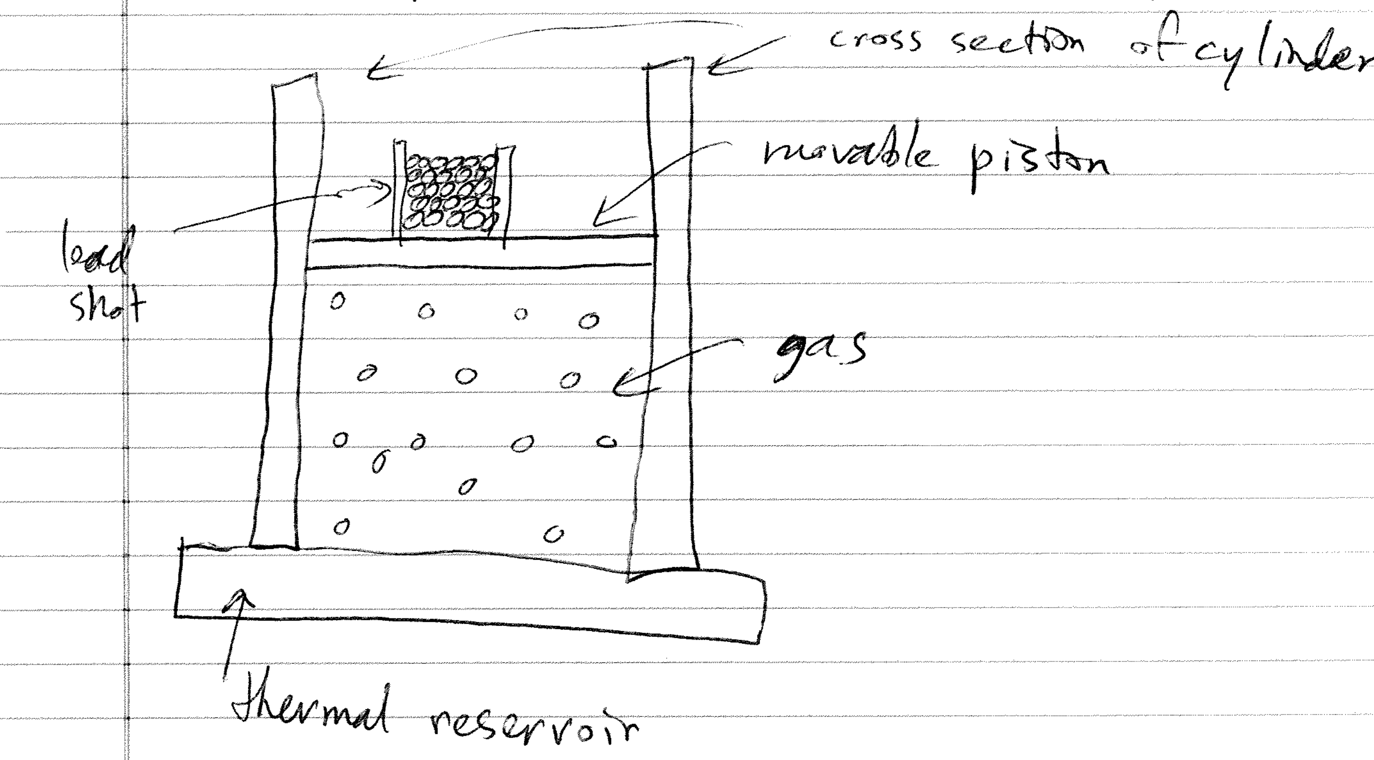
Lecture 3

First law of thermodynamics

Consider a gas confined to a cylinder w/ a movable piston:



Starting out, the upward force on piston due to gas is equal to the weight of the lead shot on top

- Walls of cylinder are made of insulating material that does not allow transfer of heat

- Bottom of cylinder rests on a thermal reservoir whose temperature T can be controlled

- Gas is the <u>system</u>

It starts out in an initial state i:

(p_i, V_i, T_i)

↑ pressure ↑ volume ↑ temperature

We can then change to a final state f:

(p_f, V_f, T_f)

- process by which we go from initial to final state is called a <u>thermodynamic process</u>

- During the process, heat energy can be transferred from the system to the reservoir (negative heat) or vice versa (positive heat) (w.r.t. system)

- Also work can be done by the system to raise piston (positive work) or lower it (negative work)

- Suppose all changes occur slowly, so that thermal equilibrium is maintained.

- Now suppose that we remove a few lead shot
- then the gas pushes the piston & remaining shot upward through a small displacement $d\vec{s}$ w/ an upward force $\vec{F}$ (assume constant)
- magnitude of $\vec{F}$ is pA where p is pressure & A is surface area of piston
- small amount of work done is then

$$dW = \vec{F} \cdot d\vec{s}$$

$$= p\underbrace{A\,ds}$$

$$= p\,dV$$

↑ differential volume

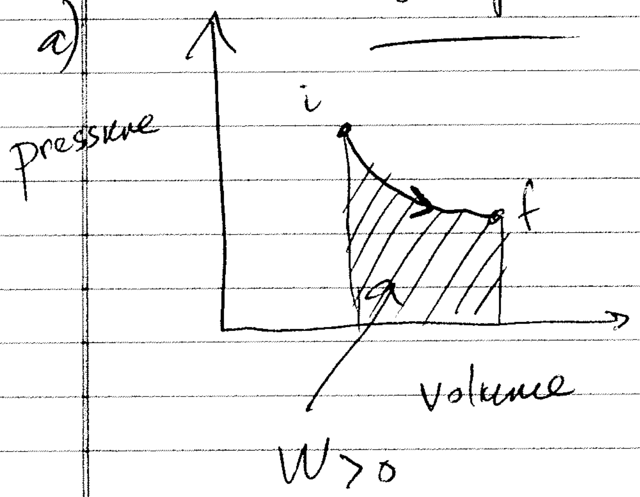
- When enough lead shot is removed, so that volume of gas changes from V_i to V_f, total work done by gas is

$$W = \int dW = \int_{V_i}^{V_f} p\,dV$$

- During volume change, pressure can change also. So we need to know how pressure changes w/ volume in order to evaluate integral.

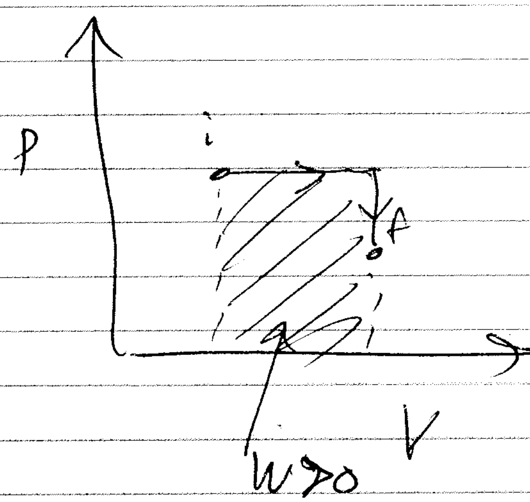
- We can use p-V diagrams for this & total work depends on path taken in this case

a) one path



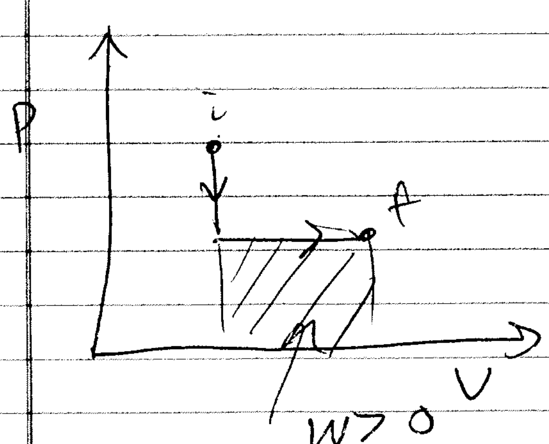
(due to direction of process)

b) another path

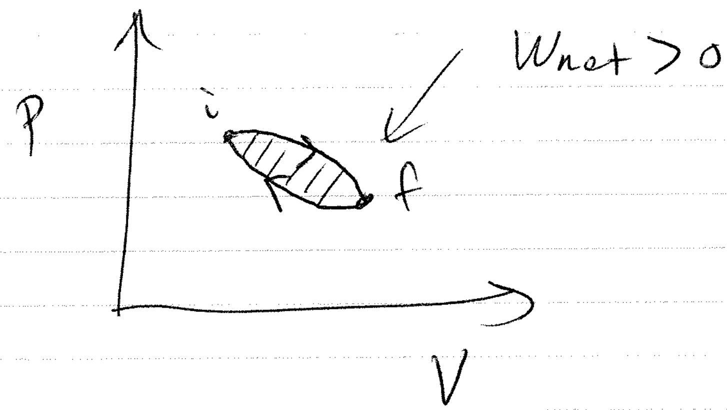


- 1st step is constant pressure, ~~[struck through]~~ done by increasing temperature.
- Next step is carried out @ constant volume by wedging piston
- Energy transferred as heat

c) reverse order of b)



⇒ heat + work are path-dependant quantities can also have a thermodynamic cycle



(direction of arrows is clockwise, so that $W_{net} > 0$)

<u>1st Law of thermodynamics</u>

$Q - W$ is <u>constant / same for all thermodynamic processes</u>

(↑ heat; ↓ work)

Also, $Q - W$ depends only on initial & final states (does not depend on path)

∴ <u>1st law</u>:

change in internal energy of a system is equal to $Q - W$:

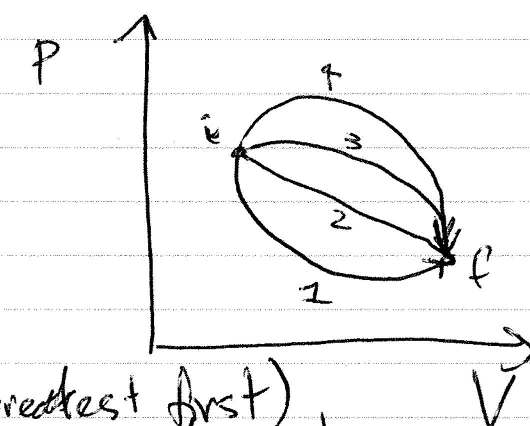
$$\Delta E_{int} = E_{int,f} - E_{int,i} = Q - W$$

<u>Differential version:</u>

$$dE_{int} = dQ - dW$$

Example:

Figure shows 4 paths on a p-V diagram along which a gas can be taken from state i to state f.



Rank paths according to (greatest first)

a) change in internal energy ΔE_{int}

$1 = 2 = 3 = 4$

b) Work done by the gas

$4 > 3 > 2 > 1$

c) magnitude of energy transferred as heat

$4 > 3 > 2 > 1$

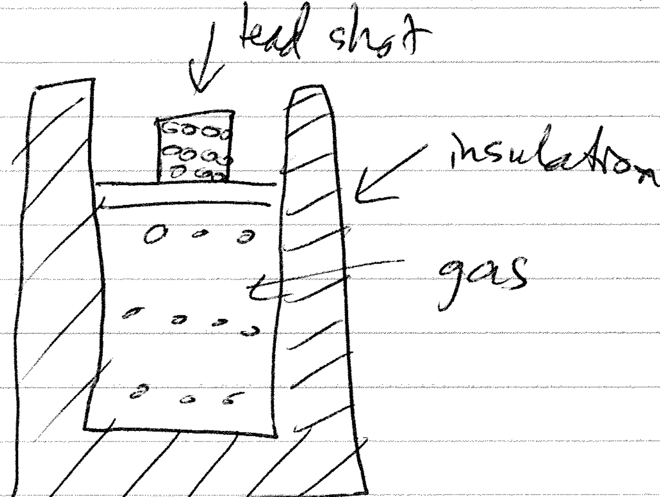
Special cases of 1st law:

1. Adiabatic process:

- occurs so rapidly or in a system so well insulated that there is no transfer of energy as heat.
- Setting $Q = 0$ gives

$$\Delta E_{int} = -W \quad \text{(adiabatic process)}$$

example:



heat cannot enter or leave due to insulation. If we remove shot, work done by system is positive + internal energy decreases

If we add shot, then
work done by system is negative
& internal energy increases.

2. Constant-volume process

If volume of a system is
held constant, it cannot do work,

$W=0$ b/c $W = \int_{V_1}^{V_2} p(V)\,dV$

$\Rightarrow$

$$\Delta E_{int} = Q$$

- If heat is absorbed, internal energy of system increases.
- If heat is lost, internal energy decreases.

3. Cyclical processes:

If the system is restored to its initial state, then internal energy does not change.

$\Rightarrow \Delta E_{int} = 0$

$\Rightarrow Q = W$ (cyclic process)

net work done = net amount of energy transferred as heat.

4. Free expansions:

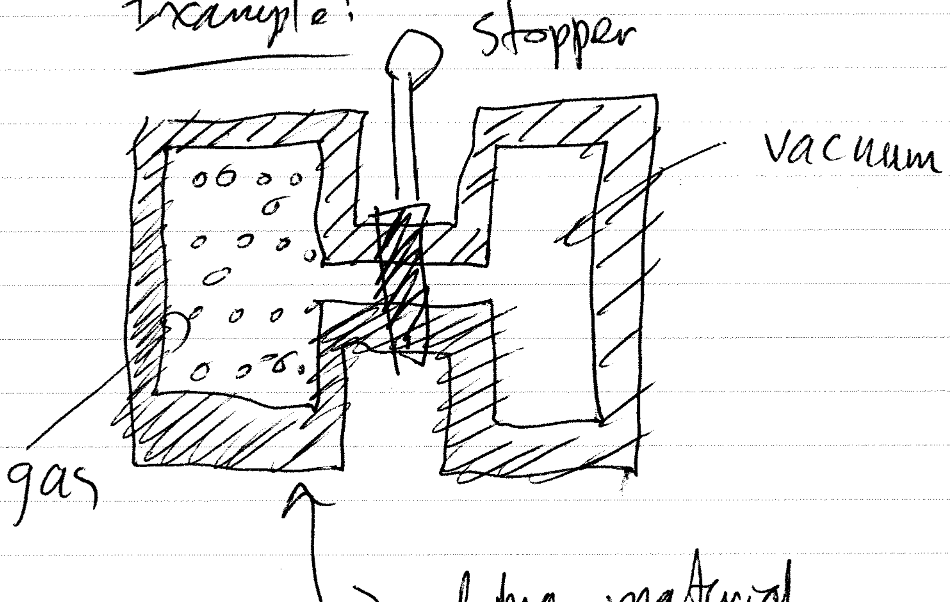
some adiabatic processes have no transfer of heat between system & environment & no work is done.

In this case, $Q = W = 0$.

then 1st law implies

$$\Delta E_{int} = 0$$

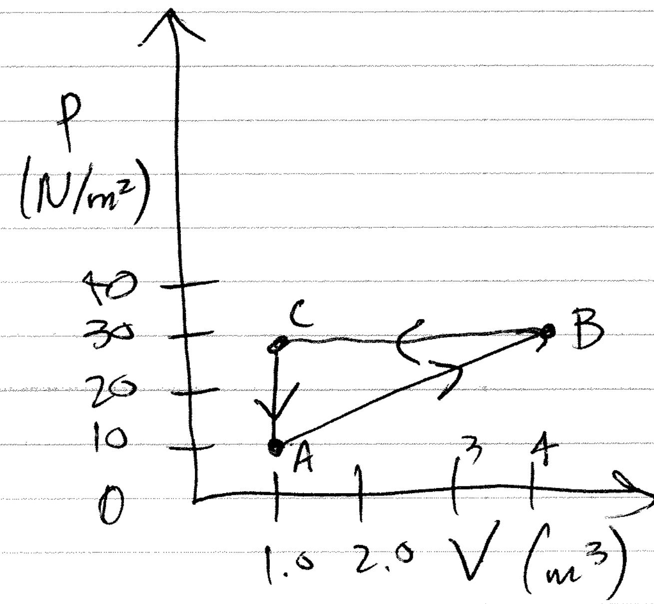
Example:



- open the stopper
- gas ~~goes~~ expands from left to right
- no heat because of insulation
- no work done b/c gas expands into vacuum & does not meet any pressure against it.

Example: (Review)

Calculate net energy added as heat during one complete cycle



$A \rightarrow B \rightarrow C$

due to cycle $\Delta E_{int} = 0$

$\Rightarrow Q = W$

so calculate W

$$W = W_{A\to B} + W_{B\to C} + W_{C\to A}$$

$$= \int_{V_A}^{V_B} P_{A\to B}\, dV + \int_{V_B}^{V_C} P_{B\to C}\, dV + \int_{V_C}^{V_A} P_{C\to A}\, dV$$

$$= \int_1^4 \left(\frac{20}{3}V + \frac{10}{3}\right) dV + \int_4^1 30\, dV$$

$$+ \int_1^1 P_{C \to A}\, dV$$

$$= \left[\frac{20}{3}\left(\frac{V^2}{2}\right) + \frac{10}{3}V\right]\Bigg|_1^4$$

$$+ \left[30V\right]\Big|_4^1 + 0 = -30\,\text{J}$$

$$\Rightarrow W = Q = -30\,\text{J}$$

Alternatively, just add up the area enclosed!

CCW - negative

CW - positive