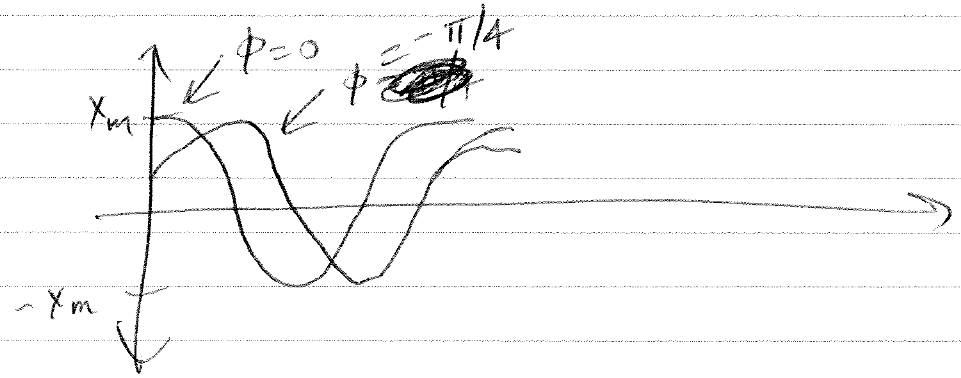
Lecture 10

Continuing w/ simple harmonic motion:

$x(t) = A\cos(\omega t + \phi)$

What if $\phi \neq 0$?

Then we have a phase shift



Values for A & t can be solved by using initial conditions

@ $t=0$, $x=x_0$ & $v=v_0$

$\Rightarrow$ $x_0 = A\cos(\phi)$

$v_0 = -\omega A\sin(\phi)$

②

$$\Rightarrow \quad \frac{x_0}{A} = \cos\phi \qquad \frac{-v_0}{\omega A} = \sin\phi$$

$\Rightarrow$ use $\quad \cos^2\phi + \sin^2\phi = 1$

$$\Rightarrow \quad \frac{x_0^2}{A^2} + \frac{v_0^2}{\omega^2 A^2} = 1$$

$$\Rightarrow \quad A = \sqrt{x_0^2 + v_0^2/\omega^2}$$

use $\quad \tan\phi = \dfrac{\sin\phi}{\cos\phi}$

$$\Rightarrow \quad \tan\phi = \frac{-v_0}{\omega x_0}$$ then use inverse tangent.

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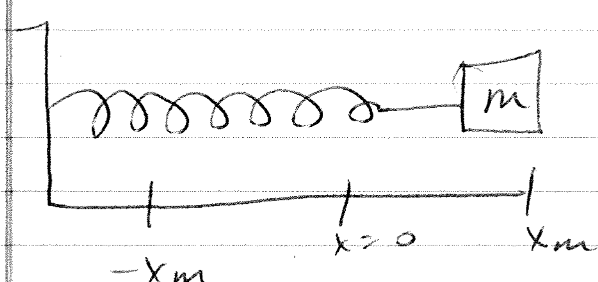
look @ example of springs



Newton's 2nd law:

$$\Sigma F = ma$$

Hooke's law for springs: $\Sigma F = -kx$

$$\Rightarrow \quad -kx = ma \quad \Rightarrow \quad kx + m\frac{d^2x}{dt^2} = 0$$

To solve this, guess solution to be

$$x(t) = A\cos(\omega t + \phi)$$

$$\Rightarrow \frac{d^2x(t)}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

Plugging in gives

$$kA\cos(\omega t+\phi) + m\left[-\omega^2 A\cos(\omega t+\phi)\right] = 0$$

$$\forall t$$

$$\Rightarrow \omega = \sqrt{\frac{k}{m}}$$ angular frequency related to mass & spring constant.

$$\Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\Rightarrow T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

frequency & period do not depend on amplitude

Energy of simple harmonic motion

KE = kinetic energy $= \frac{1}{2}mv^2$

$= \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t+\phi)$

$= \frac{1}{2}kA^2 \sin^2(\omega t+\phi)$

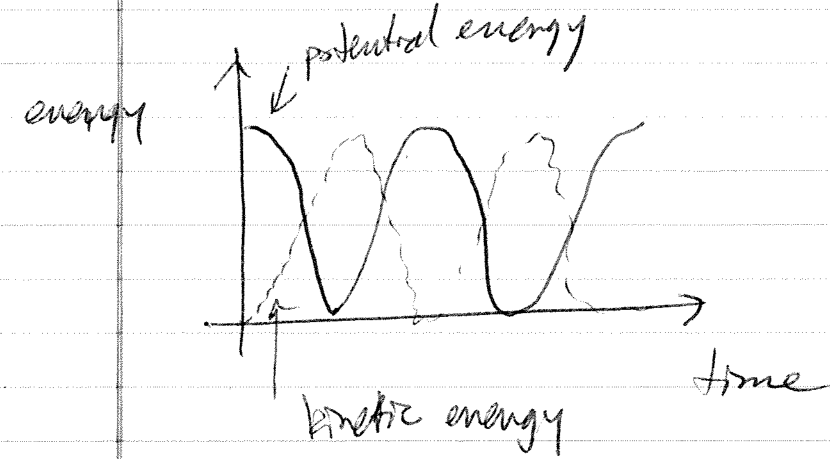
PE = potential energy $= \frac{1}{2}kx^2$

$= \frac{1}{2}kA^2\cos^2(\omega t+\phi)$

$\Rightarrow$ $KE + PE = \frac{1}{2}kA^2$

conservation of energy.

$\Rightarrow$
- in one period, energy is transferred back & forth between potential & kinetic energy
- @ equilibrium (middle) point, all energy is kinetic
- @ endpoints, all energy is potential



Angular Simple Harmonic Oscillator

twist in suspension wire leads to spring like behavior.

called a torsion pendulum

rotating disk through an angle θ introduces a restoring torque given by

$$\tau = -\kappa\theta$$

- K is called torsion constant
- recall that torque is angular force

we then have angular version of Newton's & Hooke's laws:

$$I\alpha = -K\theta$$

Compare to

$$ma = -kx$$

variable	linear	angular
inertia	m	$I = \sum_i m_i r_i^2$
displacement	x	θ
velocity	$v = dx/dt$	$d\theta/dt$
acceleration	$a = d^2x/dt^2$	$\alpha = \frac{d^2\theta}{dt^2}$
Newton 2nd law	$\sum F = ma$	$\sum \tau = I\alpha$

Newton 2nd law gives

$$I\alpha + k\theta = 0$$

$$\Rightarrow I\frac{d^2\theta}{dt^2} + k\theta = 0$$

$$\Rightarrow \theta = \theta_m \cos(\omega t + \phi)$$

angular frequency is

$$\omega = \sqrt{\frac{k}{I}}$$ (compare to spring)

period is $T = \frac{1}{f} = 2\pi\sqrt{\frac{I}{k}}$