Lecture 15

- Continuing w/ standing waves, key equation is

$$y(x,t) = \underbrace{2y_m \sin(kx)}_{\text{amplitude @ } x} \underbrace{\cos(\omega t)}_{\text{oscillatory term}}$$

- Notice that the amplitude of the standing wave varies w/ position

- At what locations x is amplitude equal to zero? i.e., for which x is $\sin(kx) = 0$?

@ $kx = n\pi$ for $n = 0, 1, 2, \ldots$

or $x = \frac{n\pi}{k}$ or $x = \frac{n\lambda}{2}$

using $k = 2\pi/\lambda$

These are called <u>nodes</u>

Adjacent nodes are separated by $\lambda/2$

- Amplitude has maximum value for some times when

$$|\sin(kx)| = 1$$

This happens when

$$kx = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi$$

or

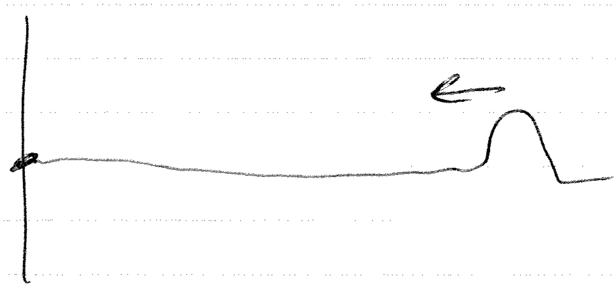
$$kx = (n + 1/2)\pi \quad \text{for } n = 0, 1, 2, \ldots$$

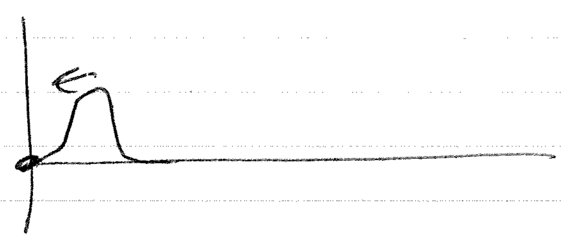
or

$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$$

These locations are called <u>antinodes</u>

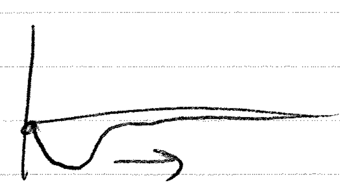
When sending a pulse for a string attached to a wall,





- as the pulse approaches the wall, it exerts an upward force on the wall.

- By Newton 3rd law, wall ~~supports~~ exerts an equal and opposite force, which generates a pulse in the opposite direction:



Question: 2 waves w/ same amplitude + wavelength interfere in 3 different situations to produce the following waves;

1) $4 \sin(5x - 4t)$

2) $4 \sin(5x) \cos(4t)$

3) $4 \sin(5x + 4t)$

In which situation is the combined wave a) traveling right $(+x)$

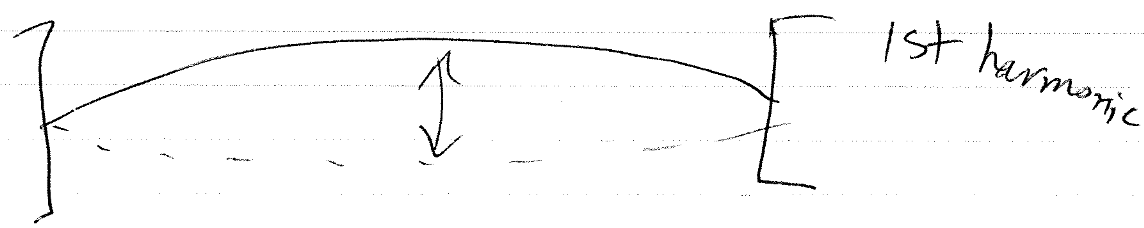
b) traveling left $(-x)$

c) in opposite directions

- Consider a string stretched between 2 clamps. If we send a continuous wave to the right, it reflects & travels left, then reflects back.
- waves are overlapping & interfere w/ each other
- for certain frequencies, interference produces a standing wave pattern, w/ nodes & antinodes
- Standing wave is produced @ <u>resonance</u> & string resonates @ <u>resonant frequencies</u>

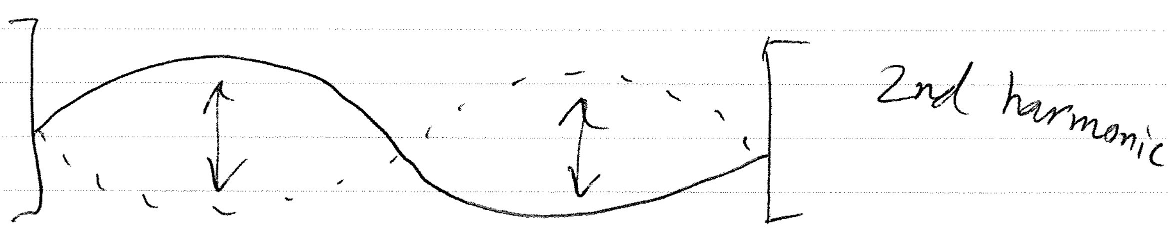
Let's find expressions for resonant frequencies

- node should exist @ each end of string
- simplest case is



one antinode @ center of string

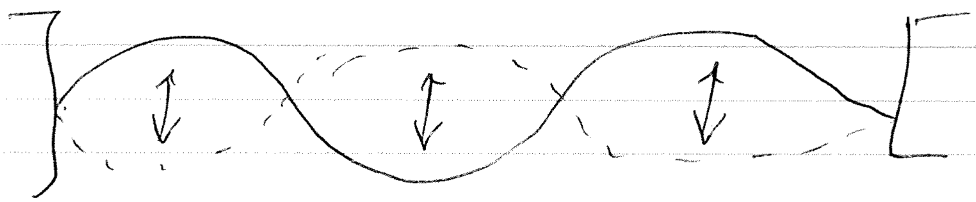
- wavelength is $\frac{\lambda}{2} = L$ or $\lambda = 2L$
- Another pattern is



3 nodes + 2 antinodes

$\lambda = L$

Another pattern:



4 nodes + 3 antinodes

$$\lambda = \frac{2}{3}L$$

More generally, standing waves

$$\lambda = \frac{2L}{n} \quad \text{for } n = 1, 2, 3$$

resonant frequencies are

$$f = \frac{v}{\lambda} = n\frac{v}{2L} \quad n = 1, 2, 3$$

- they are integer multiples of lowest resonant frequency
- show youtube videos

Problem: A string stretched between fixed supports ~~has~~ separated by 75 cm has resonant frequencies of 420 Hz + 315 Hz, w/ none in between.

What is the lowest resonant frequency?

$$f_n = n \frac{v}{2L} \quad \text{+} \quad f_{n+1} = (n+1)\frac{v}{2L}$$

$$\Rightarrow \frac{f_{n+1}}{f_n} = \frac{(n+1)\frac{v}{2L}}{n\frac{v}{2L}} = \frac{n+1}{n} = \frac{420}{315}$$

$$\Rightarrow n = 3$$

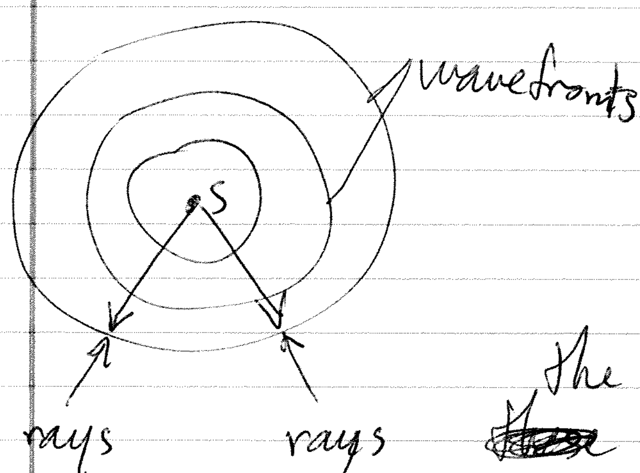
$$\Rightarrow \frac{f_3}{f_1} = \frac{3v/2L}{v/2L} = 3$$

$$\Rightarrow \frac{315}{f_1} = 3 \Rightarrow f_1 = 105\text{ Hz}$$

Ch. 17 Waves (II)

<u>Sound waves</u> – it is a longitudinal wave (direction of particle movement is parallel to source's direction)

– travels from a point source through a 3D medium



– these form spheres centered on S

the double headed arrows oscillate parallel to the rays

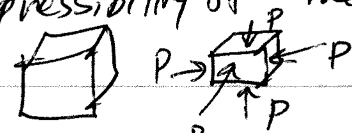
- speed of any mechanical wave depends on an inertial property of medium (kinetic energy) & elastic property (potential energy)
- for transverse wave on a string, this is

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic}}{\text{inertial}}}$$

- for a sound wave, potential energy is associated to periodic compressions & expansions of small volume elements of air.
- property that determines this for a sound wave is <u>bulk modulus</u>

$$B = -\frac{\Delta p}{\Delta V / V}$$

- B captures compressibility of medium



- $\Delta V/V$ is fractional change in volume due to a Δp change in pressure.

- units for B are <u>pascals</u> (same as pressure)

- signs of Δp & ΔV are opposite

$$v = \sqrt{\frac{B}{\rho}}$$

B ← bulk modulus

ρ ← density

for air @ 0°C $v = 331$ m/s

" @ 20°C $v = 343$ m/s

human ear can detect sound waves w/ frequencies between 20 Hz & 20 kHz