Lecture 8

Entropy, heat engines, & the Carnot cycle

- a heat engine extracts energy from its environment in the form of heat & does useful work.
- In every engine is a working substance
  1) water (liquid & vapor) for steam engine
  2) for cars, gasoline-air mixture

Engine operates in a cycle to do sustained useful work.

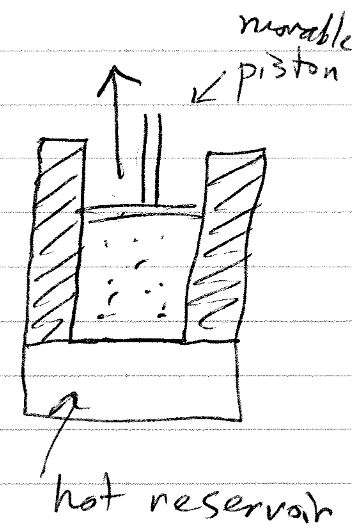
- We study an ideal engine, in which all processes are reversible & no wasteful energy transfer occurs

We study Carnot engine & Carnot cycle

Consists of 4 phases:

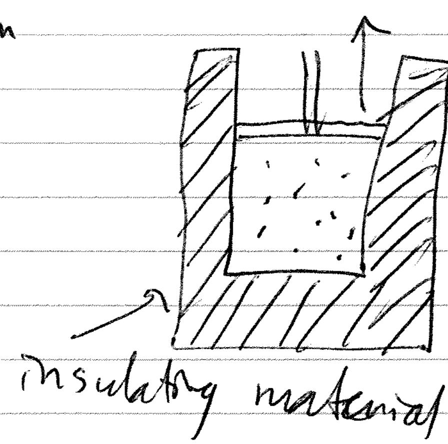
1) Isothermal expansion

work done by system



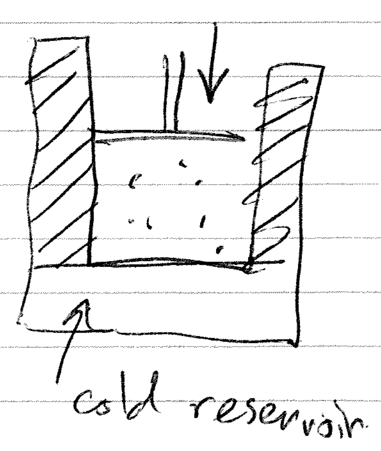
2) Adiabatic expansion

work done by system



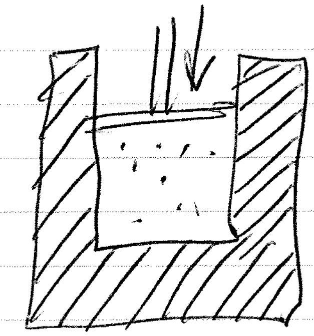
3) Isothermal compression

work done on system

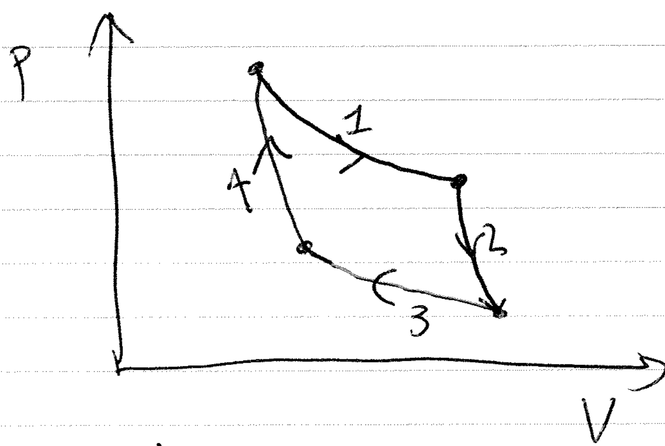


4) Adiabatic compression

work done on system



on a p-V diagram, the Carnot cycle looks like



1) + 3) occur on isotherms

work done by engine is area under curve.

Let T_H be temperature of hot reservoir & let T_L be temperature of cold reservoir

H - high , L - low

Analysis of work done in Carnot cycle:

1) heat is transferred from hot reservoir to working substance. Let $|Q_H|$ be this amount.

It is equal to $Q_H = nRT_H \ln\left(\frac{V_f'}{V_i'}\right)$

2) During adiabatic expansion, no heat is transferred.

3) During isothermal compression, heat transferred from working substance to cold reservoir

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Let $|Q_L|$ be this amount

equal to $Q_L = nRT_L \ln\left(\frac{V_f^3}{V_i^3}\right)$

4) During adiabatic compression, no heat exchanged.

What is the total work done during Carnot cycle?
by system

$\Delta E_{int} = 0$ because initial & final states are equal

$\Rightarrow$ $W = Q$ from 1st law

$= |Q_H| - |Q_L|$

What are the entropy changes?

$\Delta S = \Delta S_H + \Delta S_L$

(6)

$$= \frac{|Q_H|}{T_H} - \frac{|Q_L|}{T_L}$$

positive because heat added to system

negative b/c heat removed from system

Since entropy is a state function,

$\Delta S = 0$ (Carnot cycle)

$$\Rightarrow \frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L}$$

Since $T_H > T_L$, it follows that

$|Q_H| > |Q_L|$

Efficiency of Carnot Engine

How good is it?

thermal efficiency $\varepsilon = \dfrac{\text{work done per cycle}}{\text{heat absorbed per cycle}}$

$= \dfrac{\text{"energy we get"}}{\text{"energy we pay for"}}$

$= \dfrac{|Q_H| - |Q_L|}{|Q_H|}$

$= 1 - \dfrac{|Q_L|}{|Q_H|}$

$= 1 - \dfrac{T_L}{T_H}$

depends only on temperatures of reservoirs

Since $T_L < T_H$

efficiency is less than one.

perfect efficiency of 1 is physically impossible!

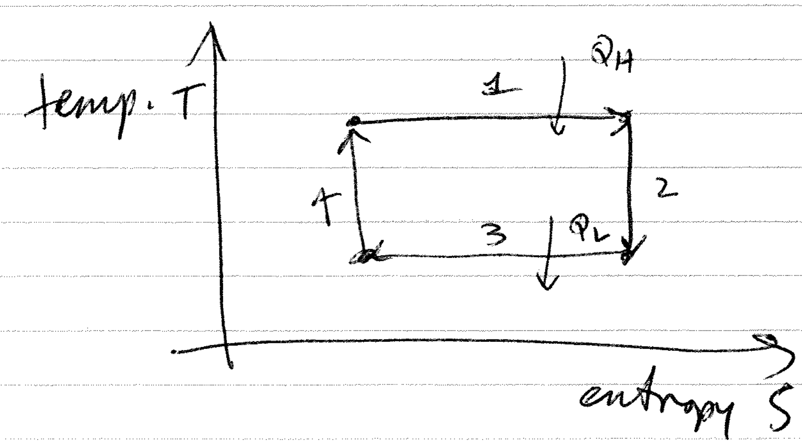
(can only happen when $T_L = 0$ or $T_H \to \infty$)

unphysical

Another way of stating 2nd law:

there is no perfect heat engine.

We can also draw the Carnot cycle on a temperature - entropy diagram



Example Problem:

Given is a Carnot engine
that operates between $T_H = 850K$
& $T_L = 300K$
engine performs 1200J of work per cycle, which takes 0.25s

1) What is the engine efficiency?

$$\varepsilon = 1 - \frac{T_L}{T_H} \approx 65\%$$

2) What is the average power?

$$P_{ave} = \frac{W}{\Delta t} = \frac{1200J}{.25s} = 4800W$$

3) How much energy $|Q_H|$ is extracted as heat from high temp reservoir per cycle?

$$\varepsilon = \frac{|W|}{|Q_H|} \Rightarrow |Q_H| = \frac{W}{\varepsilon} = \frac{1200J}{65\%} = 1855J$$

4) How much energy $|Q_L|$ delivered as heat to low temp. reservoir per cycle?

$$W = |Q_H| - |Q_L|$$

$$\Rightarrow |Q_L| = |Q_H| - W$$

$$= 1855\,J - 1200\,J$$

$$= 655\,J$$

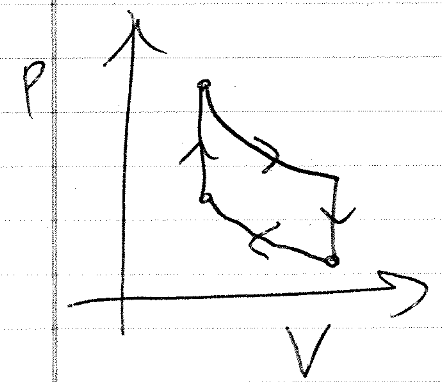
5) What is entropy change ΔS of working substance during energy transfer from high & low temp reservoir?

since isotherms,

$$\Delta S_L = \frac{Q_L}{T_L} = \frac{-655\,J}{300\,K} = -2.18\ J/K$$

$$\Delta S_H = \frac{Q_H}{T_H} = \frac{1855\,J}{850\,K} = 2.18\ \frac{J}{K}$$

Stirling engine has a different cycle,



main difference is that two isotherms are connected by constant-volume processes.

To have these, it requires changing temperature while maintaining equilibrium.

In Stirling engine, reversible heat transfers occur in all four steps.

total work done by Stirling engine cycle is

$$W = nRT_H \ln\left(\frac{V_b}{V_a}\right) + nRT_L \ln\left(\frac{V_a}{V_b}\right)$$

$$= nR \ln\left(\frac{V_b}{V_a}\right)(T_H - T_L)$$