Lecture 17

Intensity & Sound Level

- Intensity of a sound wave -

$$I = \frac{P}{A} \qquad \frac{(\text{power})}{(\text{surface area})}$$

- average rate per unit area @ which energy is transferred by the wave through or onto a surface

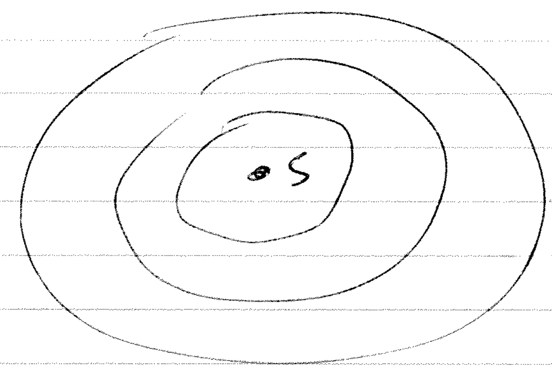
- can derive the formula

$$I = \frac{1}{2} \rho v \omega^2 s_m^2$$

density wave speed angular frequency amplitude

Variation of intensity w/ distance

Ignore echos & assume source is a point source, emitting sound waves uniformly in all directions



← concentric spheres

$$I = \frac{P_s}{4\pi r^2}$$

P_s ← power of the source

$4\pi r^2$ ← surface area of sphere of radius r

⇒ intensity @ sphere of radius r decays as inverse square law

Decibel scale

- displacement amplitude @ human ear ranges from 10^{-5} m (loudest tolerable sound) to 10^{-11} m for faintest sound
- $\Rightarrow$ ratio of intensities is 10^{12}

- deal w/ such a large range of values via logarithms

- more convenient to consider <u>sound level</u> rather than intensity

sound level $\beta = (10\,dB)\log\frac{I}{I_0}$

where I_0 is a reference intensity:

$I_0 = 10^{-12}\ W/m^2$

Simple derivation for

$$I = \frac{1}{2} \rho v \omega^2 s_m^2$$

In a tube, consider thin slice of air of thickness dx, area A, mass dm

Kinetic energy of thin slice is

$$dK = \frac{1}{2} dm \, v_s^2$$

where v_s is speed of oscillating element of air (not wave speed)

$$v_s = \frac{\partial s(x,t)}{\partial t} = -\omega s_m \sin(kx - \omega t)$$

$$dm = \rho A dx$$

$$\Rightarrow \quad dK = \frac{1}{2} (\rho A dx) (-\omega s_m)^2 \sin^2(kx - \omega t)$$

Now divide by dt:

$$\frac{dK}{dt} = \frac{1}{2}\rho A \underbrace{\frac{dx}{dt}}_{v} \omega^2 s_m^2 \sin^2(kx - \omega t)$$

Now take the time average & use the fact that average of $\sin^2$ or $\cos^2$ over one oscillation is $1/2$

$$\Rightarrow \left\langle \frac{dK}{dt} \right\rangle_{avg} = \frac{1}{4}\rho A v \omega^2 s_m^2$$

average potential energy is the same

$$\Rightarrow I = \frac{2 \left\langle \frac{dK}{dt} \right\rangle_{avg}}{A} = \frac{1}{2}\rho v \omega^2 s_m^2$$

Problem: If an earplug decreases sound level by 20dB, what is the ratio of final to initial intensity?

$$\beta_f = 10\,dB \log \frac{I_f}{I_0}$$

$$\beta_i = 10\,dB \log \frac{I_i}{I_0}$$

$$\Rightarrow \quad \beta_f - \beta_i = (10\,dB) \log \left(\frac{I_f}{I_0}\right)$$

$$-20\,dB = 10\,dB \log \frac{I_f}{I_0}$$

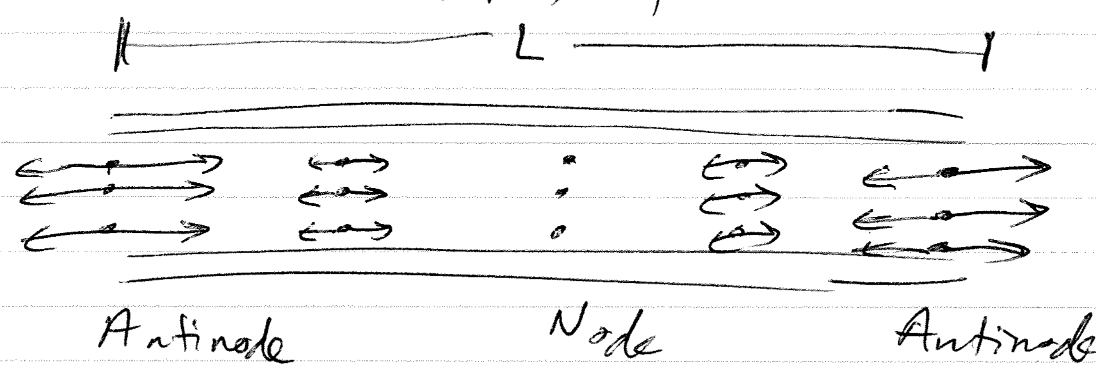
$$\Rightarrow \quad -2 = \log \frac{I_f}{I_0}$$

$$\Rightarrow \quad \boxed{10^{-2} = \frac{I_f}{I_0}}$$

Sources of musical sound

- sources include oscillating strings, membranes (drums), air columns (flute), wooden blocks, etc.
- recall standing waves for strings, oscillating @ resonant frequency
  - then causes surrounding air to oscillate w/ same frequency
- can set up standing waves of sound in an air-filled pipe
- as sound travels through air in the pipe, the waves are reflected @ each end & travel back (even if pipe is open ended)

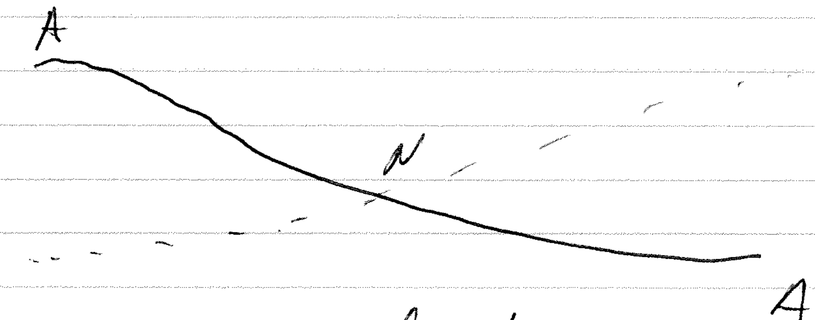
for a pipe, picture is



Arrows depict displacement of air column - max @ antinodes
none @ nodes

always have antinodes @ end for open pipes

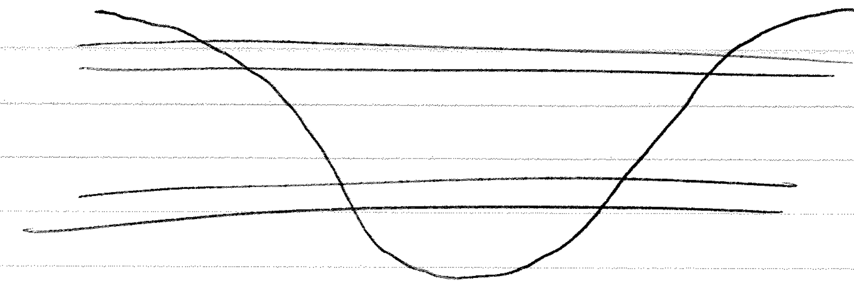
for a string, would look like



easier way of depicting wave

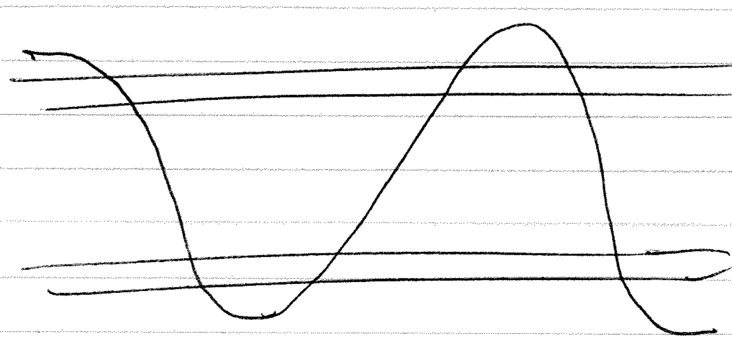
this is the first (fundamental) harmonic - $\lambda = 2L$

then other harmonics are



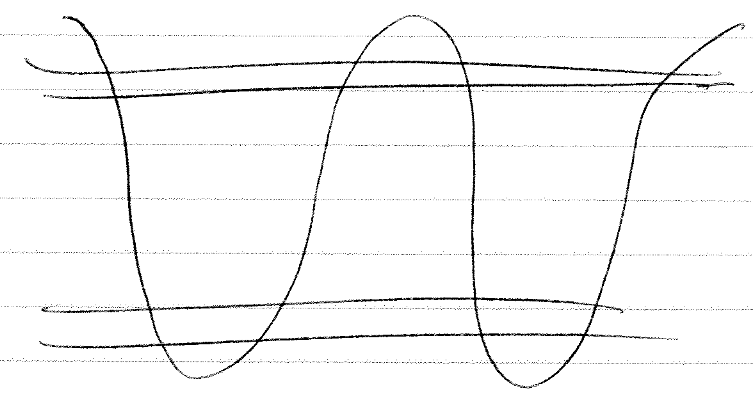
$\lambda = L$

$n = 2$



$\lambda = \frac{2L}{3}$

$n = 3$



$\lambda = \frac{2L}{4} = L/2$

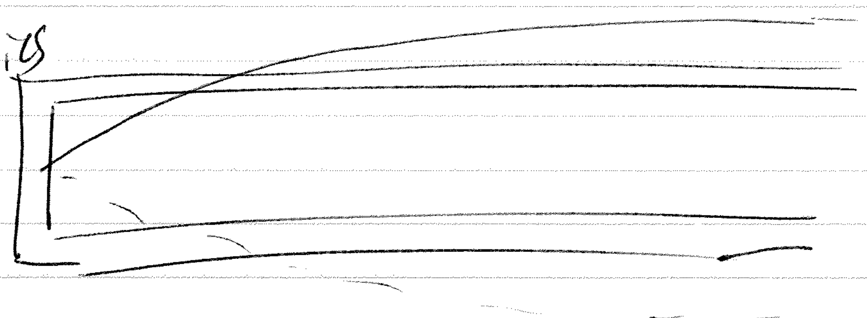
$n = 4$

wavelengths for various harmonics are $\lambda = \frac{2L}{n}$ for $n = 1, 2, 3, \ldots$

resonant frequencies are $f = \frac{v}{\lambda} = \frac{nv}{2L}$

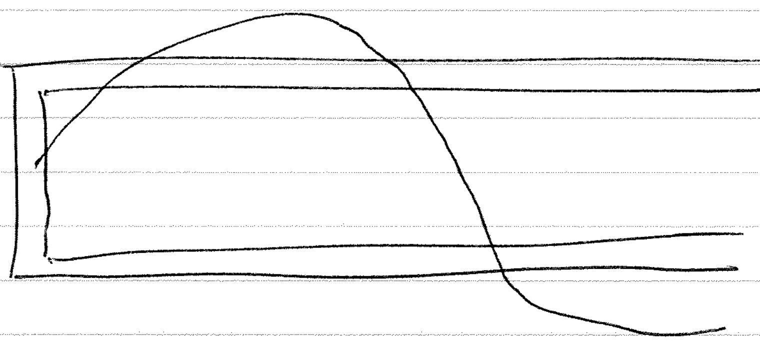
if there is only one open end, then there is antinode @ open end & node @ closed end

only odd harmonics



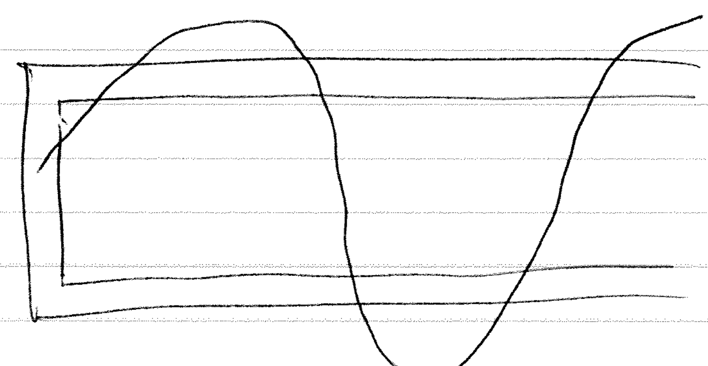
$n=1$

$\lambda = 4L$



$n=3$

$\lambda = \frac{4L}{3}$



$n=5$

$\lambda = \frac{4L}{5}$

In general, $\lambda = \frac{4L}{n}$ $n = 1, 3, 5, \ldots$

& $f = \frac{v}{\lambda} = \frac{nv}{4L}$

- cannot have even harmonics b/c these would have nodes @ open end!

- length of instrument reflects range of frequencies it can produce

- a musical instrument typically produces not only a fundamental frequency, but also one or more of the higher order harmonics

---

Problem: Pipe A is open @ both ends & has length $L_A = 0.343\,m$

Idea is to place it near 3 other pipes that already have standing waves set up, in order to produce a standing wave in pipe A.

other 3 pipes are closed @ one end

$L_B = 0.5\,L_A$

$L_C = 0.25\,L_A$

$L_D = 2\,L_A$

12

for these pipes, which of their harmonics can excite a harmonic in pipe A?

resonant freq's of pipe A are

speed of sound in air

$$f_A = \frac{n_A v}{2 L_A} = \frac{n_A \, 343 \, m/s}{2 \cdot 0.343 \, m}$$

$$= n_A \cdot 0.5 \text{ kHz}$$

resonant freq's of pipe B:

$$f_B = \frac{n_B v}{4 L_B} = \frac{n_B v}{4 \cdot 0.5 L_A} = \frac{n_B (343 \, m/s)}{2 \cdot 0.343 \, m}$$

$$= n_B \, 500 \text{ Hz} = n_B \cdot .5 \text{ kHz}$$

for $n_B =$ 1, 3, 5

there is a match

for every value of n_B,

but no even harmonic can be set up

resonant freq's of pipe C:

$$f_C = \frac{n_C v}{4L_C} = \frac{n_C v}{4 \cdot 0.25 L_A} = n_C \cdot \frac{343}{0.343}$$

$$= n_C \cdot 1000 = n_C \cdot 1\,kHz$$

for $n_C = 1, 3, 5, \ldots$

only for $n_A = 2n_C$ w/ $n_C = 1, 3, 5$ are harmonics of A excited

- can check that D does not ~~set~~ excite any harmonics of A.