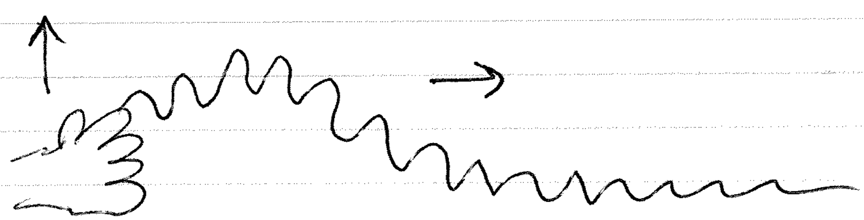
# Lecture 12

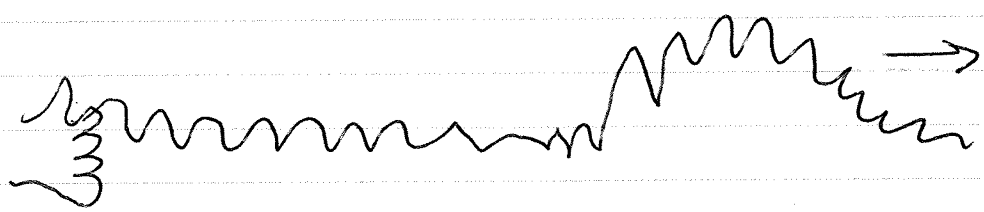
## Ch. 16 Transverse Waves

- Example is a wave sent along a stretched, taut string



hand moves up & down to generate a pulse..

time later ...

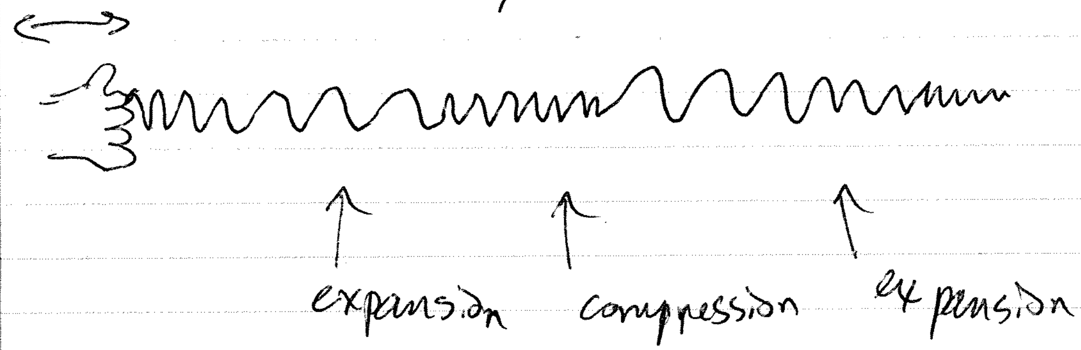


by repeating the pulse, we can generate a periodic traveling wave.

in a transverse wave, motion of particles of medium is perpendicular to the direction of the wave's travel.

## longitudinal wave

hand motion is left to right




in a longitudinal wave, displacement of particles is in the same direction as the wave

Transverse wave -

motion of particles of medium is perpendicular to direction of wave's travel.

to describe this mathematically, we need a function that describes shape of the wave

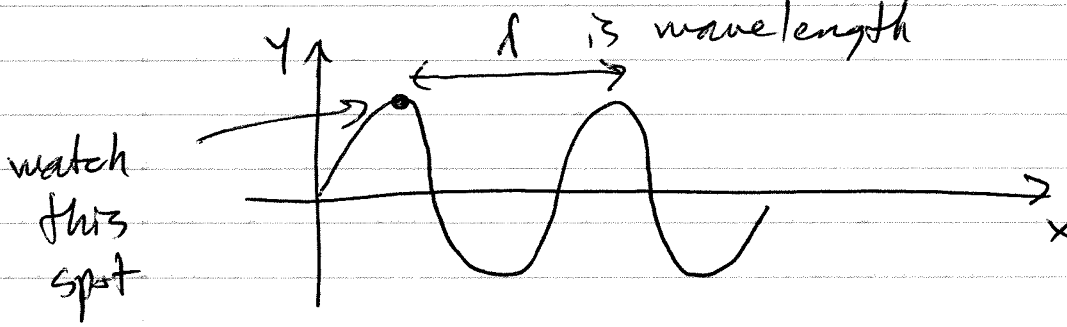
$$y = h(x, t)$$

time

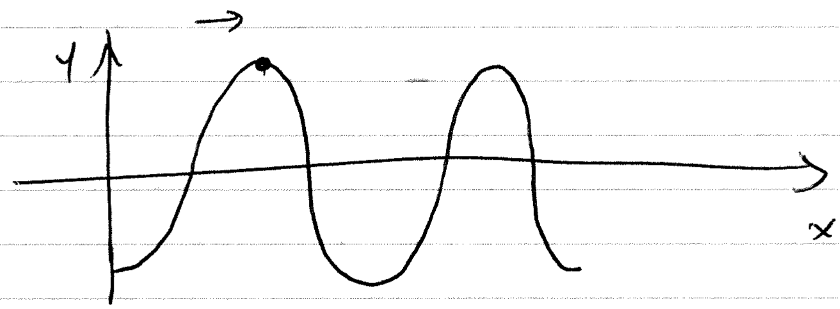
horizontal location on string

height of any part of slinky

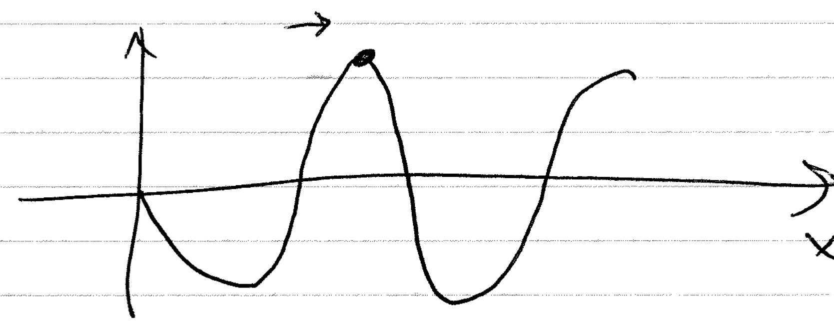
can take snapshots in time



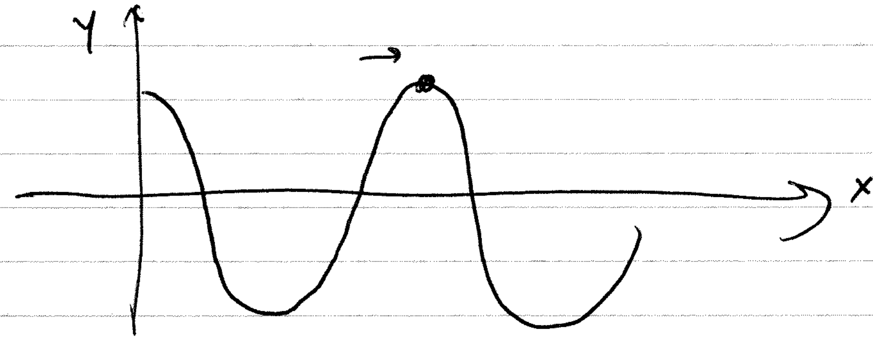

time 1




time 2




time 3




time 4

function that describes transverse wave motion is

$$y(x,t) = y_m \sin(\underbrace{k}_{\text{angular wave number}}\underbrace{x}_{\text{position}} - \underbrace{\omega}_{\text{angular frequency}}\underbrace{t}_{\text{time}})$$

$y(x,t)$: displacement, $y_m$: amplitude

we can use this equation to find displacements of all elements of the string as a function of time.

phase of the wave is the argument of sine : $kx - \omega t$

wavelength $\lambda$ of the wave is the distance between the crests (tops of waves)

"spatial period of the function"

@ time $t=0$

$$y(x,0) = y_m \sin(kx)$$

w/ $\lambda$ as wavelength, this means that $x$, $x+\lambda$, $x+2\lambda$, etc. should give the same value.

Thus

$$y_m \sin(kx) = y_m \sin(k(x+\lambda))$$
$$= y_m \sin(kx + k\lambda)$$

sine function repeats when shifting by $2\pi$, so this means that

$$k\lambda = 2\pi$$

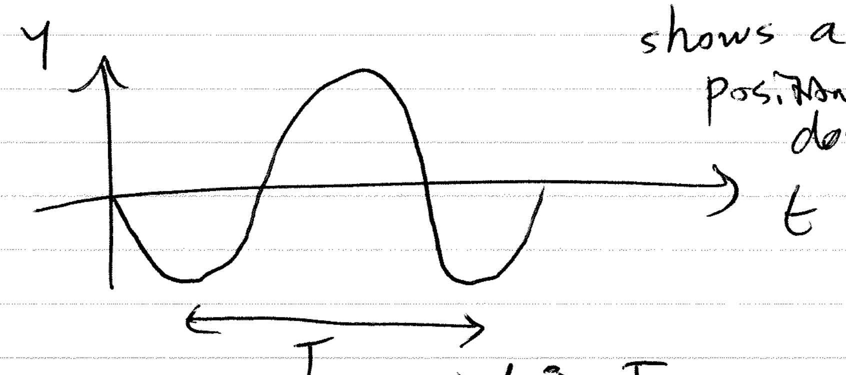
$$\Rightarrow \quad k = \frac{2\pi}{\lambda}$$

Now suppose that we fix a location / position x & we want to see how height changes w/ time.

Just fix $x=0 \Rightarrow$

$$y(0,t) = y_m \sin(-\omega t)$$
$$= -y_m \sin(\omega t)$$

graph of this function looks like




shows a single position moves down & up w/ time

period is T

similar argument as before gives

angular frequency $\rightarrow$ $\omega = \frac{2\pi}{T}$

can generalize wave function w/ a phase constant $\phi$

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

shifts the wave from its initial configuration

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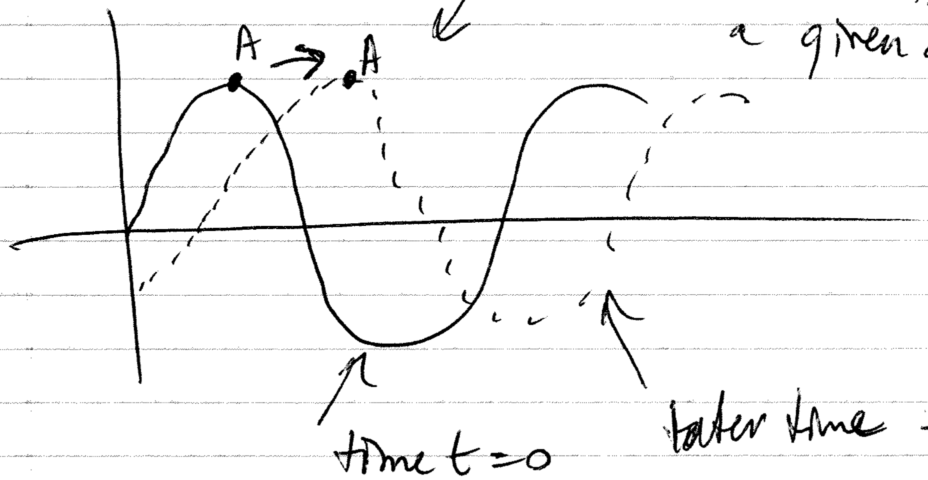
How fast does a wave travel?

we want to figure out the wave speed $v$.

Take a snapshot @ two different times




distance that A travels in a given amount of time tells us wave speed.

notice that point A is @ same height @ the initial & later ~~[illegible]~~ time.

height of A @ $t=0$ given by

$$\sin(kx_0 - \omega t_0)$$

& height of A @ later time is

$$\sin(kx_1 - \omega t_1)$$

In order for heights to be equal, we need that

$$kx_0 - \omega t_0 = kx_1 - \omega t_1$$

can generalize this arguments to all times & positions for A on ~~[illegible]~~ the wave form to conclude that

$$kx - \omega t = \text{a constant}$$

to calculate wave speed, take derivative

$$\frac{d}{dt}\left[kx - \omega t\right] = \frac{d}{dt}\text{ constant}$$

$$\Rightarrow \quad k\frac{dx}{dt} - \omega = 0$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{\omega}{k} = v$$ ← wave speed

Substituting gives $\frac{\lambda}{T}$ units: $\left(\frac{\text{distance}}{\text{time}}\right)$

$v = \frac{\lambda}{T}$ ⇒ speed is one wavelength per period.

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different kind of speed is tranverse speed: how fast does a given location on string move up & down?

this is ~~dy/dt~~

$$\frac{dy(x,t)}{dt} = \frac{d}{dt}\left[y_m \sin(kx - \omega t)\right]$$

$$= -y_m \omega \cos(kx - \omega t)$$

then maximum transverse speed is $\boxed{y_m \omega}$