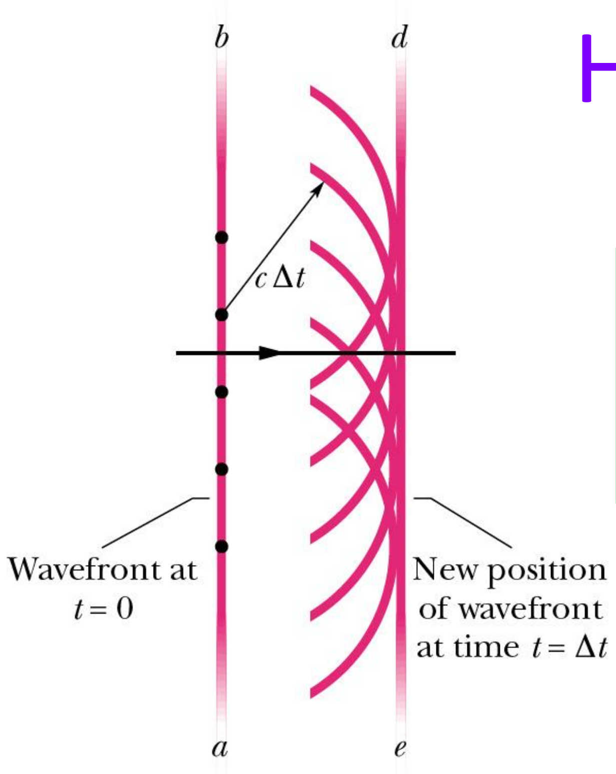
Announcements

- Dr. Marley filling in for Dr. Wilde
- Next homework due Sunday (as always...)
- Today: Continue Ch. 35
- Reminder that Exam #3 is November 19th
  (13 days away!!!)

Chapter 35: Interference



Huygen's Principle:

Light *is* a wave

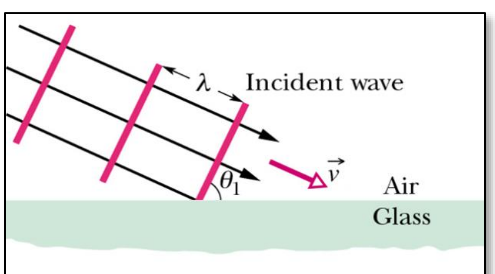
All points in a wavefront serve as point sources of spherical secondary wavelets.

Christian Huygens
1629-1695

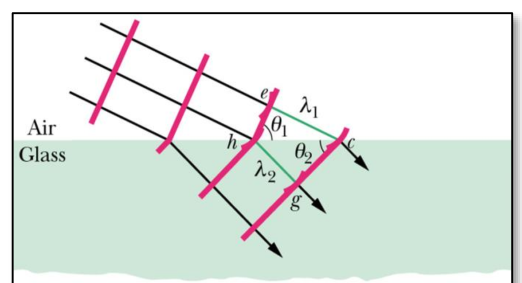
Method:

- Assign points along a wavefront
- Let the spherical waves expand for a time
- The new wavefront is the tangent line

Reflection



Refraction



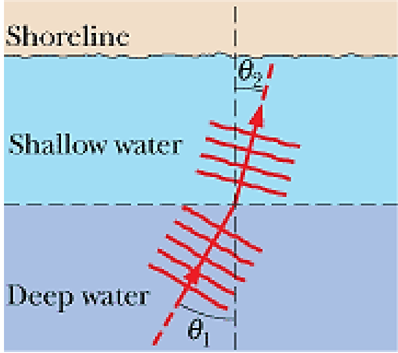
Snell's Law!!!

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$\lambda_n = \lambda_0 \frac{v_n}{c} = \frac{\lambda_0}{n}$$

$$f_n = \frac{v_n}{\lambda_n} = \frac{c/n}{\lambda_0/n} = \frac{c}{\lambda_0} = f_0$$



Ocean waves moving at a speed of 4.0 m/s are approaching the beach at an angle of 30° to the normal. The depth changes abruptly near the shore and the speed drops to 3.0 m/s.

Near the beach what is the wave propagation direction?

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

$$\sin\theta_2 = \frac{v_2}{v_1}\sin\theta_1 = \left(\frac{3.0\,\frac{\text{m}}{\text{s}}}{4.0\,\frac{\text{m}}{\text{s}}}\right)\sin 30^\circ = \frac{3}{8}$$

$$\theta_2 = \sin^{-1}\left(\frac{3}{8}\right) = 22^\circ$$

Phase Differences

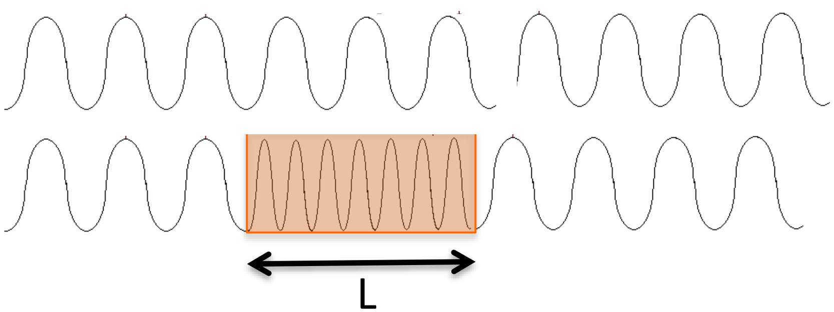
Differences in path lengths
→ interference phase differences

$$\Delta L = m\lambda \quad m = 0, 1, 2\ldots \quad \text{(CI)}$$

$$\Delta L = \left(m + \frac{1}{2}\right)\lambda \quad m = 0, 1, 2\ldots \quad \text{(DI)}$$

Another way to think of this??
→ by traveling different path lengths, waves go through a different number of wavelengths

Is there another way to travel a different number of wavelengths???



$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Number of wavelengths traveled in media *n*: $N_n = \frac{L}{\lambda_n} = \frac{L}{\left(\lambda / n_n\right)}$

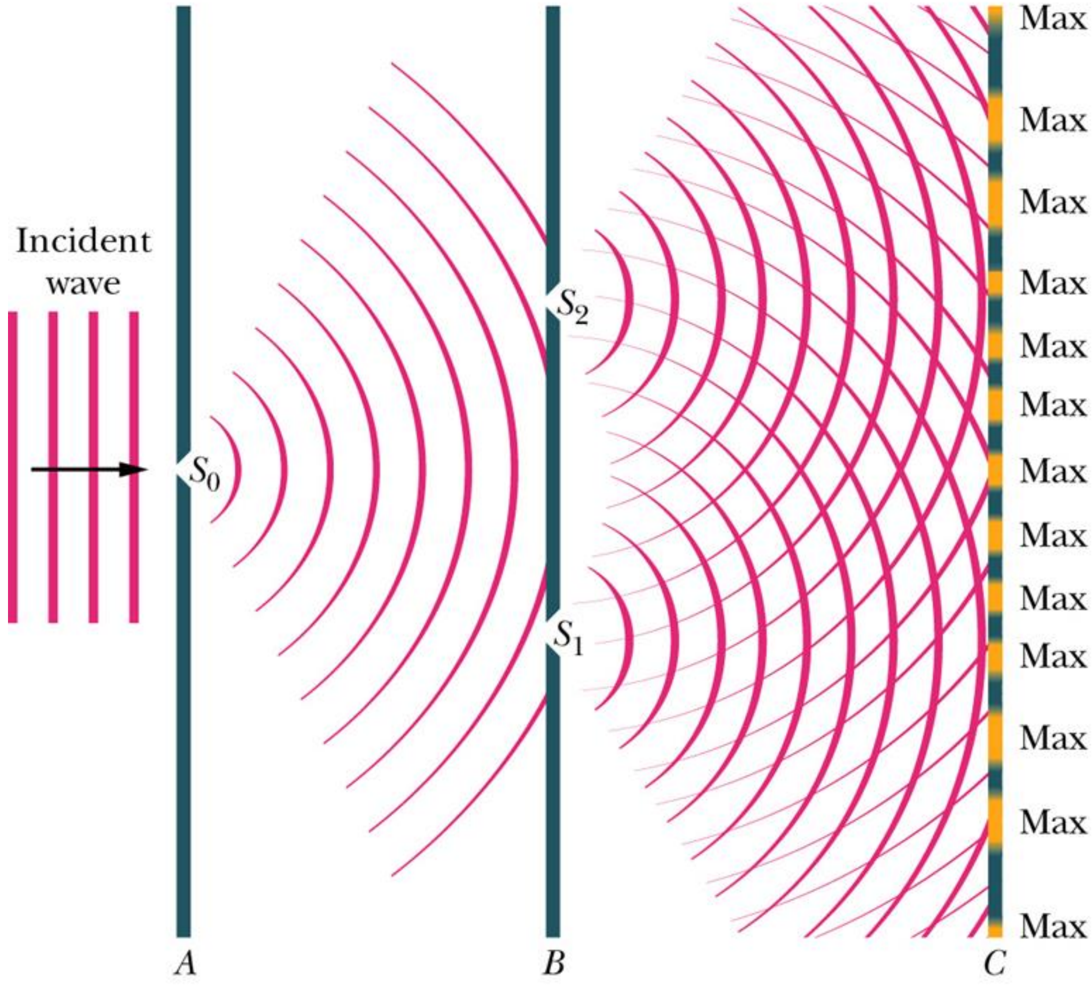
Difference in # of wavelengths over L for to initially in phase waves:

$$N_1 - N_2 = \frac{L}{\lambda}(n_1 - n_2)$$

Interference occurs both due to optical and geometric path differences!

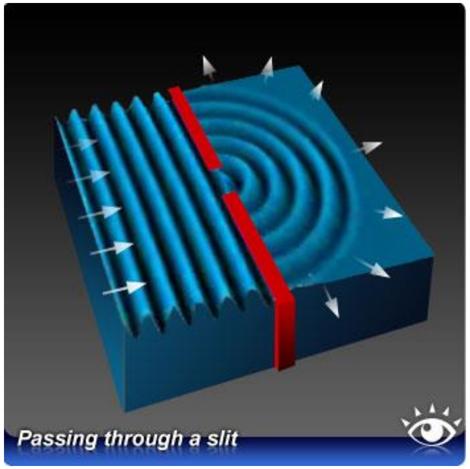
Young's Double Slit Experiment

Waves Emanating From an Aperture



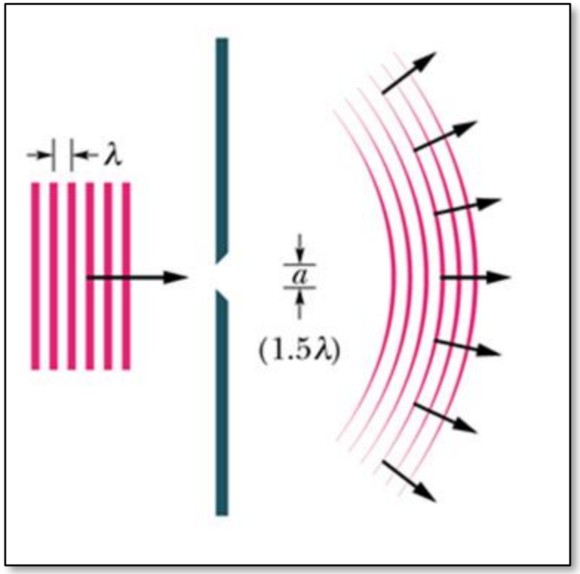
Diffraction

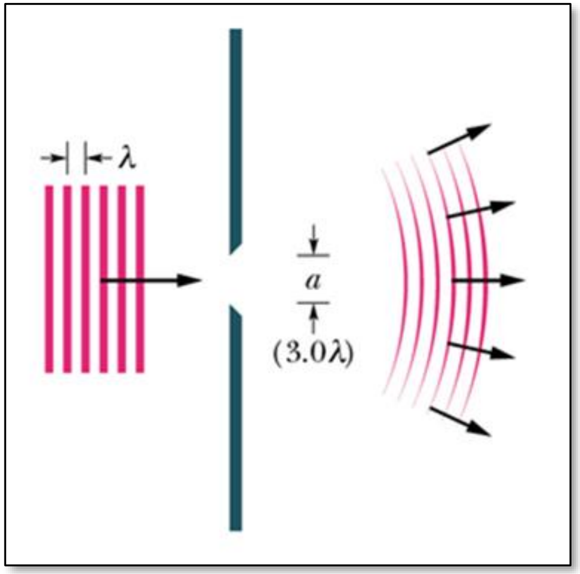
Waves Emanating From an Aperture

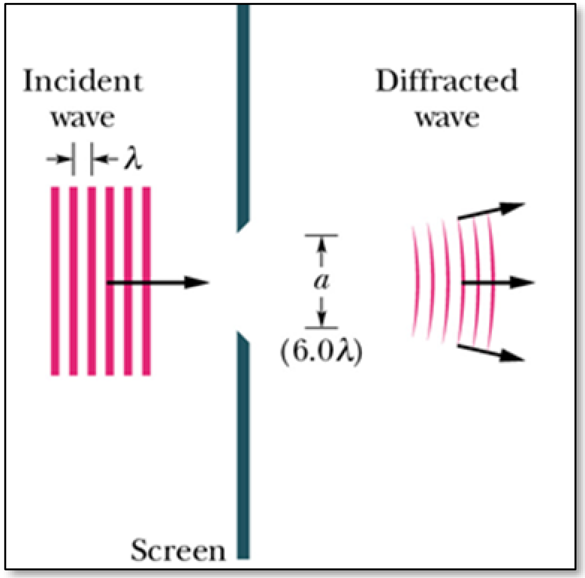


Use Huygen's Principle:

- If the opening has zero width then the outgoing wave is spherical.
- The wider the aperture is, the more planar the wave is.







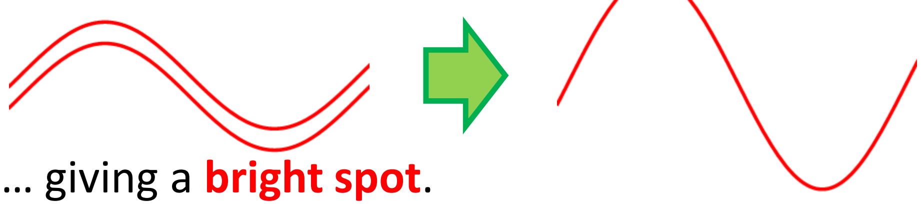
Young’s Double Split Experiment

Interference Fringes

The waves arriving at the screen will interfere constructively or destructively. The type of interference depends on the different distances they have traveled.

Constructive interference fringes:

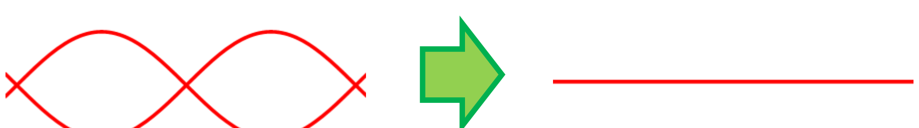
Waves arrive *in phase*…



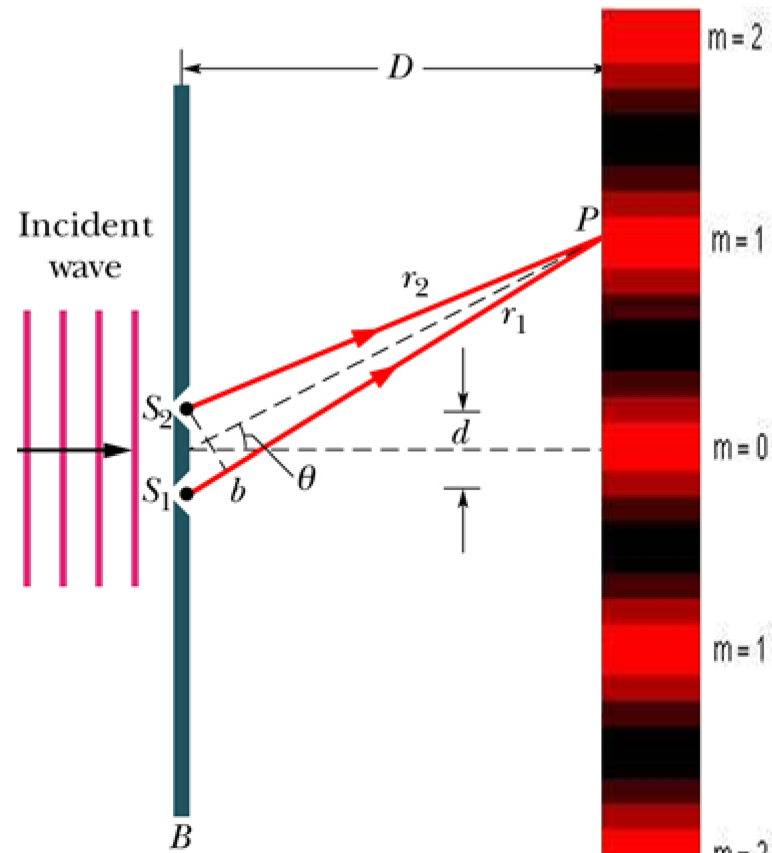
… giving a **bright spot**.

Destructive interference fringes:

Waves arrive *out of phase*…



… giving a **dark spot**.

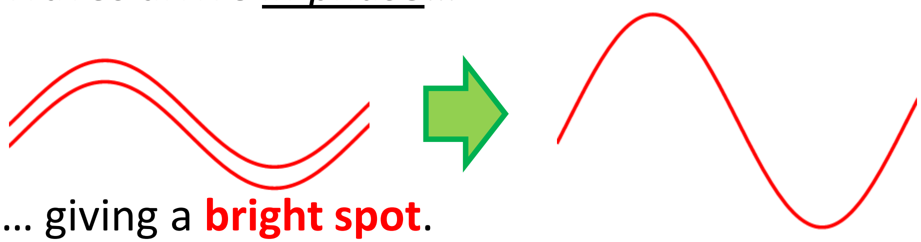


Young's Double Split Experiment

Interference Fringes

Constructive interference fringes:

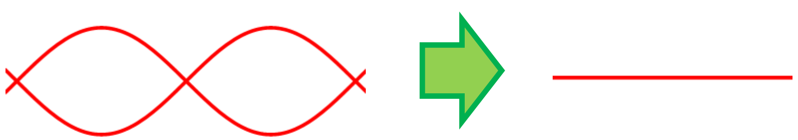
Waves arrive *in phase*...



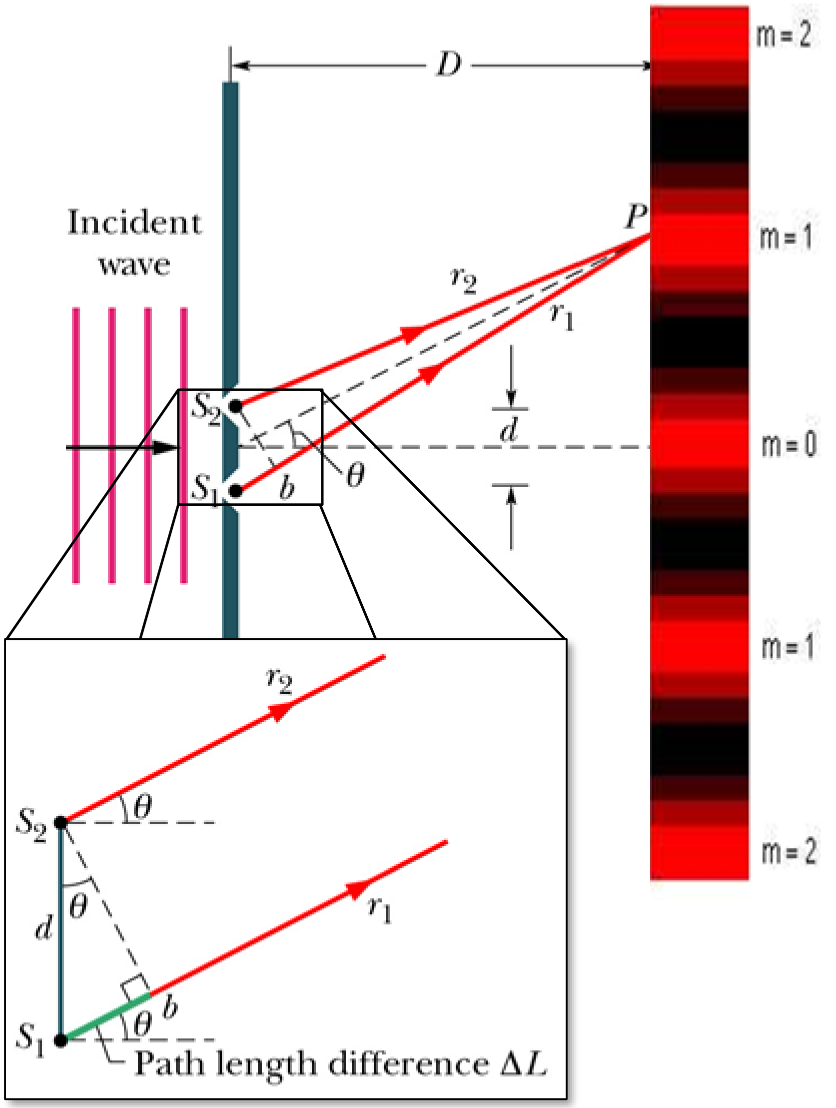
... giving a **bright spot**.

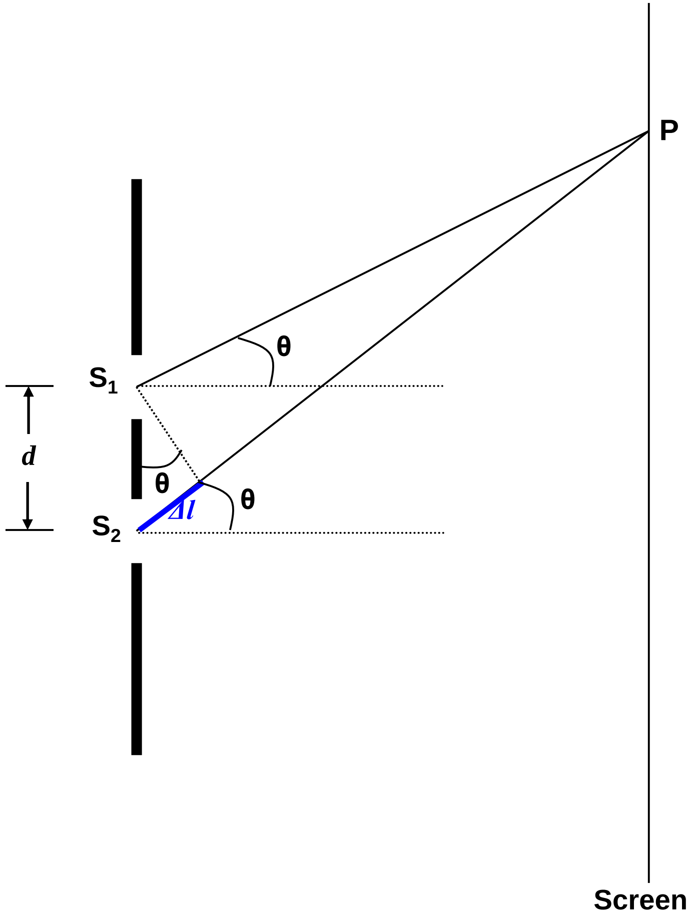
Destructive interference fringes:

Waves arrive *out of phase*...



... giving a **dark spot**.





Assume the Fraunhofer approximation:

- screen is far away from the slits
- the slits are very small

Thus, since the slits are very close together, θ is the same for each ray.

$$\sin\theta = \frac{\Delta l}{d} \quad \Rightarrow \Delta l = d\sin\theta$$

We know that for constructive interference:

$$\Delta l = m\lambda$$

Thus,

$$\boxed{d\sin\theta = m\lambda}$$

for <u>constructive interference.</u>

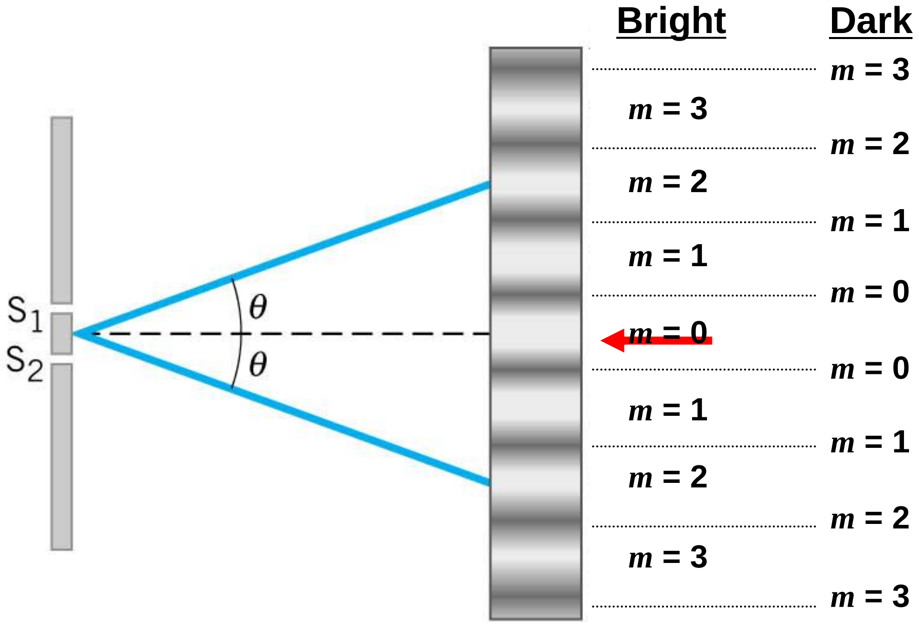
and...

$$\boxed{d\sin\theta = (m + \tfrac{1}{2})\lambda}$$

for <u>destructive interference.</u>

<u>These are the interference conditions for the double slit.</u>

Double Slit Fringe Pattern



Notice:

- alternating light and dark fringes
- the central fringe at θ = 0° is a bright fringe.
- the central fringe at θ = 0° is the brightest of the bright fringes.

The order of the bright fringes starts at the central bright fringe.

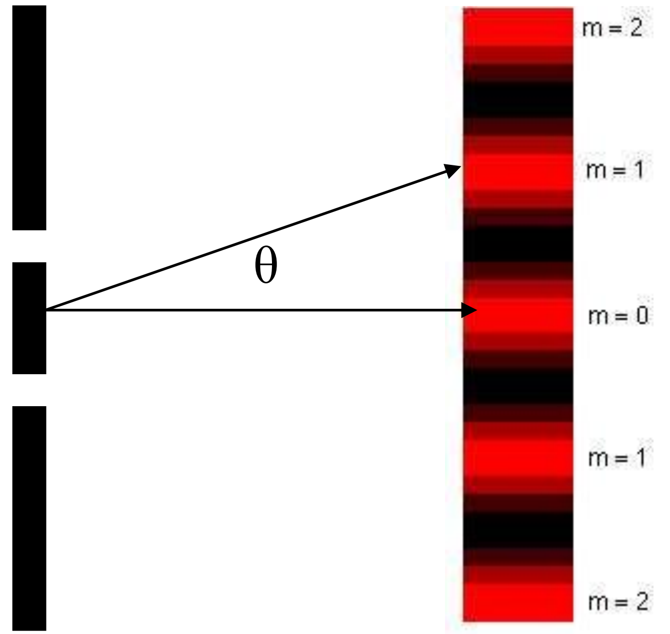
The order of the dark fringes starts right above and below the central bright fringe.

So, the first dark fringe on either side of the central bright fringe is the 1st order dark fringe, or *m* = 0, 2nd order dark fringes at *m* = 1

Remember, *m* is the order of bright fringes, *m*+1 is the order for dark fringes

In a particular double slit experiment, the separation of the two slits is 0.5 mm. The first-order interference maximum is at an inclination angle of 0.059 degrees with respect to the center of the double slit apparatus.

-What is the wavelength and color of the light used?



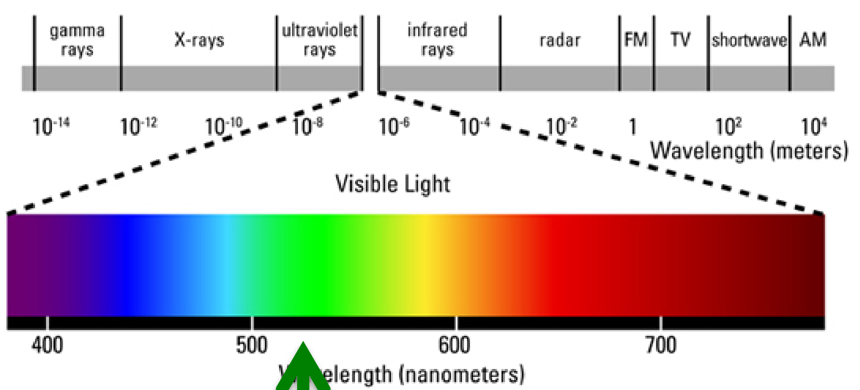
$$d \sin q = m l$$

$$l = \frac{d \sin q}{m}$$

1st order maximum → $m = 1$

$$l = \frac{0.5 \text{ mm}}{1} \sin(0.059) = 0.000514 \text{ mm}$$

$$= 514 \text{ nm}$$



GREEN

Questions

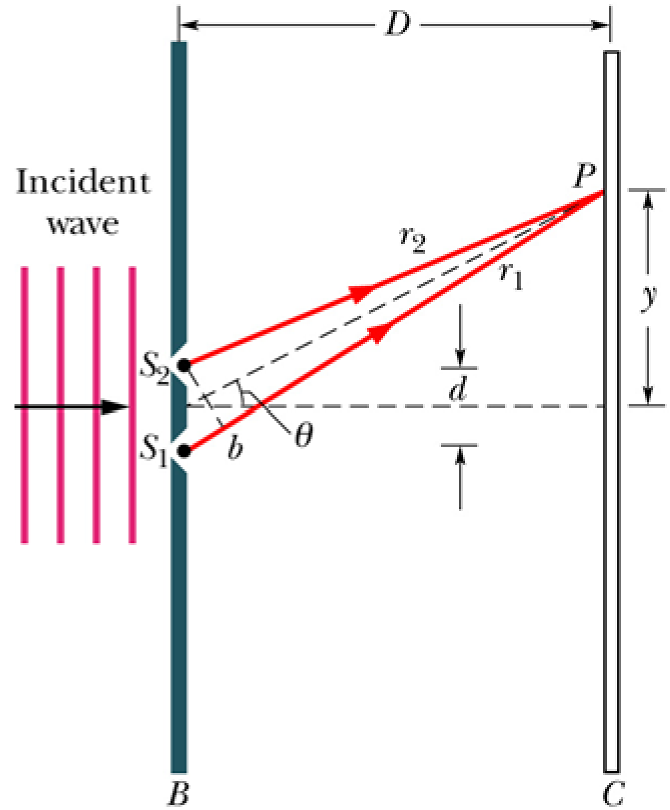
Discussion Time

Does the spacing between fringes in a two-slit interference pattern decrease, increase, or stay the same if:

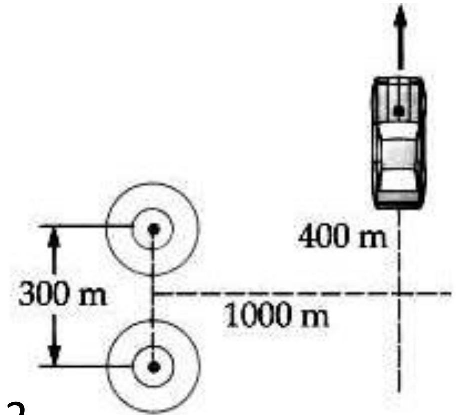
Hint: Maxima

$$d \sin q = m l$$

1. the slit separation is increased?
   - A. Increased
   - B. Decreased
2. the color of light is switched from red to blue (hint $\lambda_{red} > \lambda_{blue}$)?
   - A. Increased
   - B. Decreased
3. the whole apparatus is submerged in Grey Goose Vodka?
   - A. Increased
   - B. Decreased



Two radio antennas, separated by 300 m, as shown, broadcast identical signals at the same wavelength in phase with each other.
A car travelling due north receives the signals and at some time is at the second interference maximum.



(a) What is the wavelength of the signals? $d \sin q = m l$

In this case *d* = 300 m (the distance between the antennas) and *m* = 2 because we are at the 2nd order maxima and *m* is for order

We can find θ with geometry: $\tan\theta = \dfrac{400\text{ m}}{1000\text{ m}} \Rightarrow \theta = 21.8^\circ$

Now we can solve for wavelength:

$$\lambda = \frac{d\sin\theta}{m} = \frac{300\text{ m}\left[\sin\left(28.1^\circ\right)\right]}{2} = 55.7\text{ m}$$

(b) Where will the car find the next dead spot?

The next deadspot will be the third order minimum (*m* = 2), but will be described by constructive interference:

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \Rightarrow \theta = \sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)\lambda}{d}\right] = \sin^{-1}\left[\frac{\left(\frac{5}{2}\right)55.7\text{ m}}{300\text{ m}}\right] = 27.7^\circ$$

So the distance the car is now is:

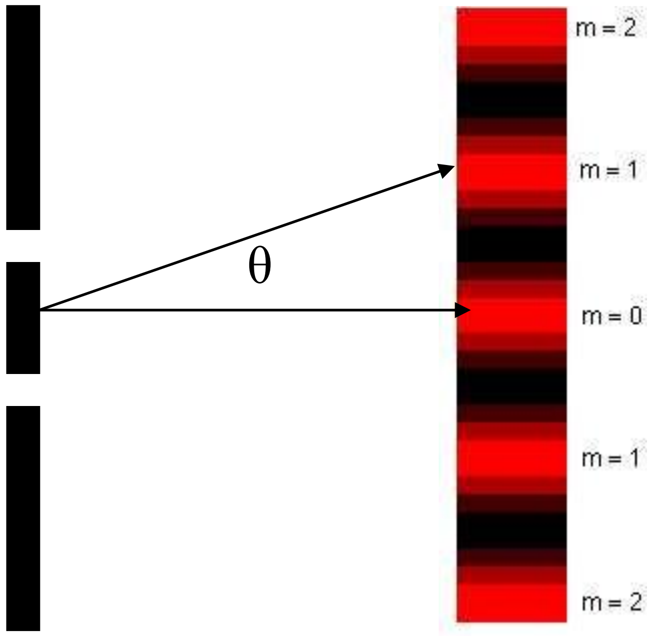
$$\tan\left(27.7^\circ\right) = \frac{y}{1000\text{ m}} \Rightarrow y = 524\text{ m}$$

Intensity

Interference fundamentally comes from two waves interfering at screen:

$$E_1 = E_0 \sin \omega t$$

$$E_2 = E_0 \sin(\omega t + \phi)$$



The intensity of fringes is given by:

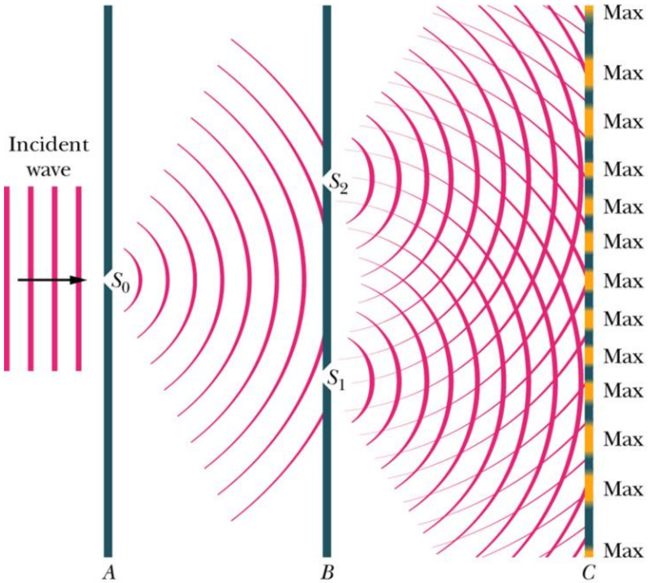
$$I = 4I_0 \cos^2\left(\frac{1}{2}\phi\right)$$

$$\phi = \frac{2\pi d}{\lambda} \sin \theta$$

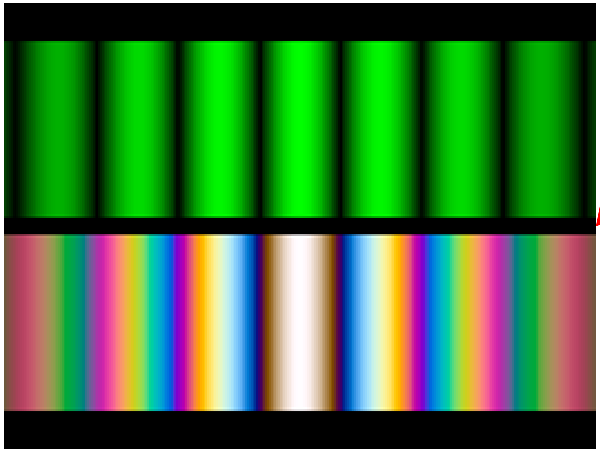
Phase shift between two waves

Young’s Double Split Experiment

Sunlight, Candle light, or What?

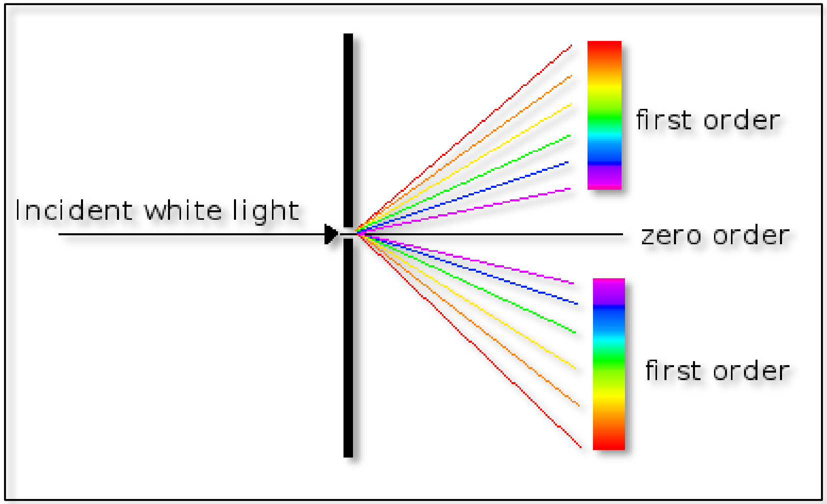


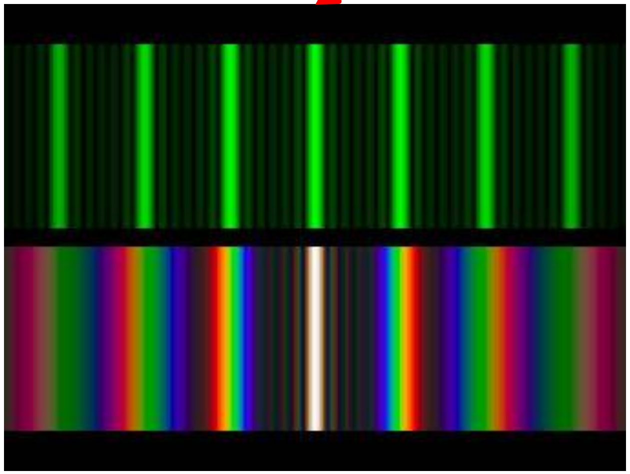
What does the pattern look like for a double slit apparatus that is illuminated with white light?



Double-slit Apparatus

Diffraction Grating





Thin Film Interference

Multicolored Thin Films

Under natural conditions, thin films, like gasoline on water or like the soap bubble in the figure, have a multicolored appearance that often changes while you are watching them. Why are such films multicolored and why do they change with time?

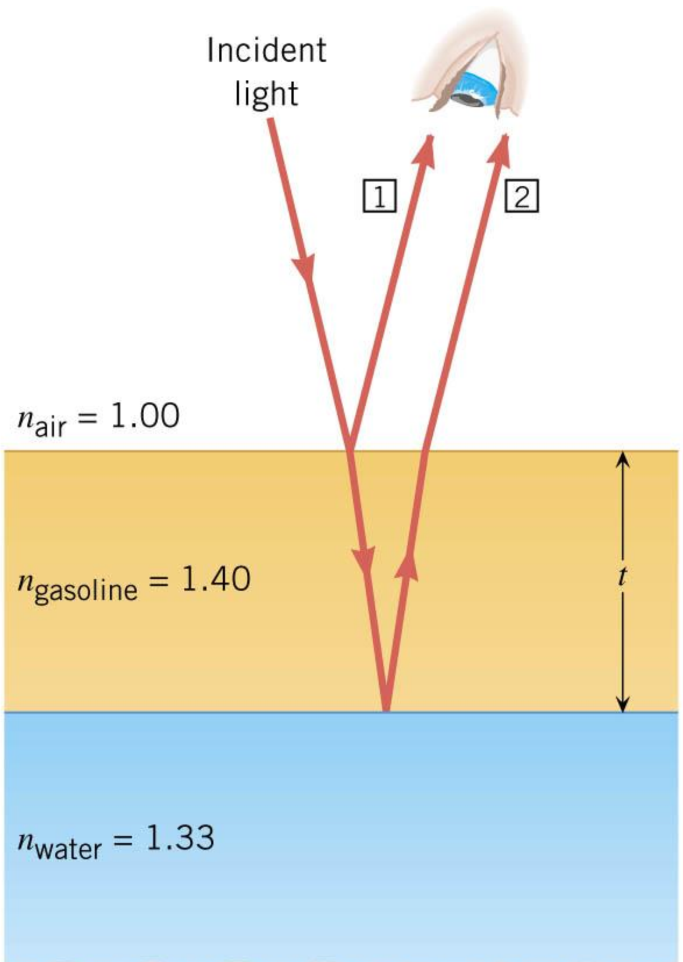
Three ways we can change phase:

1. Difference in Path length
2. Difference in *n* of media
3. Reflection

Thin Film Interference

Light waves can interfere in many situations. All we need is a difference in optical path length.

As an example, let's consider a thin film of oil or gasoline floating on the surface of water:



Part of a light ray gets reflected (1) from the surface of the film, and part gets refracted (2).

Then the refracted ray reflects back off the film/water interface and heads back into the air toward our eye.

Thus, two rays reach our eyes, and ray 2 has traveled farther than ray 1. Thus, there is a difference in the optical path length.

If the film is thin, and the ray strikes almost perpendicularly to the film, then the OPD is just twice the film thickness, or $\Delta\lambda = 2t.$

Thus, if $2t = m\lambda$, we have CI and the film appears bright (but wait….)

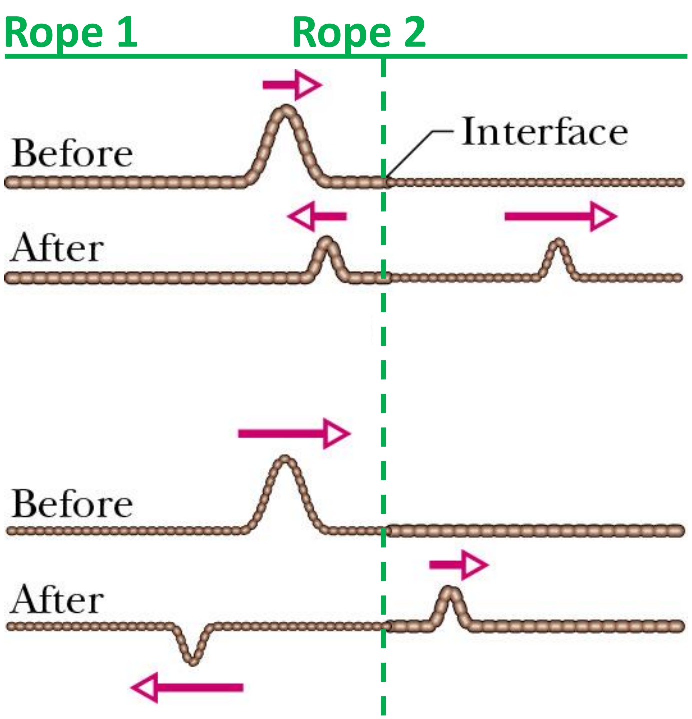
If $2t = (m + \frac{1}{2})\lambda$, we have DI and the film appears dark (but wait….)

Change of Phase

Reflection and Refraction in Rope

Consider a transverse pulse moving in rope. The pulse reaches a juncture (interface) with another rope with different mass density [remember: v_{wave} μ $1/\sqrt{r}$]

After the interface, there is a ***reflected*** and a ***transmitted*** pulse



The reflected pulse is on the **same side** as the incident pulse if the speed of propagation is **faster** in Rope 2.

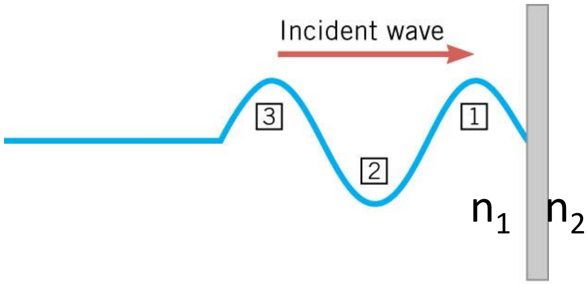
The reflected pulse is on the **opposite** side of the incident pulse if the propagation speed is **slower** in Rope 2.

Change of Phase: Reflection

The same thing that happens to our rope, happens to any wave
→ what is the analogue to mass density for light waves??

mechanical waves : mass density :: light waves : index of refraction

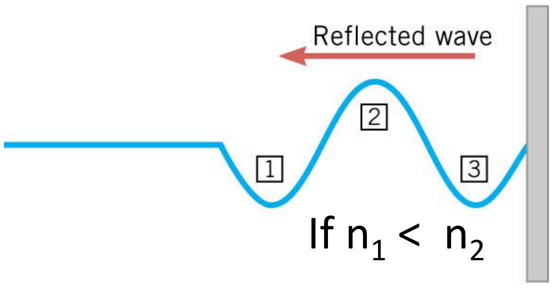
Therefore, when waves reflect from a boundary, it is possible for them to change their phase:



(a)

1. Light rays will get phase shifted by ½ λ upon reflection when they are traveling from a smaller index of refraction to a larger index of refraction.

Smaller n → larger n → Phase shift (½ λ)!



(b)

2. Light rays will experience no phase shift upon reflection when they are traveling from a larger index of refraction to a smaller index of refraction.

Larger n → smaller n → No phase shift!

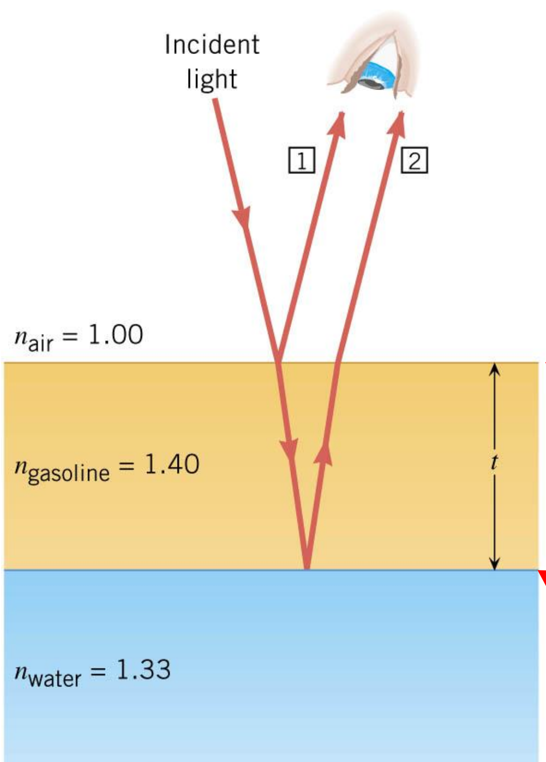
So a phase shift can occur upon *reflection*...it must be accounted for when considering thin film interference!

For thin films then the following is used:

$$Dl + (\text{any phase shifts}) = \text{Interference condition}$$

$$2t + (\text{phase shifts}) = \begin{cases} m/\,, & \textbf{Film appears bright} \\ (m+\tfrac{1}{2})/\,, & \textbf{Film appears dark} \end{cases}$$

Wavelength changes based on *n*
→ what wavelength do we choose?



Smaller *n* → larger *n* → Phase shift (½ λ)!

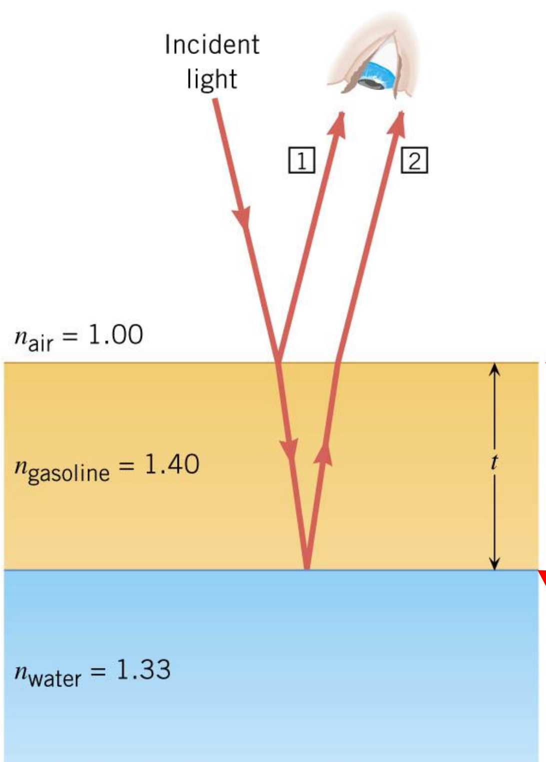
Larger *n* → smaller *n* → No phase shift!

The optical path difference occurs inside the film, so the index of refraction that is important here is n_{film}.

What is the wavelength of the light in the film (λ_{film})?

$$n_{film} = \frac{c}{v_{film}} = \frac{c}{f} \times \frac{f}{v_{film}} = \frac{\lambda_{vac}}{\lambda_{film}} \quad \Rightarrow \quad \boxed{\lambda_{film} = \frac{\lambda_{vac}}{n_{film}}}$$

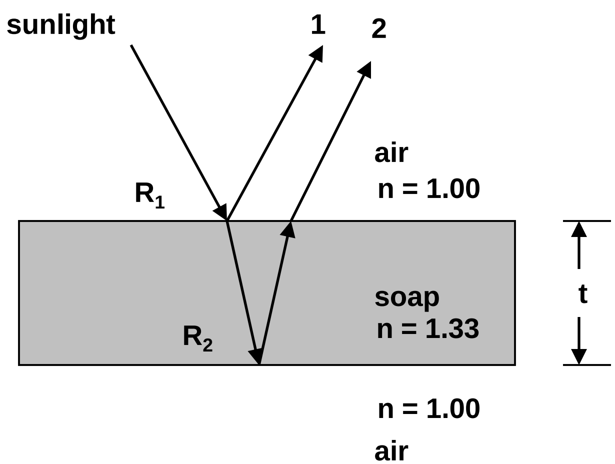
$$2t + (\text{phase shifts}) = \begin{cases} m\lambda_{film}, & \textbf{Film appears bright} \\ (m+\frac{1}{2})\lambda_{film}, & \textbf{Film appears dark} \end{cases}$$



Smaller *n* → larger *n* → Phase shift (½ λ)!

Larger *n* → smaller *n* → No phase shift!

Example A soap film (n = 1.33) is 375 nm thick and is surrounded on both sides by air. Sunlight, whose wavelengths (in vacuum) extend from 380 nm to 750 nm strikes the film nearly perpendicularly. For which wavelength(s) in this range does the film look bright in reflected light?



We want the film to appear bright, which means constructive interference:

$$\Delta\lambda + (\text{phase shifts}) = m\lambda$$

Do we have any phase shifts?

At R_1 we are going from a smaller n to a larger n → ½λ phase shift.

At R_2 we are going from a larger n to a smaller n → No phase shift.

So now we have 1 phase shift: $2t + \frac{1}{2}\lambda = m\lambda \Rightarrow 2t = (m + \frac{1}{2})\lambda$

This λ is the λ_{film}: $\lambda_{film} = \dfrac{2t}{m + \frac{1}{2}}$ But $\lambda_{vac} = n_{film}\lambda_{film}$

$$\Rightarrow \boxed{\lambda_{vac} = \dfrac{n_{film}2t}{m + \frac{1}{2}}}$$

$$\Rightarrow \lambda_{vac} = \frac{(2)(375\ \text{nm})(1.33)}{m+\frac{1}{2}} = \frac{997.5\ \text{nm}}{m+\frac{1}{2}}$$

m	λ_{vac}
0	1995 nm
1	665 nm ←
2	399 nm ←
3	285 nm

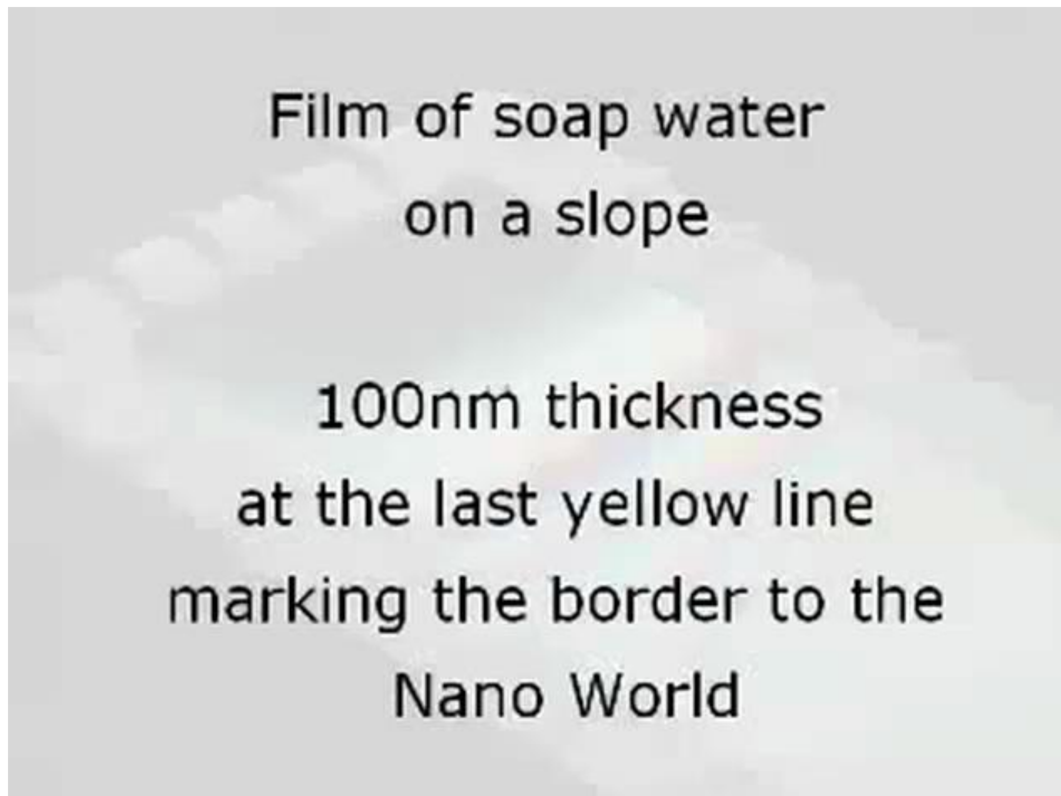
These two are in the visible spectrum (red and violet).

Thus, the film appears redish/violet.

WHAT HAPPENS IF THE THICKNESS CHANGES???

Film of soap on water on a slope

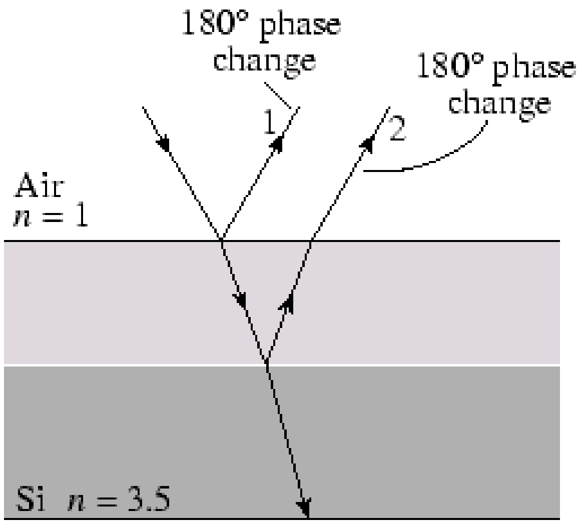
What's happening?



Example: Solar Panels

Thin Films in Real Life

Semiconductors such as silicon ($n_{Si} = 3.5$) are used to build solar cells. They are coated with a transparent thin film ($n_f = 1.45$) in order to minimize reflection and maximize absorption. What is the minimum coating thickness that will minimize reflection of light with λ = 552 nm (sodium yellow)?



Both rays under a 180º phase change upon reflection. Hence, the distance traveled in the film should be equal to one half of a wavelength in the coating for destructive interference.

$$2t = \frac{\lambda}{2n} = \frac{552\ \text{nm}}{2(1.45)}$$

$$\boxed{t = \frac{552\ \text{nm}}{4(1.45)} = 95.2\ \text{nm}}$$