Lecture 38

14-6 Equation of continuity

discuss motion of an ideal fluid, which has the following assumptions:

1) Steady flow - velocity of fluid @ any fixed point does not change w/ time.
2) Incompressible flow - density has a constant, uniform value.
3) Nonviscous flow - viscosity is a measure of how resistive fluid is to flow. nonviscous then means no friction.

4) ~~Iprotational~~ Irrotational flow-

if an insect were in fluid, it would not spin around its own center of mass.

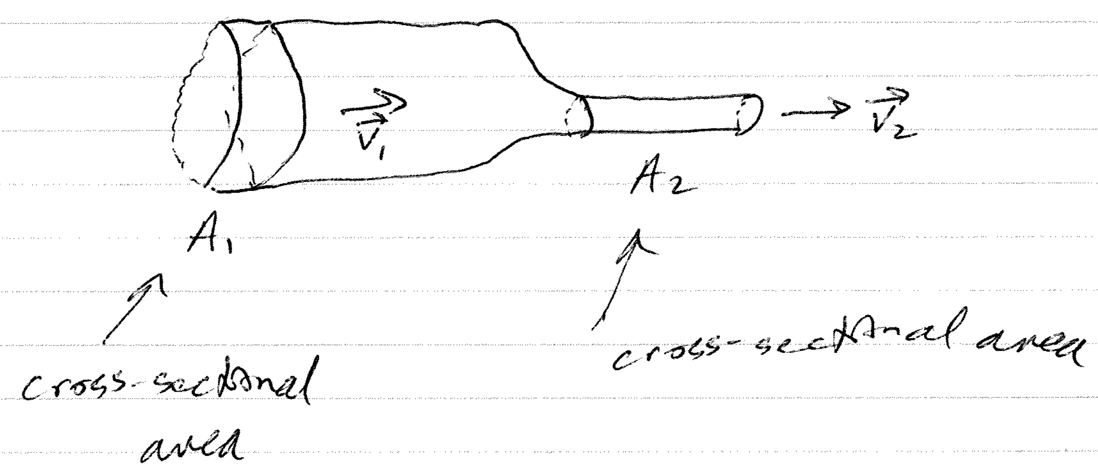
<u>equation of continuity</u>

Recall that you can increase speed of water emerging from a garden hose by partially covering hose opening w/ your thumb.

⇒ speed v of water depends on cross sectional area A through which water flows.

we can derive an equation that expresses this.

- Consider fluid flow through the following tube:

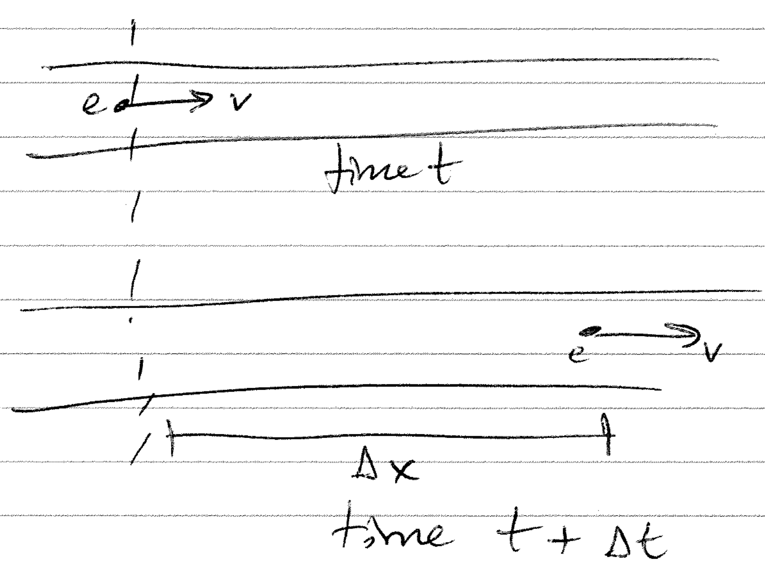


- Suppose in time interval Δt a volume ΔV enters tube @ left end.

- Since fluid is <u>incompressible</u>, an identical volume ΔV shall emerge from right end.

- can use common volume to relate speeds & areas

To figure out volume that passes through cross-sectional area in time Δt, consider how ~~far~~ a fluid element e goes in time Δt:



$$\Rightarrow \Delta V = A\,\Delta x = A\,v\,\Delta t$$

So then applying the fact that volume ΔV stays constant, we get that

$$\Delta V = A_1 v_1 \Delta t$$
$$= A_2 v_2 \Delta t$$

$$\Rightarrow A_1 v_1 = A_2 v_2$$ (eq. of continuity)

equation applies not only to a tube but also to any <u>tube of flow</u> (imaginary tube inside flow of fluid)

can rewrite previous equation as

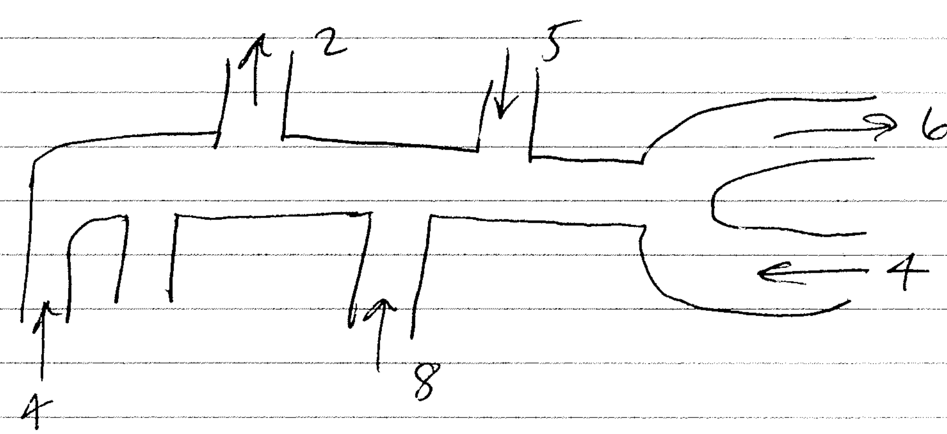
$$R_V = Av = \text{a constant}$$

↑ volume flow rate of liquid

can multiply by density to get mass flow rate:

$R_m = \rho R_v = \rho A v =$ a constant

Q:



depicted is a pipe w/ volume flow rates & directions of flow.

What is direction of flow + volume flow rate for missing tube?

$\Rightarrow$ 13 & out

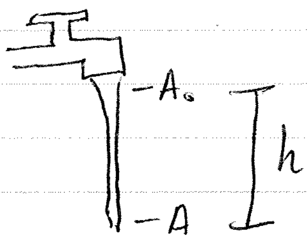
~~4+8+6+5-2~~

$4+8+4+5-2-6 = 13$

Why does water stream from faucet become narrower the farther down you look?

gravitatonal force increases speed of water the farther down it is.

What is volume flow rate from tap given A_0, A, & h?



volume flow rates should match.

$$\Rightarrow \quad A_0 v_0 = A v$$

$$v^2 = v_0^2 + 2gh \quad \text{(from 2110)}$$

$\Rightarrow$ ~~$v_0 = \sqrt{2gh - v^2}$~~

$$v = \frac{A_0 v_0}{A} \qquad \left(\frac{A_0 v_0}{A}\right)^2 = v_0^2 + 2gh$$

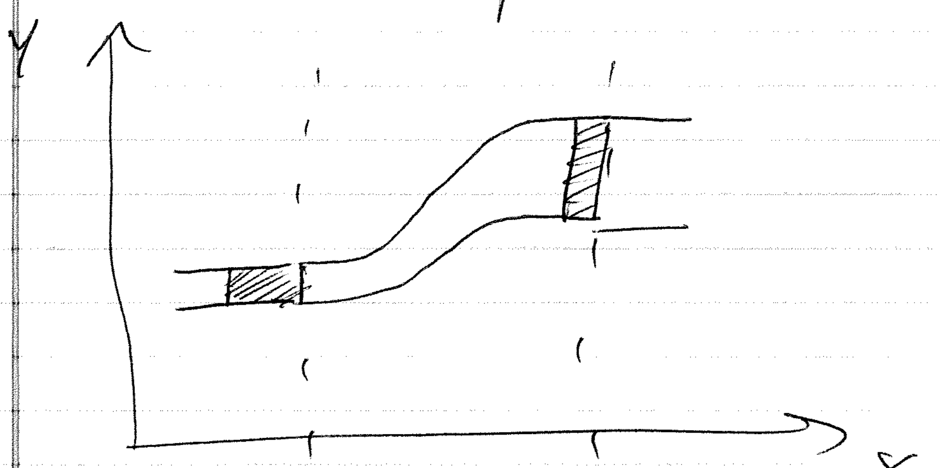
$$\left(\left(\frac{A_o}{A}\right)^2 - 1\right) v_o^2 = 2gh$$

$$\Rightarrow \quad v_o^2 = \frac{2gh}{\left(\frac{A_o}{A}\right)^2 - 1}$$

$$\Rightarrow \quad v_o = \sqrt{\frac{2gh}{\left(\frac{A_o}{A}\right)^2 - 1}}$$

Bernoulli's Equation

Consider picture of fluid flow



Let y_1 be elevation, v_1 speed, & p_1 pressure of fluid entering @ left.

y_2, v_2, p_2 are same quantities for right side.

As a consequence of conservation of energy, we have Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1$$
$$= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$\frac{1}{2}\rho v^2$ is called kinetic energy density

To get a rough sense of how it is a consequence of conservation of energy, multiply by volume

If fluid is @ rest,

then $v_1 = v_2 = 0$ &

$$p_2 = p_1 + \rho g (y_1 - y_2)$$

what we derived previously

If y is constant $y_1 = y_2$

then

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

- If speed of fluid increases along a horizontal streamline, then pressure of fluid decreases & conversely.