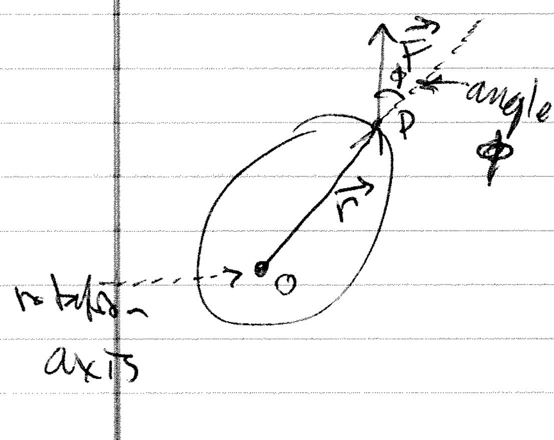
Lecture 1~~0~~

- Simple pendulum - simple harmonic oscillator in which springiness is related to gravitational force.

- To analyze pendulum, we need the concept of torque

- Torque is <u>rotational</u> force.

Consider an object



force is applied @ point P.

ability of force to rotate around axis O is called torque. depends not only on force but also distance from rotation axis.

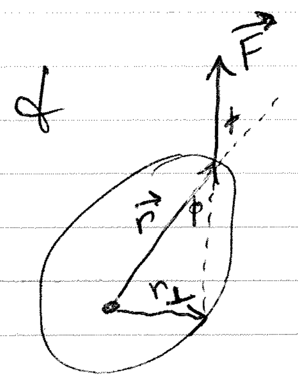
magnitude of torque

torque is given by

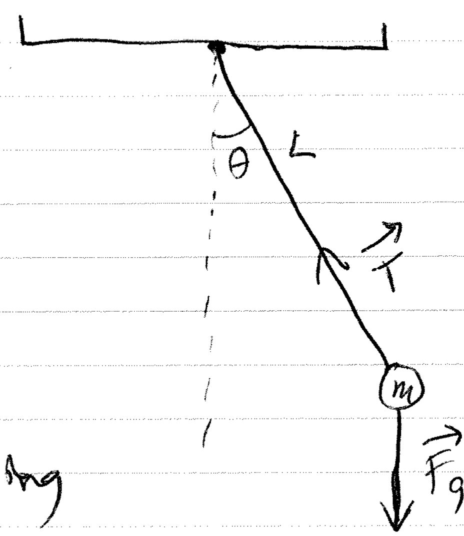
$$\tau = rF\sin\phi$$

can calculate as

$$\tau = r_\perp F \qquad \text{where } r_\perp = r\sin\phi$$



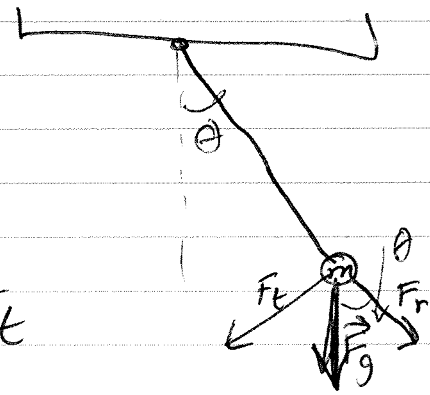
For a pendulum, the picture is given by



two forces involved

1) $\vec{T}$ - force due to tension in string

2) $\vec{F}_g$ - force due to gravity



- can resolve $\vec{F}_g$ into radial component + tangential component F_t

- tangential component F_t is a restoring torque about pendulum's pivot point. It acts in ~~a~~ a way to drive motion to the center.

- restoring torque is given by

$$\tau = -L\, F_g \sin\theta$$

L: length of string

F_g: gravitational force

minus sign there because torque acts to reduce θ.

Using Newton 2nd law for rotation

$$\tau_{net} = I\alpha$$

rotational inertia — angular acceleration

we find that

$$-L\underbrace{mg}_{F_g}\sin\theta = \tau = I\alpha$$

$$\Rightarrow \quad \alpha = \frac{-Lmg\sin\theta}{I}$$

If angle θ is small, then

$\sin\theta \approx \theta$ &

$$\alpha = -\frac{mgL}{I}\theta$$

Since $\alpha = \frac{d^2\theta}{dt^2}$, this means that

$$\frac{d^2\theta}{dt^2} + \frac{mgL}{I}\theta = 0$$

- This is the key equation for SHM, which we have already solved.

- Angular frequency ω is found by using constant $\frac{mgL}{I}$ in front of linear term.

- It is given by $\omega = \sqrt{\frac{mgL}{I}}$

for point mass, $I = mL^2$

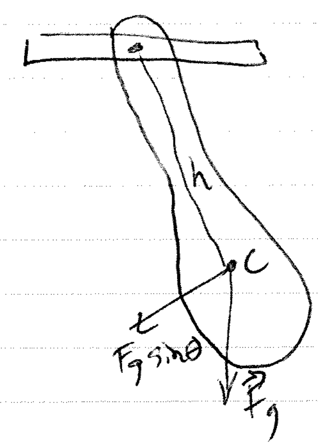
$$\Rightarrow \quad \omega = \sqrt{\frac{g}{L}}$$

just depends on length & gravitational acceleration

period is then

$$T = 2\pi \sqrt{\frac{L}{g}}$$

- for a real physical pendulum, the distribution of mass can be complicated.
- However, to analyze it, we can use center of mass & distance h to center of mass



then period becomes

$$T = 2\pi \sqrt{\frac{I}{mgh}}$$

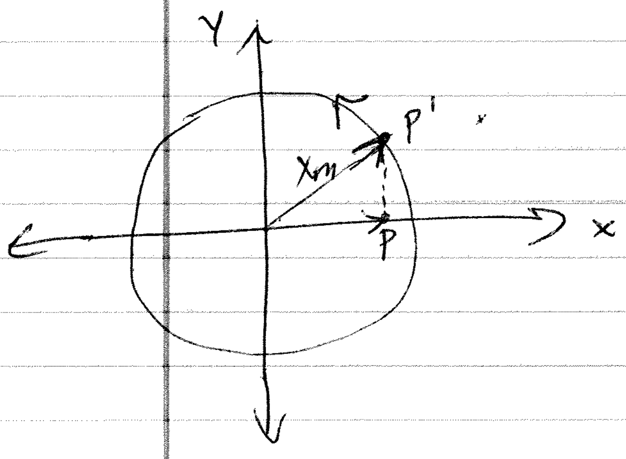
I is rotational inertia.
(not simply mh^2 in this case)

- can see that physical pendulum does not swing if center of mass is same as pivot location. This means $h \to 0$ & then period $T \to \infty$

SHM & Uniform Circular Motion

- SHM is the projection of uniform circular motion.
  - This is what Galileo observed in his records of the motion of Jupiter's moons.

Consider this diagram:



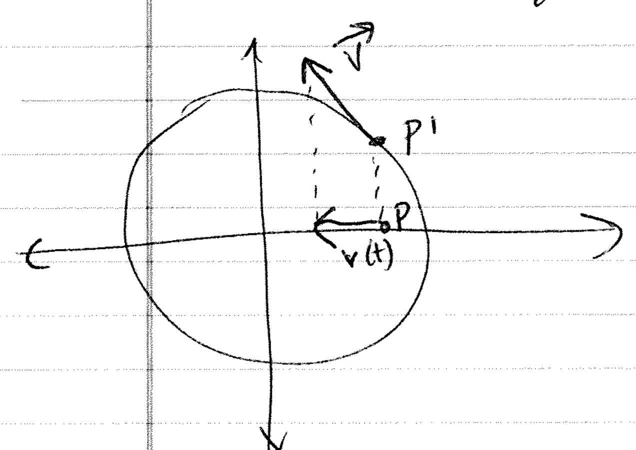
P' is particle moving on a circle

- P is "projection particle" moving on x axis.
- it is the "shadow" of P' on x axis.

coordinates of P' w/ uniform circular motion are

$$(x(t), y(t))$$
$$= \left(x_m \cos(\omega t+\phi),\ x_m \sin(\omega t+\phi)\right)$$

velocity picture is



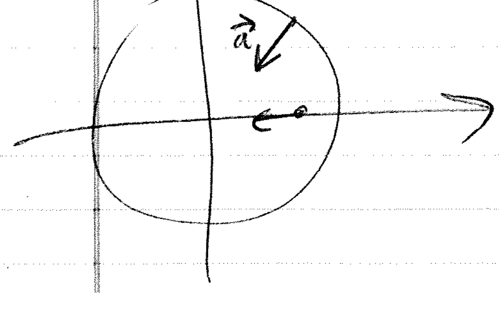
$\vec{v}$ is velocity vector of P' w/ magnitude $v = \omega r$

& projection is $v(t)$

then $v(t) = -\omega x_m \sin(\omega t+\phi)$

&

Acceleration picture is



$\vec{a}$ is radial acceleration vector of P' w/ magnitude $\omega^2 r$

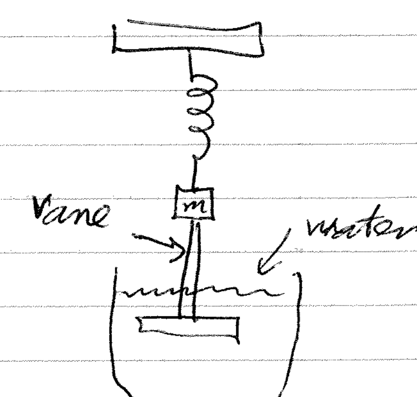
projection is

$a(t) = -\omega^2 x_m \cos(\omega t+\phi)$

Damped simple harmonic motion

- In practice, a pendulum does not swing forever b/c it has to deal w/ a drag force
- i.e., air exerts a drag force on pendulum that eventually stops its motion

- When motion is reduced by drag force, we say that oscillations are damped.
- To analyze this, consider picture



vane immersed in liquid exerts damping force

Suppose liquid exerts damping force $\vec{F}_d$ proportional to velocity of vane + block

Then $F_d = -bv$

damping constant

velocity of block & vane

force opposes motion

Newton 2nd law gives

$$-bv(t) - kx(t) = ma(t)$$

$$\Rightarrow \quad -b\frac{dx(t)}{dt} - kx(t) = m\frac{d^2x(t)}{dt^2}$$

solution is

$$x(t) = x_m \, e^{-bt/2m} \cos(\tilde{\omega} t + \phi)$$

$$\tilde{\omega} = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

motion then looks like