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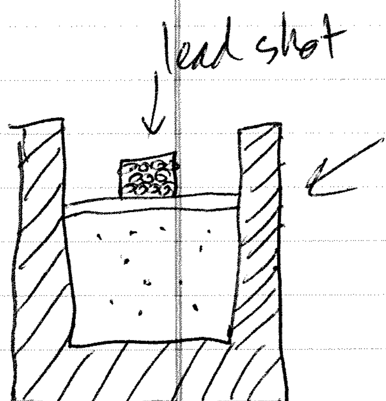
Lecture 6

Adiabatic expansion of an ideal gas

- A process during which no heat transfer occurs ($Q=0$) is called an adiabatic process. (either process occurs very quickly or container is well insulated)



- By removing some lead shot, can allow the gas to expand adiabatically.
- As volume increases, both pressure & temperature drop

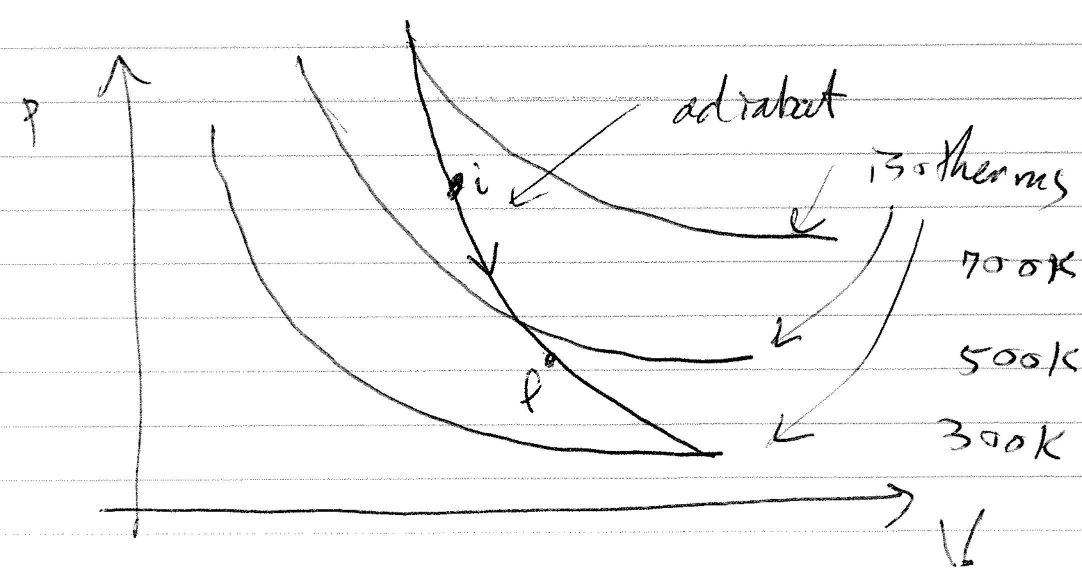
- Relation between pressure & volume during such an adiabatic process is

$$pV^{\gamma} = \text{a constant}$$

where $\gamma = C_p/C_v$ (ratio of molar specific heats)

on a p-V diagram, an adiabatic process occurs along a curve $p = \frac{\text{a constant}}{V^{\gamma}}$

called an <u>adiabat</u>



Since gas goes from initial to final state, we have that

$$p_i V_i^{\gamma} = p_f V_f^{\gamma}$$

We can also use ideal gas equation $(pV = nRT)$ to write

$$\frac{nRT}{V} \cdot V^{\gamma} = \text{a constant}$$

Since n & R are constants

$$\Rightarrow T V^{\gamma - 1} = \text{a constant}$$

$\Rightarrow$ for adiabatic process that

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

Example Question:

Consider thermodynamic system depicted previously. If the mass of lead shot is doubled, what happens to volume, pressure, + temperature? (increase, decrease, or stay the same)

Now we will derive the equation $pV^{\gamma} = $ a constant

Recall 1st law of thermodynamics

$\Delta E_{int} = Q - W$

we can use differential form of this law:

$dE_{int} = dQ - dW$

- Suppose that we remove a little amount of lead shot,
- then volume increases by a differential amount: dV

- Since volume change is small, we can assume pressure stays constant + work $W = p\,dV$
- Since process is adiabatic, there is no heat transfer + $dQ = 0$.

$$\Rightarrow \quad dE_{int} = -p\,dV$$

Now substitute $dE_{int} = nC_V\,dT$ (derived last time)

to get $$nC_V\,dT = -p\,dV$$

$$\Rightarrow \quad n\,dT = -\left(\frac{p}{C_V}\right) dV$$

Now use differential version of
ideal gas law:

$pV = nRT$

$$\Rightarrow \quad p\,dV + V\,dp = nR\,dT$$

Replace R w/ $R = C_p - C_V$

$$\Rightarrow \quad n\,dT = \frac{p\,dV + V\,dp}{C_p - C_V}$$

$$\Rightarrow \quad \frac{p\,dV + V\,dp}{C_p - C_V} = -\frac{p}{C_V}\,dV$$

$$\Rightarrow \quad p\,dV + V\,dp = \frac{C_p - C_V}{C_V}\left(-p\,dV\right)$$

$$= -\frac{C_p\,p\,dV}{C_V} + p\,dV$$

$$\Rightarrow \quad V\,dp = -\frac{C_p}{C_V}\,p\,dV$$

$$\Rightarrow \quad \frac{dp}{p} + \frac{C_p}{C_V}\frac{dV}{V} = 0$$

integrating this equation gives

$$\ln p + \gamma \ln V = \text{a constant}$$

where $\gamma = \frac{C_p}{C_V}$

$$\Rightarrow \quad \ln\left(pV^{\gamma}\right) = \text{a constant}$$

$$\Rightarrow \quad pV^{\gamma} = \text{a constant}$$

- Previously, we discussed <u>free expansions</u>, which are particular adiabatic processes in which <u>no work</u> is done & there is <u>no change</u> in internal energy

- The equations we have just derived do not apply to free expansions.

For a free expansion, it holds that

$$T_i = T_f \quad \text{(free expansion)}$$

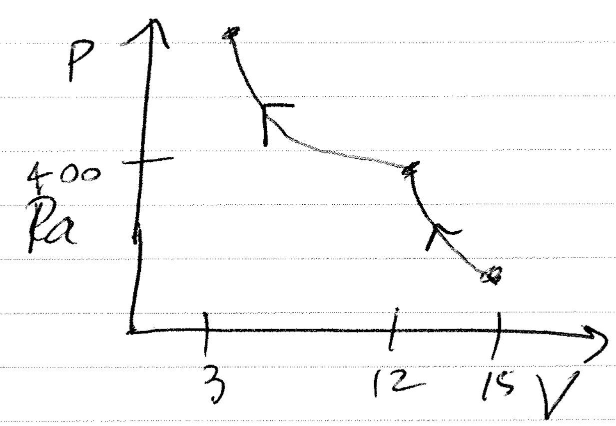
b/c $\Delta E_{int} = 0$

Then for an ideal gas, we conclude that

$$p_i V_i = p_f V_f$$

Example:

Figure depicts an <u>adiabatic</u> compression of 1.7 mole of an ideal gas from $15 m^3$ to $12 m^3$, followed by isothermal compression to a final volume of $3 m^3$.



What is total energy transferred as heat?

Two part process: adiabatic & isothermal

heat transferred by adiabatic is equal to zero

then calculate it for isothermal part

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$\Delta E_{int} = Q - W$ for isothermal $\Delta E_{int} = 0$

$\Rightarrow$ $Q = W_{by} = nRT \ln\left(\frac{V_f}{V_i}\right)$

comes from

$p = \frac{nRT}{V}$ + $W = \int_{V_i}^{V_f} p\,dV$

for isothermal process, temp. is constant

$\Rightarrow T = \frac{P_i V_i}{nR}$

$\Rightarrow W_{by} = nRT \ln(V_f/V_i)$

$= nR\left(\frac{P_i V_i}{nR}\right) \ln\left(\frac{V_f}{V_i}\right)$

$= P_i V_i \ln\left(\frac{V_f}{V_i}\right)$

Another example

Ideal monatomic gas w/ initial pressure p_0 expands isothermally until its volume is twice the initial volume. Then it is slowly & adiabatically compressed back to its initial volume.

$\boxed{\text{What is its final pressure?}}$

two parts 1) isothermal $i \to b$ ← intermediate state

2) adiabatic $b \to f$

1) $PV = nRT$ ← T is constant

$$\Rightarrow P_i V_i = P_b V_b$$

$$\Rightarrow P_b = \frac{P_i V_i}{V_b} = \frac{P_i}{2}$$

2) $$P_f V_f^{\gamma} = P_b V_b^{\gamma} \Rightarrow P_f = P_b \left(\frac{V_b}{V_f}\right)^{\gamma} = \frac{P_i}{2}\left(\frac{2V_i}{V_i}\right)^{\gamma} = P_i 2^{\gamma-1}$$