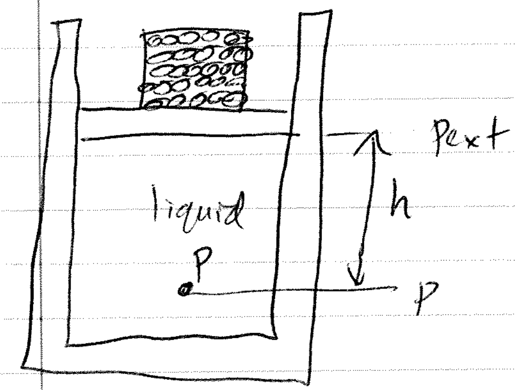
Lecture 37

14-4 Pascal's principle

Change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid & to the walls of its container

Example: squeezing one end of toothpaste tube to get it out the other end.

Consider incompressible fluid contained in a tall cylinder.



Atmosphere, container, + lead shot exert pressure p_{ext} on piston.

Pressure p @ point P is

$$p = p_{ext} + \rho g h$$

where ρ is density of liquid + g is gravitational constant (formula derived last time)

If we increase pressure p_{ext} by amount Δp_{ext}, then pressure change @ P is

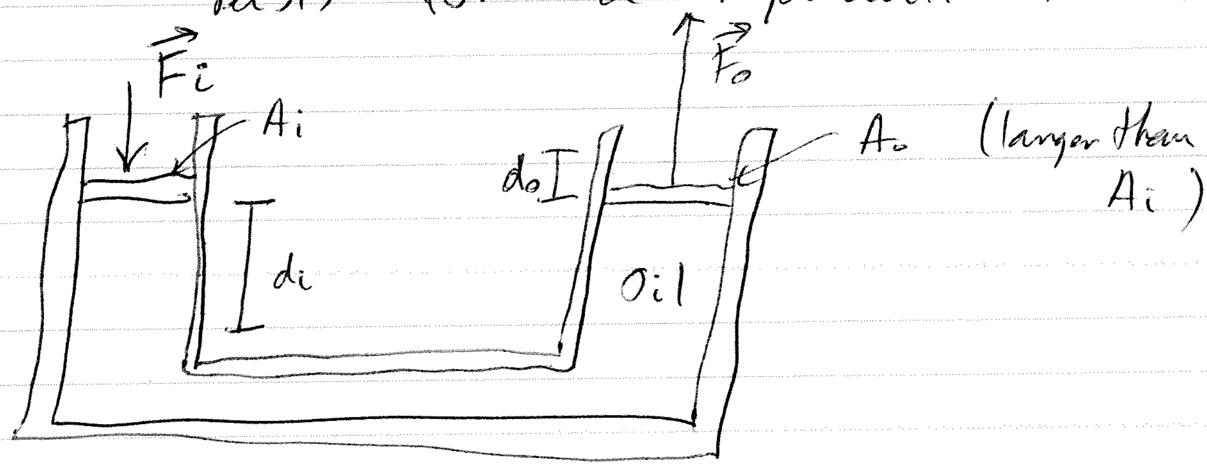
$$\Delta p = \Delta p_{ext}$$

(pressure change is independent of h)

So Pascal's principle holds.

Pascal's principle & hydraulic lever

use Pascal's principle as basis for a hydraulic lever.



direct force $\vec{F}_i$ downward

- on left piston w/ area A_i

- incompressible liquid produces upward force F_o on right piston, w/ area A_o

Applying force on left produces a change in pressure given by

$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o}$$

↑ apply Pascal's principle

$$\Rightarrow F_o = \frac{F_i}{A_i} \cdot A_o$$

If $A_o > A_i$, then output force is greater.

If input piston moves downward a distance d_i, then output piston moves upward a distance d_o.

- same volume of incompressible liquid is displaced @ both pistons

$$V = A_i d_i = A_o d_o$$

$$\Rightarrow \quad d_o = d_i \frac{A_i}{A_o}$$

- Allows to figure out the distance moved.
- If $A_o > A_i$, then output piston moves a smaller distance.

Q: We can also calculate work done on output:

Q: $$W = F_o d_o = \left(F_i \frac{A_o}{A_i}\right)\left(d_i \frac{A_i}{A_o}\right)$$

$$= F_i d_i$$ (same as work done on input)

With hydraulic lever, a force applied over a distance can be transformed to a greater force applied over smaller distance.

can use a hydraulic jack to lift a car!

Archimedes principle (14-5)

Answers why some objects float & others don't.

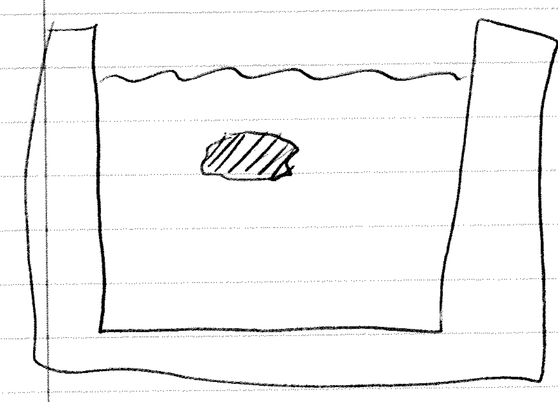
- When a body is fully or partially submerged in a fluid, a buoyant force $\vec{F}_b$ from surrounding fluid acts on the body.

Force is directed upward & has magnitude equal to weight $m_f g$ of fluid that has been displaced by the body.

$$F_b = m_f g$$

↑ mass of fluid displaced by body.

To see this, consider a stone submerged in water



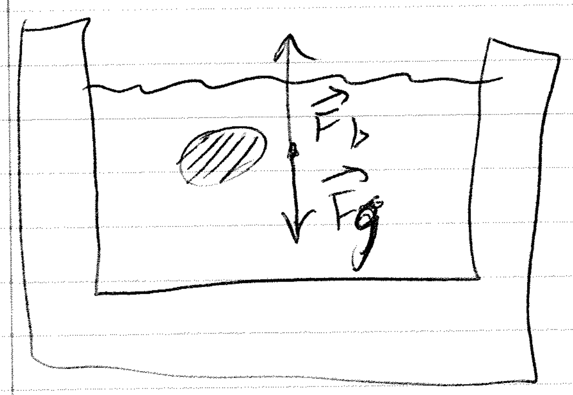
Now ~~not~~ imagine a hole where the stone is

pressure near bottom of stone is greater than pressure near top of stone

Add all forces around hole & net force is upward, called buoyant force $\vec{F_b}$

- Its magnitude is equal to $\vec{F_b} = m_f g$, which comes about from analyzing a static situation of equilibrium

- For a stone, downward force due to gravity is larger than buoyant force, so it sinks



Whether it rises or sinks depends on F_b vs. F_g.

Floating - when a body floats to the top of water or liquid, magnitude of buoyant force equals magnitude of gravitational force:

$$F_b = F_g$$

Apparent weight in a fluid:

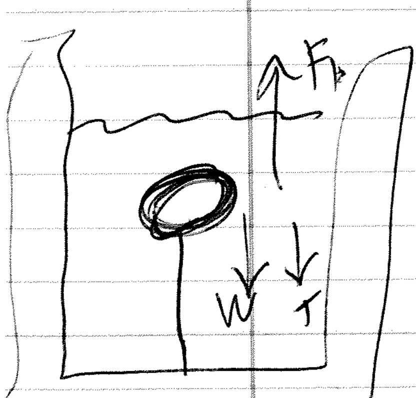
$$\text{weight}_{app} = \text{weight} - F_b$$

↑ actual weight

easier to lift something when it is underwater

Example problem:

A solid foam sphere w/ density 706 kg/m³ & radius of 0.15 m is tied w/ string to bottom of a water filled tank. Sphere is submerged. What is tension in string?



$$F_b - W - T = 0$$

$$\Rightarrow T = F_b - W$$

$$= W_f - W$$

↑ weight of displaced fluid

$$\Rightarrow \quad T = (m_f - m)\, g$$

$$= (\rho_f V_f - \rho V)\, g$$

$V_f = V$ since sphere submerged

$$\Rightarrow \quad T = (\rho_f - \rho)\, g\, V$$

$$= (\rho_f - \rho)\, g \cdot \frac{4}{3}\pi r^3$$

$$= (1000\ kg/m^3 - 706\ kg/m^3)$$

$$\cdot\ 9.8\ m/s^2 \cdot \frac{4}{3}\pi\ (0.15)^3\ m^3$$

$$= 41\ N$$