$\boxed{\text{Lecture 4}}$

Kinetic theory of Gases (Ch. 19)

- focus more on the physics of gases

- gas consists of atoms that fill their container's volume & exert pressure on container's walls.

- can associate (p, V, T) w/ gas & these are a consequence of motion of atoms

- kinetic theory of gases relates atomic motion to (p, V, T)

1st question: How to measure the amount of gas in a sample?
Use Avogadro's number!

$$N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$$

tells us how many atoms or molecules per mole.

$\Rightarrow$ number n of moles obeys

$$n = \frac{N}{N_A}$$

N: number of molecules in sample

N_A: number of molecules per mole

n: number of moles

can also use mass of sample & molar mass M (the mass of 1 mol)

$$n = \frac{M_{sam}}{M} = \frac{M_{sam}}{m N_A}$$

M: molar mass

m: molecular mass

Ideal Gases

- we want to explain macroscopic behavior of gas (pressure & temperature) in terms of molecular behavior.

How to do so?

Assume low density of gas

Then all real gases obey the $\boxed{\text{ideal gas law}}$: (found experimentally)

$$pV = nRT$$

absolute pressure, volume, # of moles, gas constant, temp.

$$R = 8.31 \ \frac{J}{mol \cdot K}$$

law holds for any single gas or mixture of gases (@ low density) can write this in an alternative way in terms of Boltzmann constant k_B:

$$k_B = \frac{R}{N_A} = \frac{8.31\ J/mol\cdot K}{6.02\cdot 10^{23}\ mol^{-1}} = 1.38\times 10^{-23}\ \frac{J}{K}$$

also write as "k"

$$\Rightarrow \quad nR = Nk$$

\# of molecules

$\Rightarrow$

$$pV = nkT$$

(alternate way to write ideal gas law)

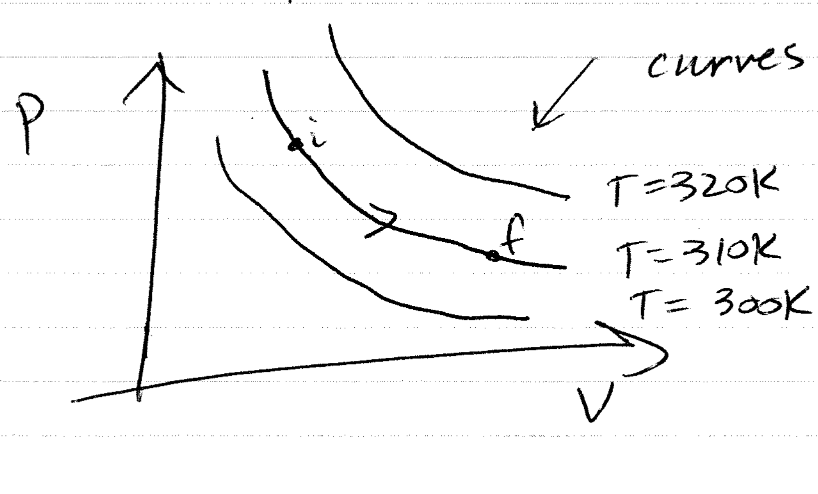
Work done by an Ideal gas @ constant temp.

- Put an ideal gas in piston-cylinder arrangement from before.
- Allow gas to expand from initial to final volume @ constant temp. (isothermal expansion)

then pressure @ a given instant is

$$p = \frac{nRT}{V}$$

on a p-V diagram, this looks like



curves are called "isotherms"

What is work done by gas during expansion?

$$W = \int_{V_i}^{V_f} p\, dV$$

$$= \int_{V_i}^{V_f} \frac{nRT}{V}\, dV$$

$$= nRT \left[\ln V \Big|_{V_i}^{V_f} \right]$$

$$= \boxed{nRT \ln \frac{V_f}{V_i}}$$

key formula for work done by gas during expansion

What about sign during compression?

What about work done during a constant volume process?

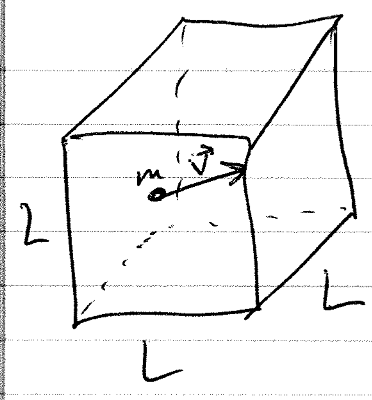
$$W = 0$$

constant pressure?

$$W = p(V_f - V_i) = p\,\Delta V$$

Pressure, Temp., & RMS Speed

- Develop kinetic theory to determine connection between pressure exerted by gas & speed of molecules
- Consider a single gas molecule of mass m & velocity $\vec{v}$ in a box



Suppose molecule collides w/ right wall & collision is elastic

molecule's change in momentum is along the x axis & is equal to

$$\Delta p_x = (-mv_x) - (mv_x) = -2mv_x$$

↑ final momentum ↑ initial momentum

⇒ momentum delivered to wall is

$$2mv_x$$

molecule will hit wall repeatedly & time Δt is time between collisions (travel back to right wall after hitting)

speed is v_x & distance is $2L$

$$\Rightarrow \Delta t = \frac{2L}{v_x}$$

average rate at which momentum is delivered is

$$\frac{\Delta p_x}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$

From Newton 2nd law, rate @ which momentum is delivered is the force ~~along~~ acting on the wall.

Suppose now that there are many molecules. Then pressure is total force divided by area

$$p = \frac{F_x}{L^2} = \frac{\frac{mv_{x1}^2}{L} + \cdots + \frac{mv_{xN}^2}{L}}{L^2}$$

$$= \frac{m}{L^3}\left(v_{x1}^2 + \cdots + v_{xN}^2\right)$$

N = # of molecules

$$= \frac{mN}{L^3}\left(\frac{V_{x_1}^2 + \cdots + V_{xN}^2}{N}\right)$$

$$= \frac{mN}{L^3}\left(V_x^2\right)_{avg.}$$

$$N = n N_A$$

$$\Rightarrow \quad p = \frac{n\, m\, N_A}{L^3}\left(v_x^2\right)_{avg}$$

molar mass

$$= \frac{n M}{V}\left(v_x^2\right)_{avg}$$

volume

Now take into account all directions of velocity $(x, y, \& z)$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

& since directions are random when averaged, we get

$$v_x^2 = \frac{1}{3} v^2$$

$$\Rightarrow \quad p = \frac{nM(v^2)_{avg}}{3V}$$ root-mean-square speed of molecules

also write

$$v_{rms} = \sqrt{(v^2)_{avg}}$$

$$\Rightarrow \quad p = \frac{nM\, v_{rms}^2}{3V} \Rightarrow pV = \frac{nM\, v_{rms}^2}{3}$$

Now use ideal gas law to get

$$pV = nRT$$

$$\Rightarrow \quad \frac{nM\, v_{rms}^2}{3} = nRT$$

$$\Rightarrow \quad v_{rms}^2 = \frac{3RT}{M}$$

$$\Rightarrow \quad v_{rms} = \sqrt{\frac{3RT}{M}}$$

Translational Kinetic Energy

$$K_{avg} = \left(\frac{1}{2}mv^2\right)_{avg} = \frac{1}{2}m(v^2)_{avg}$$

$$= \frac{1}{2}m\,v_{rms}^2$$

average translational kinetic energy

$$\Rightarrow K_{avg} = \frac{1}{2}m\,\frac{3RT}{M}$$

$$= \frac{3RT}{2N_A}$$

using Boltzmann constant

$$K_{avg} = \frac{3}{2}kT$$

All ideal gas molecules have the same average translational kinetic energy!