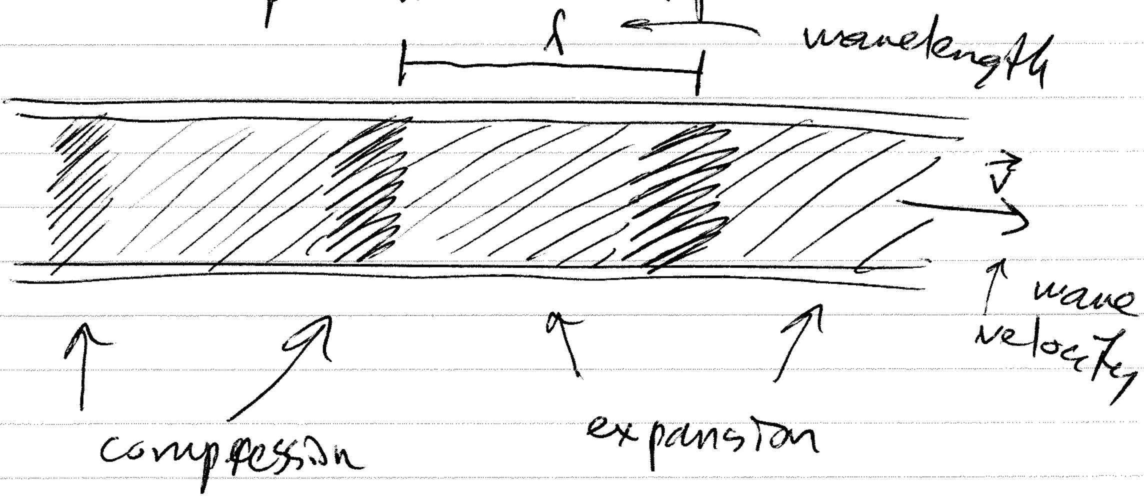
Lecture 16

Traveling Sound Waves

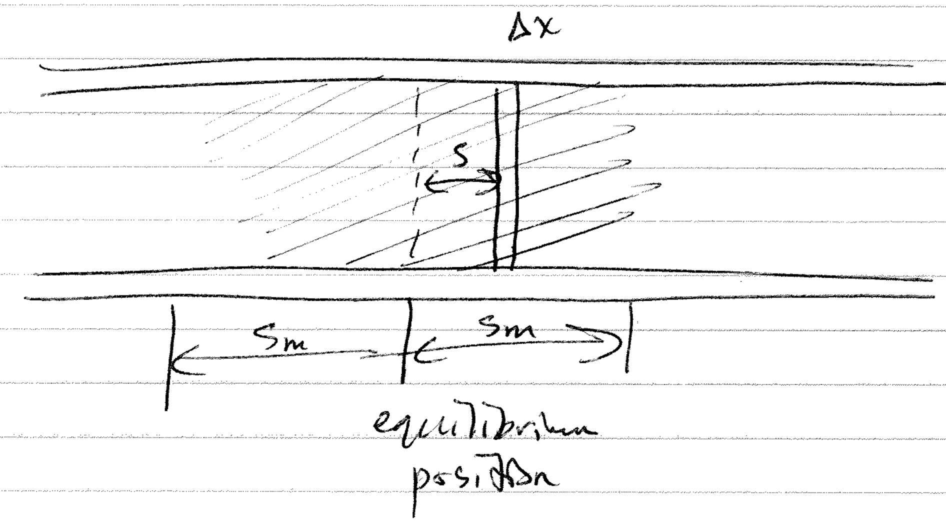
- Now we examine displacements & pressure variations associated w/ sound waves.

- can produce such a wave by moving a piston left & right @ the end of a ~~tube~~ tube.

Consider this figure depicting compressions & expansions of air



consider this figure



- vertical column depicts a small oscillating fluid element

- element of air oscillates in simple harmonic motion about its equilibrium position.

- air element oscillates longitudinally

- we want to understand the function $s(x,t)$, which captures the horizontal movement /displacement of an air element @ equilibrium position x away from this equil. position as a function of location x + time t.

- It is given by

$$s(x,t) = \underbrace{s_m}_{\text{displacement amplitude}} \underbrace{\cos(kx - \omega t)}_{\text{oscillating term}}$$

All variables are the same as before:

s_m - maximum displacement

k - angular wave number $k = \frac{2\pi}{\lambda}$

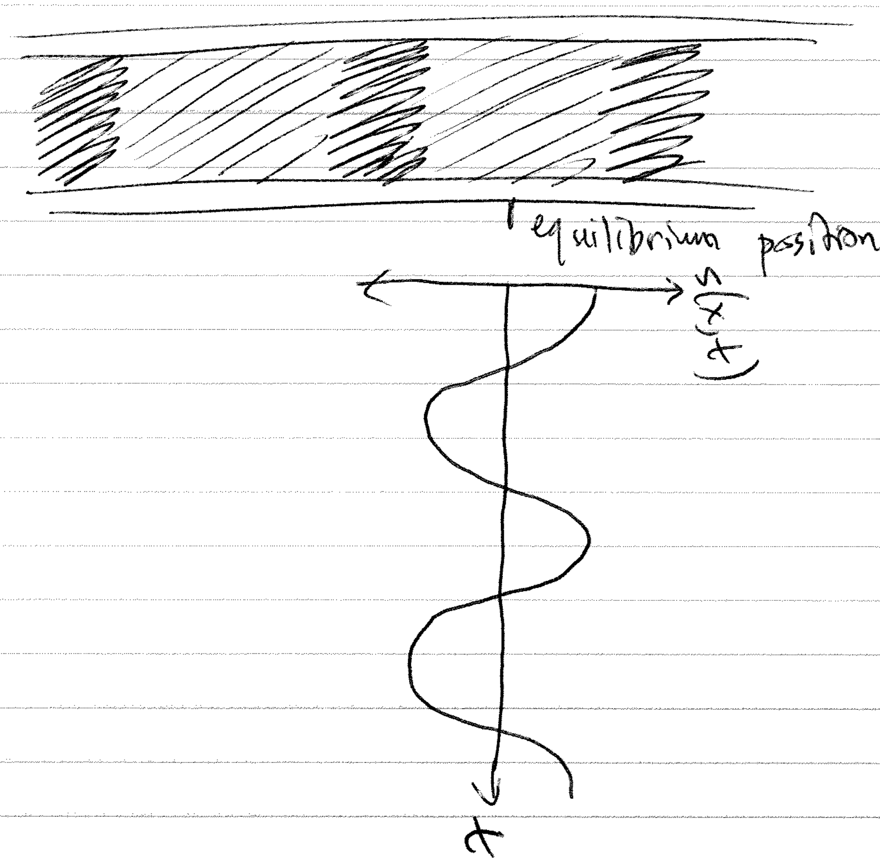
λ - wavelength (distance @ which pattern of compression + expansion repeats)

ω - angular frequency

f - frequency $\qquad \omega = 2\pi f$

T - period $\qquad \omega = \dfrac{2\pi}{T}$

so picture to have in mind is



~~can also~~

can also describe pressure variation @ a given location x as

$$\underbrace{\Delta p(x,t)}_{\text{pressure variation}} = \underbrace{\Delta p_m}_{\text{pressure amplitude}} \sin \underbrace{(kx - \omega t)}_{\text{oscillating term}}$$

pressure amplitude ↓ max. increase in pressure due to wave

relation to max. displacement is

$$\Delta p_m = (v \rho \omega) s_m$$

v ← wave speed

ρ ← density

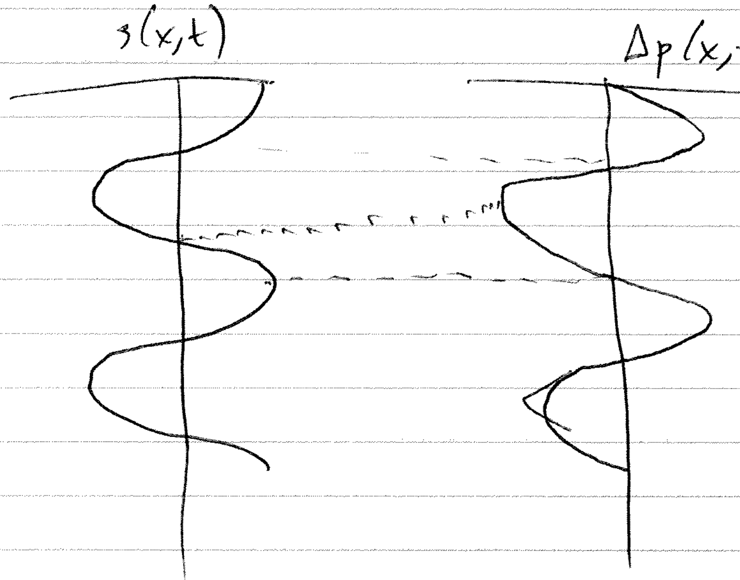
ω ← angular frequency

s_m ← max. displacement

$\Delta p(x,t)$ is $\pi/2$ out of phase w/ $s(x,t)$.

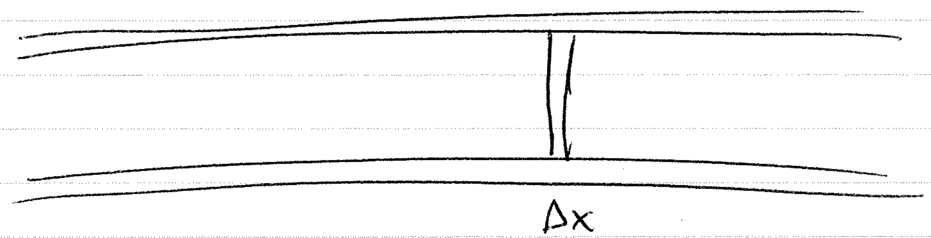
Question: When an oscillating air element is moving rightward through the point of zero displacement is the pressure in the element beginning to increase or decrease?

$\boxed{\text{increase}}$



We can derive relation between

$\Delta p(x,t)$ & $s(x,t)$



consider element of air w/ thickness Δx & cross-sectional area A.

Now use definition of bulk modulus to get

$$\Delta p = -B \frac{\Delta V}{V}$$

volume of air element is $V = A\Delta x$

ΔV is change in volume due to displacement
- comes about because 2 faces of air element will be different under displacements

$$\Delta V = A\,\Delta s$$

Substitute in to get (8)

$\Rightarrow$ $$\frac{\Delta V}{V} = \frac{A \Delta s}{A \Delta x} = \frac{\Delta s}{\Delta x}$$

$$\Delta p = -B \frac{\Delta s}{\Delta x}$$

Now take partial derivative to get

$$\Delta p = -B \frac{\partial s}{\partial x}$$

$$= B k s_m \sin(kx - \omega t)$$

Then $\Delta p_m = B k s_m$

use $\sqrt{\frac{B}{\rho}} = v$ & $\frac{\omega}{k} = v$

to get $B = v^2 \rho$ & $k = \frac{\omega}{v}$

&

$$\Delta p_m = v^2 \rho \frac{\omega}{v} s_m$$

$$= v \rho \omega s_m$$

- The superposition principle applies to sound waves also
  (think of a choir)

- sound waves can interfere w/ each other

- Suppose 2 sound waves are traveling positive x direction w/ same amplitude, wavelength, & frequency, but w/ different phase ϕ

$$s_1(x,t) = s_m \cos(kx - \omega t)$$

$$s_2(x,t) = s_m \cos(kx - \omega t + \phi)$$

Then

$$s_1(x,t) + s_2(x,t) =$$

$$2 s_m \cos\left(\tfrac{1}{2}\phi\right) \cos\left(kx - \omega t + \tfrac{1}{2}\phi\right)$$

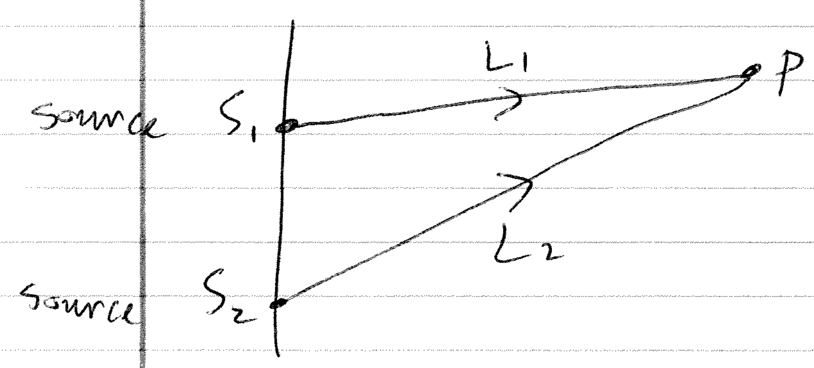
Amplitude is

$$\left|2 s_m \cos\left(\tfrac{1}{2}\phi\right)\right|$$

value of ϕ determines whether interference is constructive, destructive, or intermediate.

- Taking the same sound signal, duplicating it, & varying the phase of one, creates the <u>phaser effect</u>, heard in many Van Halen songs!

we can control ϕ by sending waves along different paths:



interference @ P depends on difference in path lengths

- Suppose sources have identical amplitude, wavelength, frequency, & phase.
- Suppose distance to P is much greater than distance between sources, so it is as if waves are traveling in same direction.
- path traveled by S_2 is longer than path traveled by S_1 & so they arrive @ P out of phase
- This depends on <u>path length difference</u> $\Delta L = |L_2 - L_1|$

Recall that phase diff. of 2π radians corresponds to one wavelength

$$\Rightarrow \quad \frac{\phi}{2\pi} = \frac{\Delta L}{\lambda} \quad \left(\text{equating proportions}\right)$$

$$\Rightarrow \quad \phi = \frac{\Delta L}{\lambda} 2\pi$$

Fully constructive interference occurs when $\phi = m\, 2\pi$ for $m = 0, 1, 2, \ldots$

$\Rightarrow$

~~occurs when~~ $\dfrac{\Delta L}{\lambda} = 0, 1, 2, \ldots$

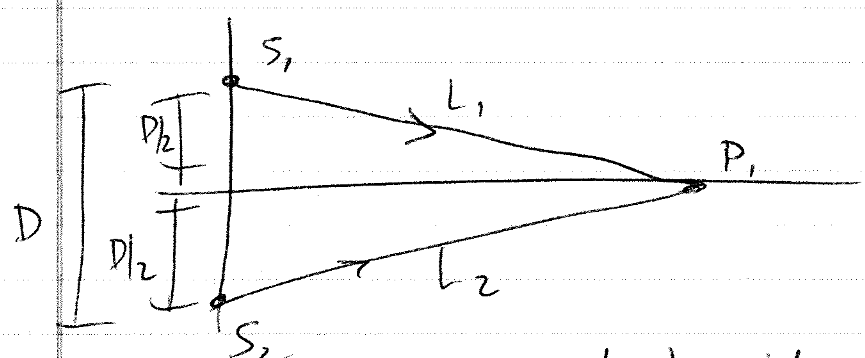
Fully destructive interference occurs when

$$\phi = (2m+1)\pi \quad \text{for } m = 0, 1, 2, \ldots$$

$$\Rightarrow \quad \frac{\Delta L}{\lambda} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$$

Example:

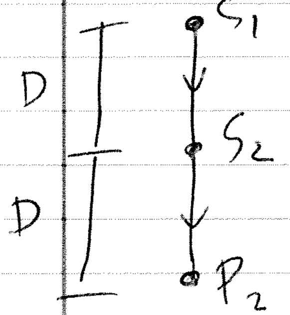
Suppose two sources S_1 & S_2 emit identical sound waves of wavelength λ. Suppose they are separated by distance $D = 1.5\lambda$



What is path length difference of waves from S_1 & S_2 @ point P_1?

What type of interference occurs @ P_1

$\Delta L = 0$, fully constructive interference



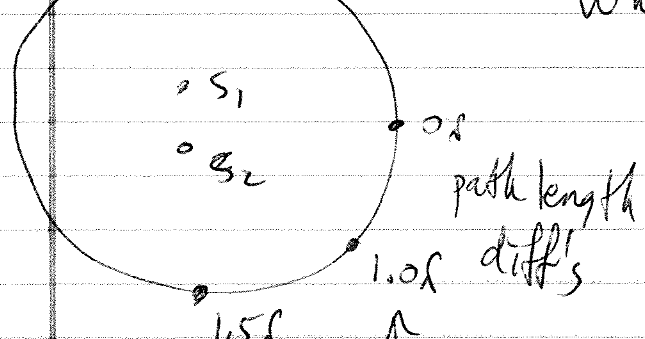
What is path length difference of waves from S_1 & S_2 @ P_2?

What kind of interference occurs?

$\Delta L = 1.5\lambda$, fully destructive b/c

$$\frac{\Delta L}{\lambda} = 1.5$$

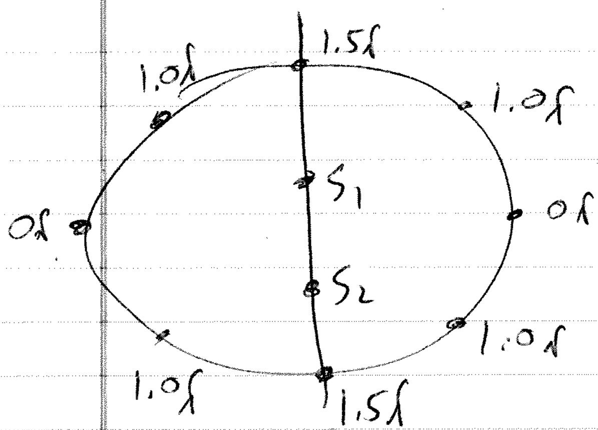
Now consider moving along a circle w/ radius much greater than D.



Where are points along circle w/ fully constructive interference?

↑ there must be one point in between these two w/ fully constructive int.

By similar reasoning



Six points of fully constructive interference