# Lecture 31

## 36-5 Diffraction Gratings

- diffraction grating is like setup of double slit, but has a much greater number $N$ of slits (show slides)

- when monochromatic light is sent through slits, it forms narrow interference fringes that can be used to determine wavelength $\lambda$ of light.

- If we gradually increase # of slits from two to $N$, intensity plot changes from double-slit plot to much more complicated & then a simple graph (show slides)

- maxima as seen on screen are narrow lines & they are separated by wide dark regions

<u>GOAL</u>: Find locations of bright lines

Make same assumption as before: screen is far enough away so that rays reaching a point P are $\approx$ parallel.

Apply same reasoning from double slit for path length differences

- separation $d$ between rulings is called <u>grating spacing</u>

- If $N$ rulings occupy total width $w$, then $d = w/N$.

- Path length difference between adjacent rays is then <u>$d \sin\theta$</u>

<u>Q:</u> - Since we are interested in constructive interference for bright lines, they occur @

$$d \sin\theta = m\lambda \quad \text{for } m = 0, 1, 2, \ldots$$

(same as for double slit)

- each integer $m$ represents a different line (bright line)

Integers are then called order numbers

$m=0$ — central line

$m=1$ — first-order line

$m=2$ — second-order line

We can then use the formula to determine wavelength of light

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

or $$\lambda = \frac{d\sin\theta}{m}$$

## Width of lines

- grating's ability to resolve lines of different wavelengths relies on width of lines (show slides) half-width of central line:

angle $\Delta\theta_{hw}$ from center of line @ $\theta = 0$ outward to where line effectively ends & darkness effectively begins w/ 1st minimum.

Actual width of central line is $2(\Delta\theta_{hw})$

How to calculate the half-width?

- Need to figure out angle to 1st minimum (this is $\Delta\theta_{hw}$)

- use same reasoning from single slit diffraction

- use rule of thumb from before that minimum are related to path length difference between top & bottom rays being equal to $\lambda, 2\lambda, 3\lambda, \ldots$

(show slides)

- Then path length difference for top & bottom rays is

$$Nd \sin \Delta\theta_{hw}$$

comes from geometry & fact that total width of diffraction grating is $Nd$.

So condition is then

$$Nd \sin \Delta\theta_{hw} = \lambda$$

Since $\Delta\theta_{hw}$ is small, we have

$$\Delta\theta_{hw} = \frac{\lambda}{ND}$$ (half-width of central line)

State without justification that

$$\Delta\theta_{hw} = \frac{\lambda}{Nd\cos\theta}$$ (half width of line @ angle $\theta$)

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- It is not just half-width that determines what we can resolve but also <u>dispersion</u>
- we can use diffraction grating for polychromatic light
- grating is useful if it spreads apart diffraction lines associated w/ different wavelengths

this spreading or dispersion is defined as

$$D = \frac{\Delta\theta}{\Delta\lambda}$$

- greater D is, greater is the distance between two emission lines whose wavelengths differ by $\Delta\lambda$

- can show that dispersion @ angle $\theta$ is

$$D = \frac{m}{d\cos\theta}$$

where m is the order of line,
d is grating spacing

- resolving power:
to resolve lines of different wavelength, the line should be as narrow as possible.

resolving power $R = \dfrac{\lambda_{avg}}{\Delta \lambda}$

$\lambda_{avg}$ = mean wave-length of two emission lines that can barely be recognized as separate

$\Delta \lambda$ is wavelength difference

Q: What criterion do we need to apply to figure this out?
Rayleigh

follow derivation + get that

$R = Nm$

resolving power increases w/ # of rulings.

b) What is angular separation between the two lines?

$$D = \underbrace{\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}}_{\text{use dispersion equation}}$$

$$\Delta\theta = \frac{m\,\Delta\lambda}{d\cos\theta} = \frac{1\cdot(589.59-589_{nm})}{2016nm\cdot\cos(16.99)}$$

$$= 3.06\times10^{-4}\ \text{rad}$$