Lecture 18

Beats – occurs when waves of close frequencies combine

– show demonstration w/ tone generator

To see how this works, let time-dependent variations of two sound waves w/ equal amplitudes be

$$s_1 = s_m \cos(\omega_1 t)$$

$$s_2 = s_m \cos(\omega_2 t)$$

use trig. identity

$$\cos\alpha + \cos\beta = 2\cos\left[\tfrac{1}{2}(\alpha-\beta)\right] \cdot \cos\left[\tfrac{1}{2}(\alpha+\beta)\right]$$

then

$$s_1 + s_2 = 2 s_m \cos\left[\tfrac{1}{2}(\omega_1 - \omega_2)t\right] \cdot \cos\left[\tfrac{1}{2}(\omega_1 + \omega_2)t\right]$$

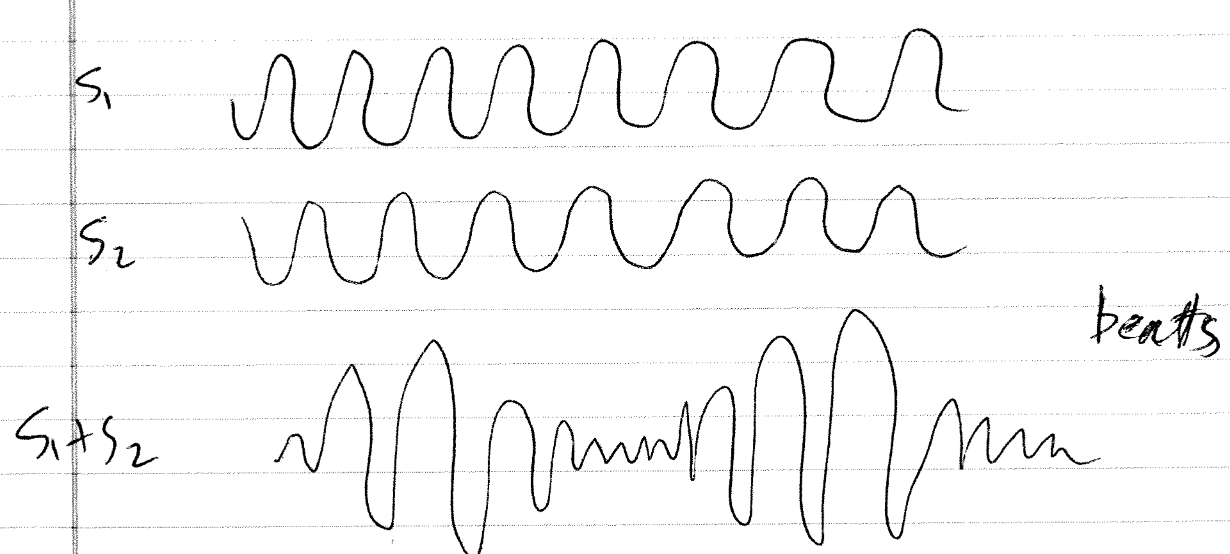
Set $\omega' = \frac{1}{2}(\omega_1 - \omega_2)$

$\omega = \frac{1}{2}(\omega_1 + \omega_2)$ (average)

then $s_1 + s_2 = \left[2 s_m \cos(\omega' t)\right] \cos(\omega t)$

If $\omega_1 \approx \omega_2$ then $\omega \gg \omega'$

Picture is



What is the frequency of the beats?

Since cosine has maximum amplitude whenever it is +1 or −1, & this occurs twice in every repetition of cosine function,

beat frequency is

$$\omega_{beat} = 2\omega' = 2\left(\tfrac{1}{2}\right)(\omega_1 - \omega_2)$$

$$= \omega_1 - \omega_2$$

in Hertz, this is

$$f_{beat} = f_1 - f_2$$

Where is this phenomenon useful?

- Musicians use it in tuning instruments.
- Change frequency / note of string until the beats go away. When this happens, the frequencies are the same (i.e., no beats).

Doppler effect

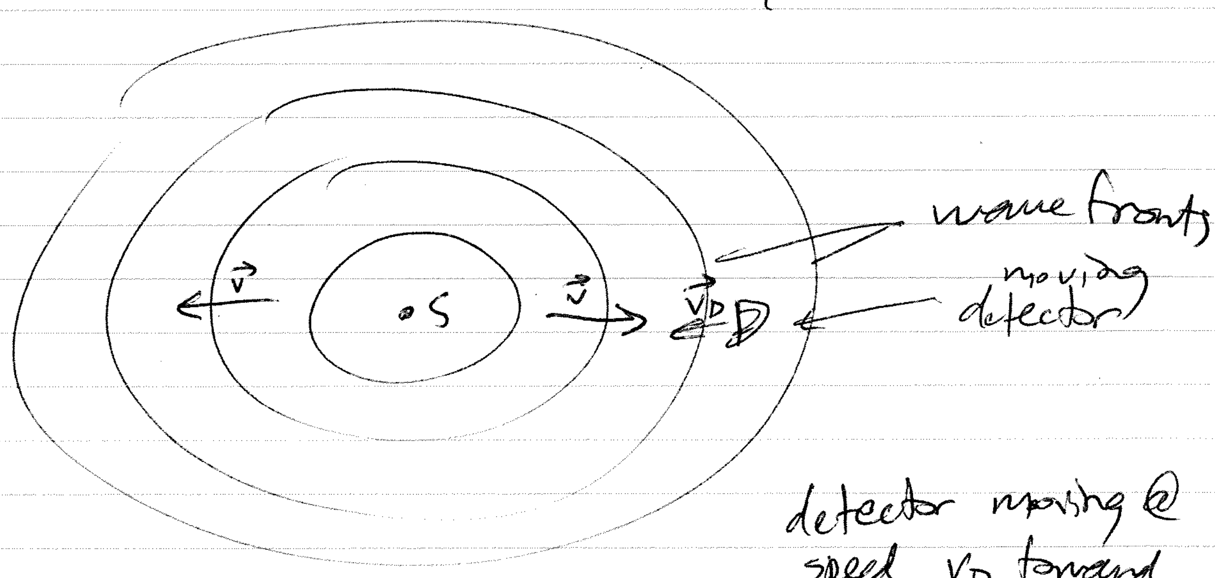
- Suppose a police car is parked ~~by~~ & sounding its 1000 Hz siren
- If you are parked, then you hear the same frequency.
- But if there is relative motion between you & police car, then you hear a different frequency

- if you are driving toward the police car @ 75 mi/hr, you will hear a higher frequency 1096 Hz

- if you are driving away, you will hear a lower frequency 904 Hz

Why does this happen?

Consider a stationary source

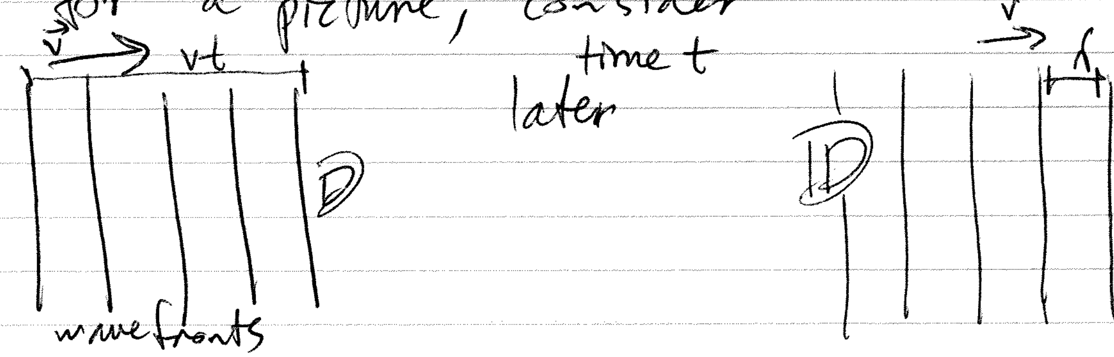


detector moving @ speed v_D toward source wave fronts have wavelength λ & frequency f, moving @ speed v of sound.

- Suppose @ 1st that detector is not moving. Then in time t, the wave fronts move to the right a distance vt.
- the number of wavelengths in the distance vt is equal to the number of wavelengths intercepted by detector. Equal to

$$vt/\lambda$$

for a picture, consider



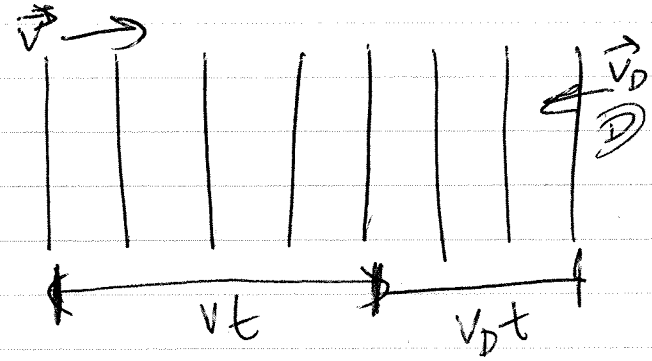
The rate @ which D intercepts wavelengths is frequency detected by D:

$$f = \frac{vt/\lambda}{t} = \frac{v}{\lambda}$$ (arrive @ usual expression)

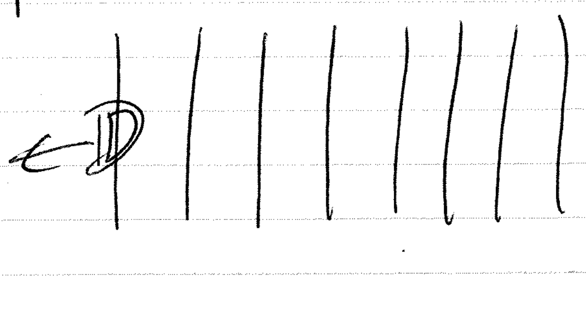
# of wavelengths intercepted by D in time t

Now suppose that the detector is moving to the left. Then it detects more wave fronts!

Picture is now



later time t is



# of wavelengths intercepted in time t is

$$(vt + v_D t)/\lambda$$

then frequency @ detector is

$$f' = \frac{(vt+v_D t)/\lambda}{t} = \frac{v+v_D}{\lambda}$$

Since $\lambda = \frac{v}{f}$ this implies that

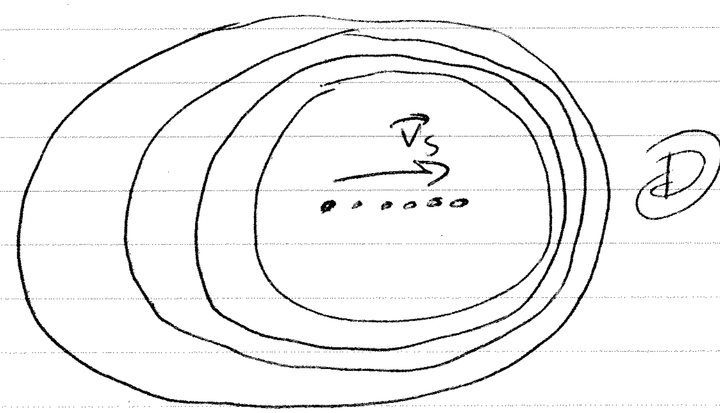
$$f' = \frac{v+v_D}{v} \cdot f$$

& frequency f' detected is now higher

If detector is moving away from source, then there is a sign flip

$$f' = \frac{v-v_D}{v} \cdot f$$ & frequency is lower

Now suppose that the source is moving, but the detector is stationary



"source is chasing its own wavefronts"
reduces wavelength of wavefronts
& higher frequency is detected

let's figure out the change in frequency

- Suppose T is the time between the emission of successive wavefronts

- During time T, wavefront W_1 moves a distance vT + source moves a distance $v_s T$

- At end of time T, next wavefront W_2 is emitted.

- Distance between W_1 + W_2 is $vT - v_s T$. This is wavelength λ' of waves moving in that direction.

- Frequency detected is

$$f' = \frac{v}{\lambda'} = \frac{v}{vT - v_s T} = \frac{v}{v/f - v_s/f} = f\frac{v}{v - v_s}$$

(speed of waves is same

If direction of moving source is flipped, then there is a sign flip:

$$f' = f \frac{v}{v + v_s}$$

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general equation for Doppler effect is

$$f' = f \frac{v \pm v_D}{v \pm v_s}$$

Key to remembering signs:

if source or detector are moving toward each other, there should be an increase in frequency.

if opposite, then there should be a decrease in frequency.

Example: Bats detect prey using echolocation.

- Suppose a bat is flying to the right @ speed $v_b = 9$ m/s after a moth flying to the right @ speed 8 m/s

- Suppose bat emits ultrasound @ freq. 82.52 kHz

What freq. does moth detect?

use $f' = f \dfrac{v - v_D}{v - v_S}$

signs: moth is moving away & corresponds to v_D, so this tends to decrease freq & then minus sign

bat is moving toward & this tends to increase freq. & so minus sign