Lecture 19

Ch. 33 – Electromagnetic Waves

- biggest achievement of Maxwell was to prove mathematically that a beam of light is a traveling wave of electric + magnetic fields
- Heinrich Hertz proved experimentally that electromagnetic waves exist, validating the theory of Maxwell (thinking about the question on + off for a decade) 1879 - 1889
- Guglielmo Marconi was the first to use EM waves for communication.
(Italian gov't officials referred him to an insane asylum for claiming this, "to the Longara" at which point he moved to England)

EM waves include

visible light, infrared, ultraviolet,
radio waves, X-rays, gamma rays.

EM waves have a wavelength
& frequency

wavelength (m)	10^8	10^2	10^{-3}	10^{-4} 10^{-5}
	long waves	radio waves	$\boxed{\text{infrared}}$	
frequency (Hz)	10^2	10^6	10^{12}	10^{13}

(m)	10^{-6} 10^{-7}	10^{-8}	10^{-10}
	visible	Ultraviolet	Xrays
(Hz)	10^{14} 10^{15}	10^{16}	10^{18}

All EM waves travel @ same speed $c \approx 3.0 \times 10^8\ m/s$

- nothing travels faster than c
- c is independent of frame of reference (principle of relativity)

Maxwell came to his conclusions about EM waves and light by purely mathematical considerations

Consider Maxwell equations in free space w/ no charges or currents

Then

$$\oint \vec{E} \cdot dA = 0, \quad \oint \vec{B} \cdot d\vec{A} = 0,$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A},$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A},$$

changing E-field gives B-field +
changing B-field gives E-field

equivalent form of the equations
in this case are

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

these two are coupled

take curl of 1st to get

$$\nabla \times \nabla \times \vec{E} = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$= -\frac{\partial}{\partial t}\left[\nabla \times \vec{B}\right]$$

$$= -\frac{\partial}{\partial t} \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$= -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla \times \nabla \times \vec{B} = \nabla \times \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$= \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \left[\nabla \times \vec{E} \right]$$

$$= -\mu_0 \varepsilon_0 \frac{\partial}{\partial t} \left[\frac{\partial \vec{B}}{\partial t} \right]$$

$$= -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

can then use identity $\nabla \times \nabla \times \vec{B} = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B}$ to get

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

simple versions of these are

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

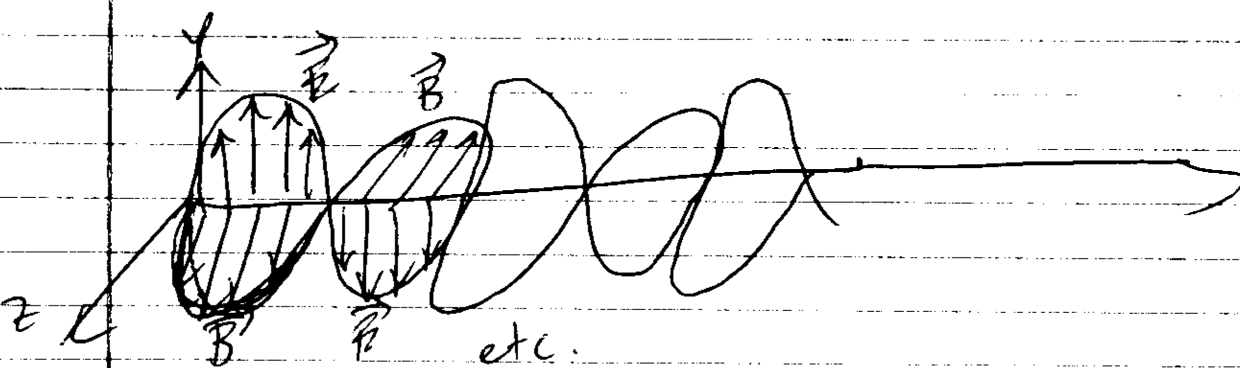
$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

solutions are of the form of propagating waves

$$E = E_m \sin(kx - \omega t)$$

$$B = B_m \sin(kx - \omega t)$$

picture of a propagating EM wave is



$$\frac{\omega}{k} = \frac{\text{time frequency}}{\text{wave number}} = \text{speed of propagation}$$

(call this c for now)

take the x derivative twice to get

$$-E_m k^2 \sin(kx - \omega t)$$

take the t derivative twice to get

$$-E_m \omega^2 \sin(kx - \omega t)$$

plug in to get

$$-E_m k^2 \sin(kx - \omega t) = \mu_0 \varepsilon_0 \left[-E_m \omega^2 \sin(kx - \omega t)\right]$$

$$\Rightarrow k^2 = \mu_0 \varepsilon_0 \omega^2$$

$$\Rightarrow \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 3.0 \times 10^8 \, m/s$$

Maxwell then suggested that since the speed of propagation matches the speed of light that light itself should be an EM wave. (he thought it was much more than just a coincidence.)

proven correct much later...

key facts about EM waves:

1. E- + B-fields are always perpendicular to the direction in which the wave is traveling. Transverse wave
2. E-field is always perpendicular to B-field
3. cross product $\vec{E} \times \vec{B}$ gives direction of travel

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4. fields vary sinusoidally &
are in phase w/ each other.

Energy Transport by EM waves
& the Poynting Vector

EM waves can transport energy from one location to another (from Sun to our skin. this is also the basis for solar energy.)

Power transported by the wave is quantified by the Poynting vector (magnitude and direction)

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

Since $\vec{E}$ & $\vec{B}$ are perpendicular the magnitude $|\vec{S}| = \frac{1}{\mu_0} EB$

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direction is always in the direction of wave propagation

For an EM wave,
we have that $\frac{E}{B} = c$

so we can write $|\vec{S}| = \frac{1}{\mu_0 c} E^2$

units of S are $\frac{\text{power}}{\text{unit area}}$

Since magnitudes of $\vec{E}$ + $\vec{B}$ change w/ time, so does $\vec{S}$.
It goes from a minimum of zero to a maximum value

So $|\vec{S}| = \frac{1}{\mu_0 c} E_m^2 \sin^2(kx - \omega t)$

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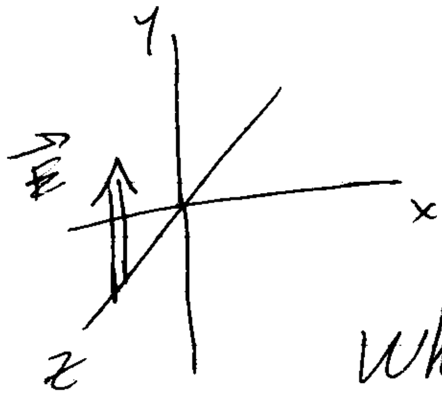
QUESTION:

Suppose EM wave is propagating in $-z$ direction & picture of it @ a given time and instant is



What is direction of magnetic field?

Since Poynting vector is changing w/ time, a better measure of energy in an EM wave is just to average it over one cycle of the wave - then we get intensity $I = \bar{S} = S_{avg} =$

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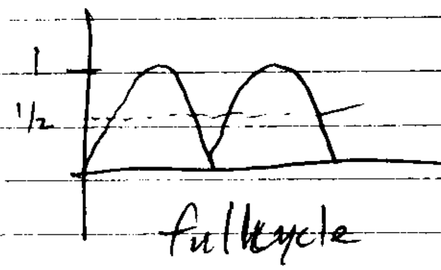
$$S_{avg} = \frac{1}{c\mu_0}\left[E_m^2 \sin^2(kx - \omega t)\right]_{avg}$$

average of $\sin^2$ over 1 cycle is just $1/2$



$$\Rightarrow \quad S_{avg} = \frac{1}{c\mu_0}\frac{E_m^2}{2}$$

energy density of E-field is (energy per unit volume) $\frac{1}{2}\varepsilon_0 E^2$

that for B-field is

$$\frac{1}{2\mu_0}B^2$$

these are the same for an EM wave

Why? $E = cB \Rightarrow \frac{1}{2}\varepsilon_0 E^2 = 1/2\,\varepsilon_0 (cB)^2$

using $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$$\Rightarrow \quad 1/2\, \varepsilon_0 (c^2 B^2) = \frac{1}{2\mu_0} B^2$$

which is B-field energy density

Example: Solar Energy

Light from the Sun ~~measured~~ on Earth has an intensity

$\approx 1\,\mathrm{kW/m^2}$

What is the total power incident on a roof of dim. 8m x 20m?

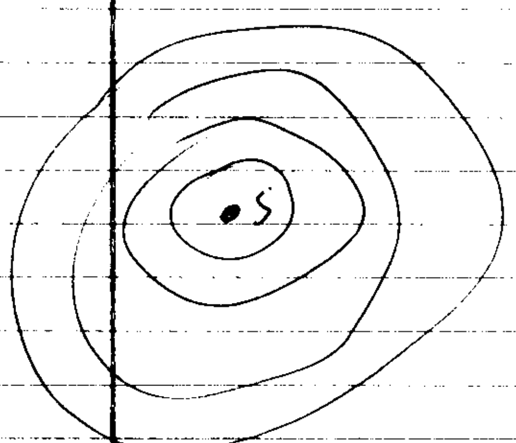
$$P = IA = (10^3\, \mathrm{W/m^2}) \cdot 8\mathrm{m} \cdot 20\mathrm{m}$$

$$= 0.16 \text{ MegaWatt}$$

solar panels @ the moment are about 10-20% efficient

Often it is the case that
energy company pays you
for generating electricity

Intensity varies w/ distance
from the source of radiation.
This can be complicated, but
for a point source, it is
easy to model.



Assume that energy
of EM waves is conserved
going outward from
the source.

power emitted goes
through spheres of larger
& larger radius.

Since intensity is power per unit area
it should be the power of the source

divided by surface area of of the spheres which is @ distance r

$$\Rightarrow \quad I = \frac{P_s}{4\pi r^2}$$

$\frac{1}{r^2}$ law for intensity

QUESTION: radio station transmits a 10kW signal @ frequency 100MHz. @ distance 1km from antenna, find electric & magnetic field strengths & energy incident on a square plate of side 10cm for 5min.

$$I = \frac{P_s}{4\pi r^2} = \frac{10\text{ kW}}{4\pi(1\text{ km})^2} = .8\frac{\text{mW}}{\text{m}^2}$$

$$I = \frac{1}{2c\mu_0}E_m^2 \Rightarrow E_m = \sqrt{2c\mu_0 I}$$

$$= .775\text{ V/m}$$

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$B_m = E_m / c = 2.58\,nT$

Received energy $I = \frac{P}{A} = \frac{\Delta U / \Delta t}{A}$

$\Rightarrow \Delta U = I A \Delta t =$

$\left(.8 \frac{mW}{m^2}\right) \cdot (10cm)^2 (300s)$

$= 2.4\,mJ$