}}\big(|\Phi_{00}\rangle - |\Phi_{10}\rangle\big)\Big]|1\rangle_B\Big)$$

(5)

$$= \frac{1}{2}\Big(|\Phi_{00}\rangle_{A'A} \otimes (\alpha|0\rangle_B + \beta|1\rangle_B)$$

$$+ |\Phi_{10}\rangle_{A'A} \otimes (\alpha|0\rangle_B - \beta|1\rangle_B)$$

$$+ |\Phi_{01}\rangle_{A'A} \otimes (\alpha|1\rangle_B + \beta|0\rangle_B)$$

$$+ |\Phi_{11}\rangle_{A'A} \otimes (\alpha|1\rangle - \beta|0\rangle)\Big)$$

$$= \frac{1}{2}\Big(|\Phi_{00}\rangle_{A'A} \otimes |\psi\rangle_B +$$

$$|\Phi_{10}\rangle_{A'A} \otimes Z_B|\psi\rangle_B +$$

$$|\Phi_{01}\rangle_{A'A} \otimes X_B|\psi\rangle_B +$$

$$|\Phi_{11}\rangle_{A'A} \otimes X_B Z_B|\psi\rangle_B \Big)$$

After Alice measures $A'A$ in Bell basis, state becomes

$|\Phi_{00}\rangle_{A'A} \otimes |\psi\rangle_B, \ldots, |\Phi_{11}\rangle_{A'A} \otimes X_B Z_B |\psi\rangle_B$

w/ equal probability 1/4.

- Alice communicates measurement outcomes x & z using two classical bit channels

- Bob then performs X^x & followed by Z^z on system B.

final state is $|\psi\rangle_B$, so that teleportation is complete.

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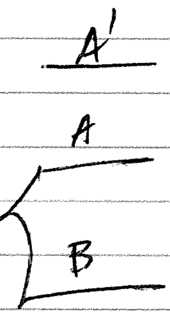
remarks: Before receiving classical bits, reduced state of Bob is maximally mixed.

If an eavesdropper gains access to classical bits, they contain no information about q. state.

There is no instantaneous teleportation: classical bits are needed to reconstruct state.

No violation of no-cloning theorem. State is "disembodied" & reconstructed.

protocol can be generalized to qudits. Consider diagram:



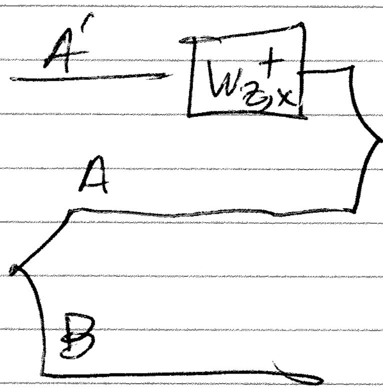
Measure in Bell basis

$$\{|\Phi_{z,x}\rangle\langle\Phi_{z,x}|\}_{z,x}$$

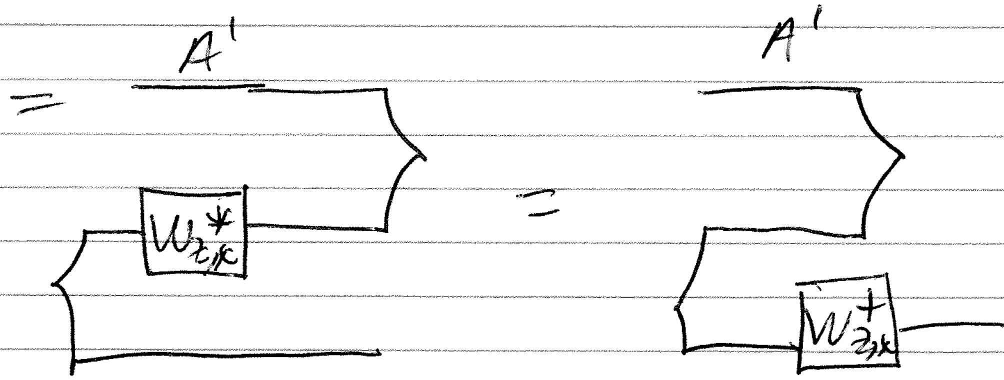
where

$$|\Phi_{z,x}\rangle = (W_{z,x}\otimes I)|\Phi\rangle.$$

Diagram becomes

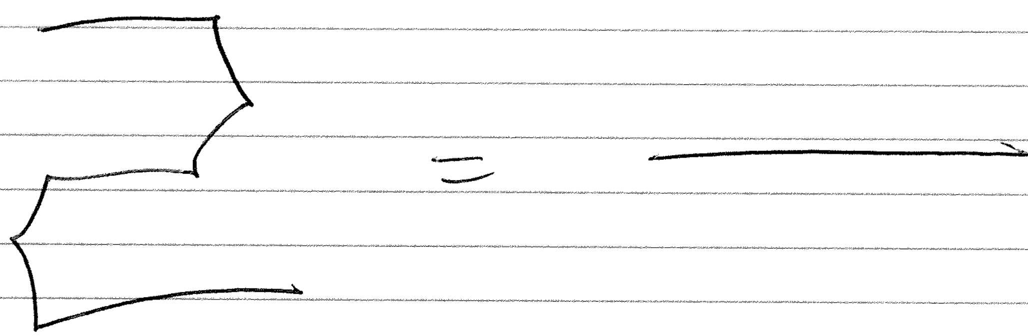


use transpose trick



then Bob applies $W_{z,x}$

to undo $W_{z,x}^{\dagger}$.



can analyze mathematically