Lecture 36

Heisenberg's uncertainty principle

- It is not possible to assign deterministic values to both the position & momentum of a particle
- recall that the state of a quantum particle is not described by a list of numbers but instead a wavefunction from which we can compute probabilities

In quantitative terms, uncertainty principle is

$$\Delta x \cdot \Delta p_x \geq \hbar = \frac{h}{2\pi}$$

Planck constant

uncertainty (standard deviation) when measuring x component of position

uncertainty (standard deviation) when measuring x component of momentum

~~uncer~~ lower bound is small but it matters when dealing w/ effects @ quantum scale.

To interpret, suppose that we can make uncertainty of position x very small.

Then necessarily uncertainty of momentum p_x must be large.

There is also uncertainty principle for y + z coordinates:

$$\Delta y \cdot \Delta p_y \geq \hbar$$

$$\Delta z \cdot \Delta p_z \geq \hbar$$

We won't derive them here b/c we need more background on quantum mechanics to do that!

Q: Suppose that an electron is moving along an x axis and that you measure its speed to be 2.05×10^{6} m/s & w/ precision of .5%.

What is minimum uncertainty w/ which you can measure its position?

Note that $\Delta p_x = (.005)\, p_x$.

A:

$$p_x = m \cdot v_x = (9.11 \times 10^{-31}\,\text{kg})(2.05 \times 10^{6}\,\text{m/s})$$

$$= 1.87 \times 10^{-24}\,\text{kg}\cdot\frac{\text{m}}{\text{s}}$$

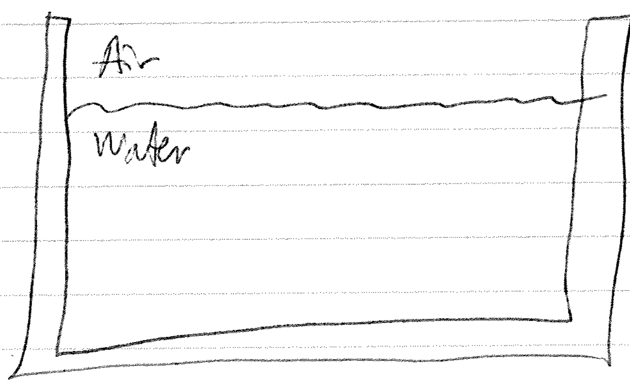
$$\Rightarrow \Delta p_x = 9.35 \times 10^{-27}\,\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$\Rightarrow \Delta x = \frac{\hbar}{\Delta p_x} \approx \frac{6.63 \times 10^{-34}\,\text{J}\cdot\text{s}/2\pi}{9.35 \times 10^{-27}\,\text{kg}\cdot\text{m/s}}$$

$= 11\,nm$ ($\tilde{100}$ atomic diameters)

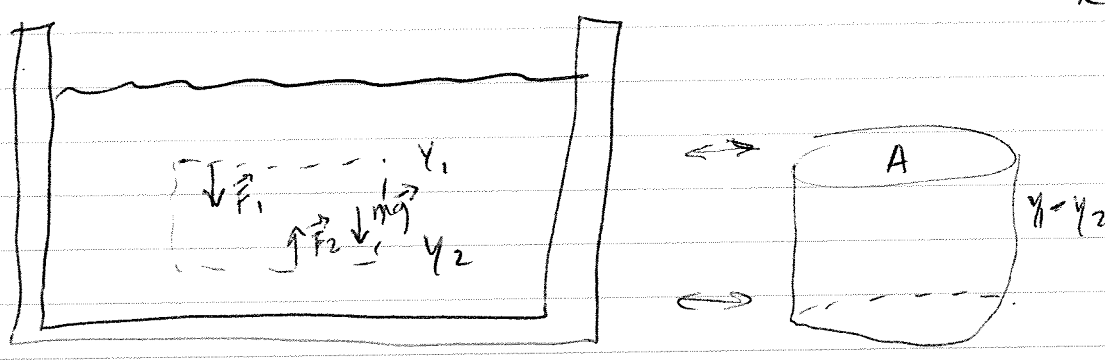
14-2 Fluids at rest

Consider a tank of water



- pressure increases w/ depth below air-water interface.
- we are interested in hydrostatic pressure, due to fluid @ rest, as a function of depth.

How to analyze? Consider forces acting on a cylindrical region in tank



- downward force $\vec{F}_1$ due to water above cylinder
- upward force $\vec{F}_2$ due to water below cylinder
- $m\vec{g}$ is downward gravitational force due to mass of water in cylinder.

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balance of forces is (sum of forces = 0 for static scenarios)

$$F_2 = F_1 + mg$$

In terms of pressures, this is

$$F_1 = p_1 A \quad , \quad F_2 = p_2 A$$

mass of water in cylinder is

$$m = \rho V$$

ρ: density of water

V: volume of water

so mass of water

$$m = \rho \underbrace{A (y_1 - y_2)}_{\text{volume of cylinder}}$$

$$\Rightarrow \quad p_2 A = p_1 A + \rho A (y_1 - y_2) g$$

$$\Rightarrow \quad p_2 = p_1 + \rho (y_1 - y_2) g$$

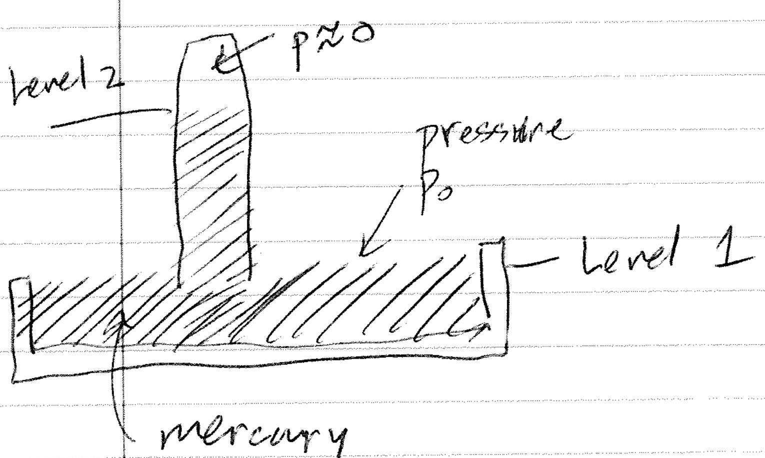
can use this to find pressure below surface.

can also use it to find pressure in atmosphere.

pressure @ a point in a static fluid depends only on depth + not on horizontal dimension.

Measuring Pressure

Mercury Barometer



Can use previous equation to calculate atmospheric pressure p_0 in terms of height h of mercury column.

$y_1 = 0 \,,\; p_1 = p_0$

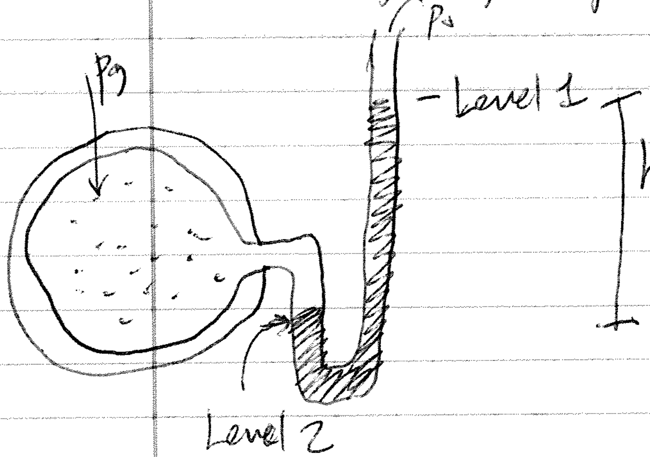
$y_2 = h \,,\; p_2 = 0$

$\Rightarrow \quad p_0 = \rho g h$

where ρ is density of mercury.

Another way to measure

open-tube manometer measures gauge pressure of a gas



called a "U-tube"

can again use equation to measure pressure p_g.

$y_1 = 0 \qquad p_1 = p_0$

$y_2 = -h \qquad p_2 = p$

$\Rightarrow \quad p_g = p - p_0 = \rho g h$

density of liquid

gauge pressure can be positive or negative (depending on whether $p > p_0$ or $p < p_0$)