36-2

Lecture 30

Intensity in single-slit diffraction

- Goal now is to consider an expression for the intensity of diffraction pattern appearing in single-slit diffraction
- There is a long procedure for doing this. At a high level, it involves dividing slit into N zones of equal widths Δx & assuming each zone acts as a point source of Huygens' wavelets.
- Then use superpposition principle to add E-field of each point source, then square this to get intensity

the result of this procedure is that intensity $I(\theta)$ as a function of angle θ from central axis is

$$I(\theta) = I_m \left(\frac{\sin(\alpha(\theta))}{\alpha(\theta)} \right)^2$$

where

$$\alpha(\theta) = \frac{\pi a}{\lambda} \sin\theta$$

a - slit width

λ - wavelength of light

I_m - greatest value of intensities in pattern.

we can consider special cases:

Q: What is value when $\theta = 0$?

Since $\alpha(\theta) \to 0$ as $\theta \to 0$

& $\sin\theta \approx \theta$ for small θ, $I(\theta = 0) = I_m$

Q: Where do minima occur?
(for what values of α?)

when $\alpha = m\pi$ for $m = 1, 2, 3, \ldots$

b/c $\sin(m\pi) = 0$

$\Rightarrow$ minima occur when

$$m\pi = \frac{\pi a}{\lambda}\sin\theta$$

$\Rightarrow$ when $m\lambda = a\sin\theta$

which is exactly what we found the last time.

(show slides)

As slit width increases, central diffraction maximum decreases

Example: Consider a single slit experiment. $\lambda = 580\,nm$

Screen is 2m away from slit.

Slit width is 300 μm.

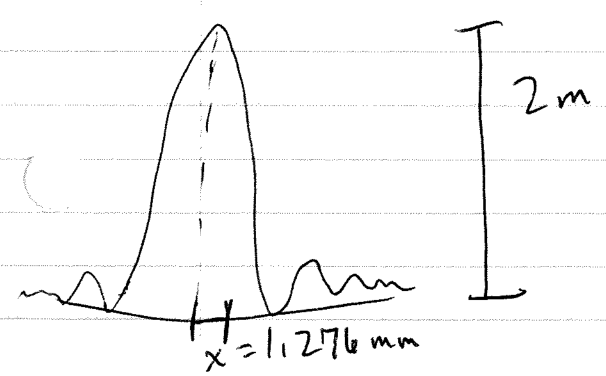
Relative to intensity I_m @ center $(x=0)$, compute intensity on screen @ $x = 1.276\,mm$

$$\frac{I}{I_m} = \left(\frac{\sin\left(\frac{\pi}{\lambda} a \sin\theta\right)}{\frac{\pi}{\lambda} a \sin\theta} \right)^2$$

figure out θ

$$\Rightarrow \tan\theta = \frac{1.276\,mm}{2\,m}$$

$$\Rightarrow \theta = 6.30 \times 10^{-4}$$



$$\Rightarrow \frac{I}{I_m} = 0.689$$

Where is this point w.r.t. dark fringes?

Dark fringes are @ $a \sin\theta = m\lambda$

1st dark fringe @

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$$

$$= 0.0019333$$

$$\Rightarrow \quad D \tan\theta = 2m \cdot \tan(0.0019333)$$

$$= 3.867 \text{ mm}$$

Diffraction by a double slit

In previous analysis, we assumed that $a \ll \lambda$

(a ↑ slit width; λ ↑ wavelength)

- In practice, this condition is often not met.
- Then intensities of fringes produced by double-slit interference are modified by diffraction of light.

(show slides)

Intensity for double-slit experiment is

$$I(\theta) = I_m \left(\cos^2(\beta(\theta))\right)\left(\frac{\sin\alpha(\theta)}{\alpha(\theta)}\right)^2$$

$$\beta(\theta) = \frac{\pi d}{\lambda}\sin\theta$$

$$\alpha(\theta) = \frac{\pi a}{\lambda}\sin\theta$$

d - distance between centers of slits

Main new contribution is
interference factor $\cos^2\beta(\theta)$
due to interference of
two slits

Diffraction by a circular aperture

- light passes through a circular opening (like circular lens) + then diffraction occurs.
- Image formed by laser is not a point, but a circular disk surrounded by secondary rings. (show slides)
- Involved analysis shows that 1st minimum for diffraction pattern occurs @

$$\sin\theta = 1.22 \frac{\lambda}{d}$$

where d is diameter of circular aperture

+ angle θ is angle from central axis to any point on circular minimum.

can compare this w/ previous expression for slit: $\sin\theta = \frac{\lambda}{a}$

- main difference is 1.22 factor, which is due to circular aperture.

Resolvability

- Diffraction places limitations on <u>resolving</u> point sources in an image.

(show slides)

<u>Rayleigh's condition for resolvability</u>:

angular separation of 2 point sources is such that central maximum of diffr. pattern of one source is centred on 1st minimum of diffr. pattern of the other.

$$\Rightarrow \quad \theta_R = \sin^{-1}\left(\frac{1.22\lambda}{d}\right)$$

where θ_R is angular separation of point sources.

Since angles are small, can use $\sin\theta_R \approx \theta_R$ to get

$$\theta_R = 1.22\frac{\lambda}{d} \quad \text{(Rayleigh's criterion)}$$

Q: Sps. you can barely resolve two red dots b/c of diffraction by your eye's pupil.

If we increase illumination around you so that your pupil decreases in diameter, does resolvability of dots improve or diminish? diminish