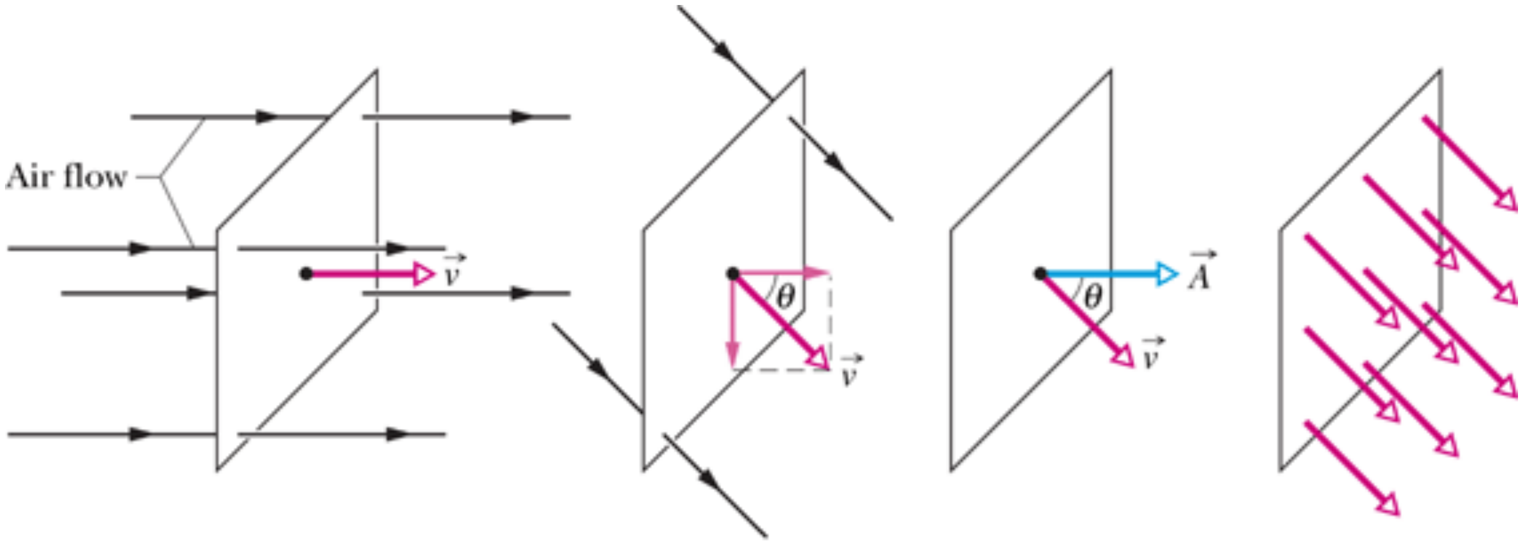
23.1-23.3

#8: Flux and Gauss' Law

Flux (in physics) refers to the product of a field crossing an area

Consider air flowing through a window (volume flux)

$$\Phi = vA\cos\theta$$



If we define an area vector, where the magnitude is equal to the area of a surface and the direction is perpendicular to the surface, then

$$\Phi = \vec{v} \bullet \vec{A}$$

The electric field flux is similarly defined: $\Phi = \vec{E} \bullet \vec{A}$

The Gaussian surface

Electric field is often not uniform across an area.

To find the total flux we have to add up the contribution to the flux from small elements all over a flux area.

The Gaussian surface

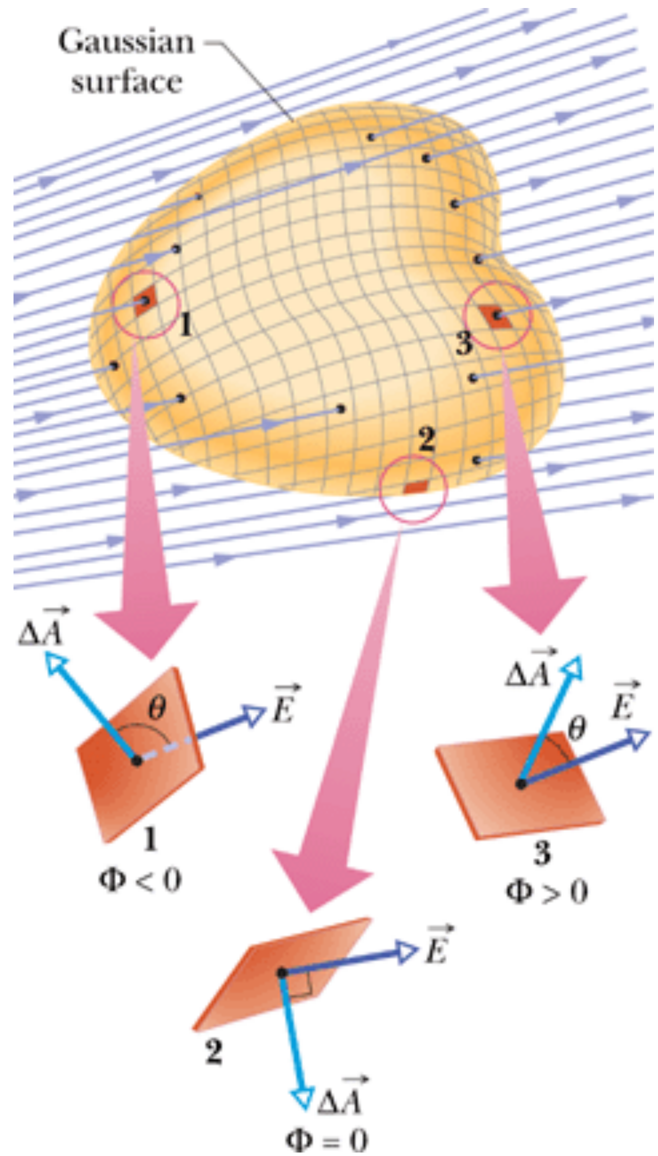
Imaginary surface that is completely enclosed (hollow with no holes)

A Gaussian surface can have any shape

Choosing a Gaussian surface that reflects the symmetry of a problem can greatly simplify your life

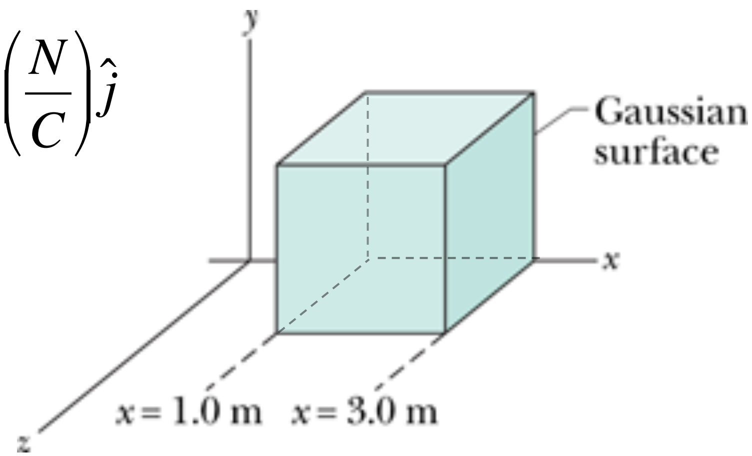
$$\Phi = \oint \vec{E} \bullet d\vec{A}$$

The electric flux through a surface is proportional to the number of electric field lines passing through the surface



Note: Direction of an area element always points “out”

A nonuniform electric field of $\vec{E} = 3x\left(\frac{N}{Cm}\right)\hat{i} + 4\left(\frac{N}{C}\right)\hat{j}$ exists everywhere. What is the electric flux through each face of the cube shown? Total?



$$\Phi_i = \int\left(3x\hat{i} + 4\hat{j}\right)\bullet\left(dA\hat{i}\right)$$
$$\Phi_i = \int 3xdA = 9\int dA = 9A = 36\, Nm^2/C$$

$$\Phi_{-i} = \int\left(3x\hat{i} + 4\hat{j}\right)\bullet\left(-dA\hat{i}\right)$$
$$\Phi_{-i} = -\int 3xdA = -3\int dA = -3A = -12\, Nm^2/C$$

$$\Phi_j = \int\left(3x\hat{i} + 4\hat{j}\right)\bullet\left(dA\hat{j}\right)$$
$$\Phi_j = \int 4dA = 4\int dA = 4A = 16\, Nm^2/C$$

$$\Phi_{-j} = \int\left(3x\hat{i} + 4\hat{j}\right)\bullet\left(-dA\hat{j}\right)$$
$$\Phi_{-j} = -\int 4dA = 4\int dA = 4A = -16\, Nm^2/C$$

$$\Phi_k = \int\left(3x\hat{i} + 4\hat{j}\right)\bullet\left(dA\hat{k}\right)$$
$$\Phi_k = 0$$

$$\Phi_{-k} = \int\left(3x\hat{i} + 4\hat{j}\right)\bullet\left(-dA\hat{k}\right)$$
$$\Phi_{-k} = 0$$

$$\Phi_{tot} = \Phi_i + \Phi_{-i} + \Phi_j + \Phi_{-j} + \Phi_k + \Phi_{-k}$$

$$\Phi_{tot} = 24\, Nm^2/C$$

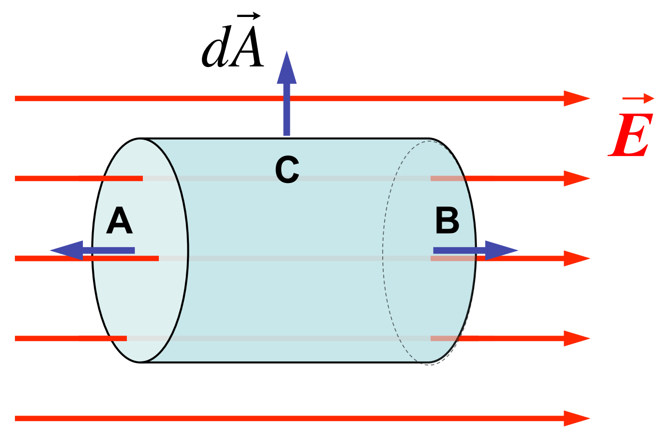
Gauss' Law

Gauss' Law: The integral of the electric flux over a Gaussian surface is proportional to the charge the surface encloses

$$\varepsilon_0 \Phi = \varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

This implies that charges external to a Gaussian surface do not contribute to the electric flux passing through the surface.

Consider a cylindrical Gaussian surface in a uniform electric field:



$$\oint \vec{E} \bullet d\vec{A} = \int_A \vec{E} \bullet d\vec{A} + \int_B \vec{E} \bullet d\vec{A} + \cancel{\int_C \vec{E} \bullet d\vec{A}}$$

$$\int_A \vec{E} \bullet d\vec{A} = EA\cos(\pi) = -EA$$

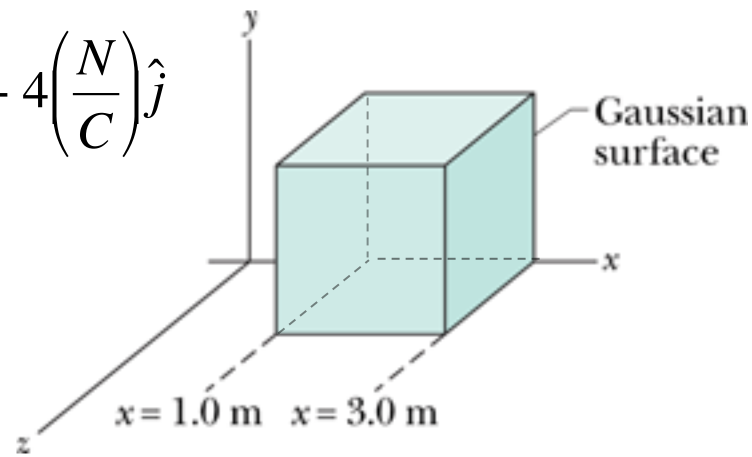
$$\int_B \vec{E} \bullet d\vec{A} = EA\cos(0) = EA$$

$$\oint \vec{E} \bullet d\vec{A} = 0$$

No charge enclosed

What's in the box?

A nonuniform electric field of $\vec{E} = 3x\left(\frac{N}{Cm}\right)\hat{i} + 4\left(\frac{N}{C}\right)\hat{j}$ exists everywhere. What is the electric flux through each face of the cube shown? Total?



$$\Phi_{tot} = \oint \vec{E} \bullet d\vec{A} = 24 \, Nm^2/C$$

Gauss' Law: $\varepsilon_0 \Phi = \varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$

$$q_{enc} = \varepsilon_0 \Phi = \left(8.85 \times 10^{-12} \frac{C^2}{Nm^2}\right)\left(24 \frac{Nm^2}{C}\right)$$

$$q_{enc} = 2.1 \times 10^{-10} C$$

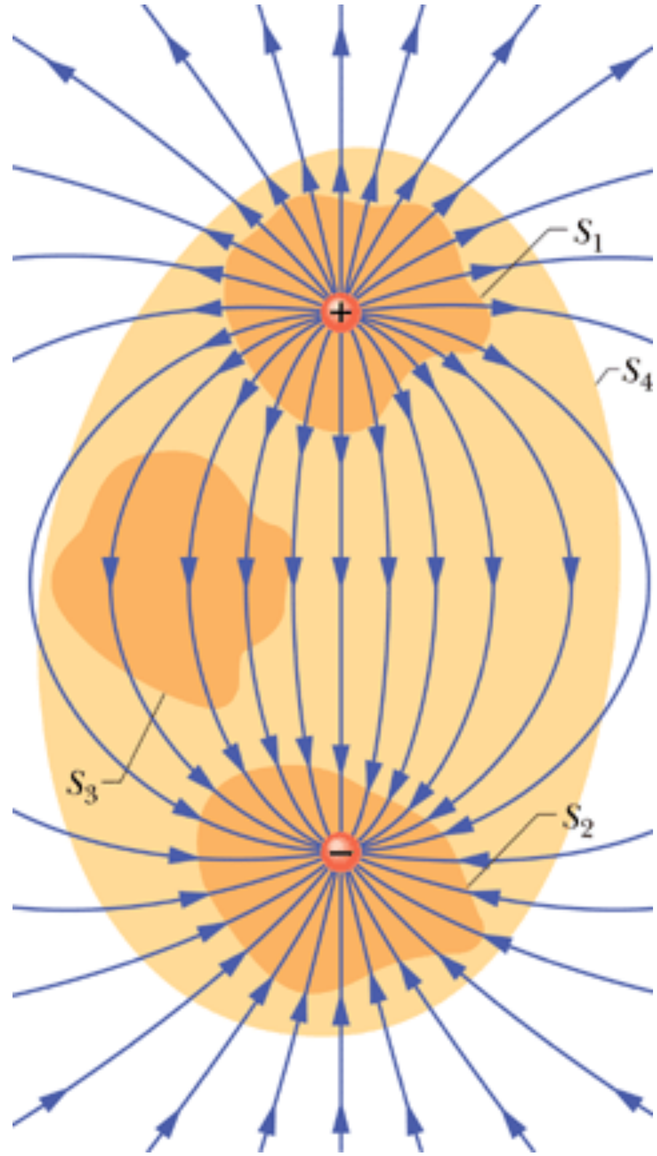
There is a net charge of +0.2 nC in the box!

A positive (outward) flux means that a net positive charge is enclosed.

A negative (inward) flux means that a net negative charge is enclosed.

The figure to the right shows an electric dipole and 4 Gaussian surfaces S_1, S_2, S_3 and S_4.

What can we say about the charge/flux through/within each surface?



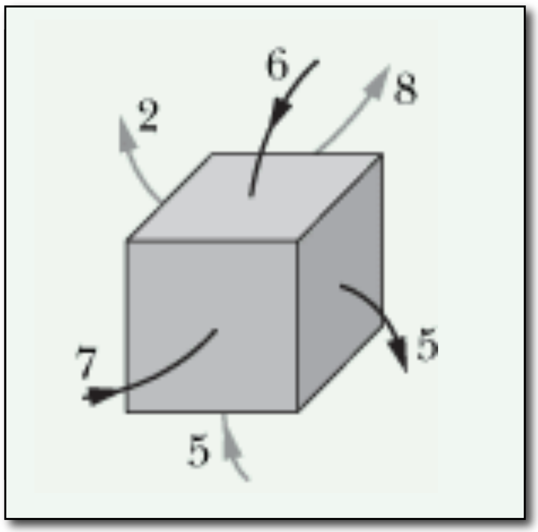
S_1 The flux is outward and a postive charge is enclosed.

S_2 The flux is inward and a negative charge is enclosed.

S_3 There is no net flux and no charge enclosed.

S_4 There is no net flux and no charge enclosed.

The figure below shows the flux into/out of the six faces of a cube in units of Nm²/C. What's in the box?



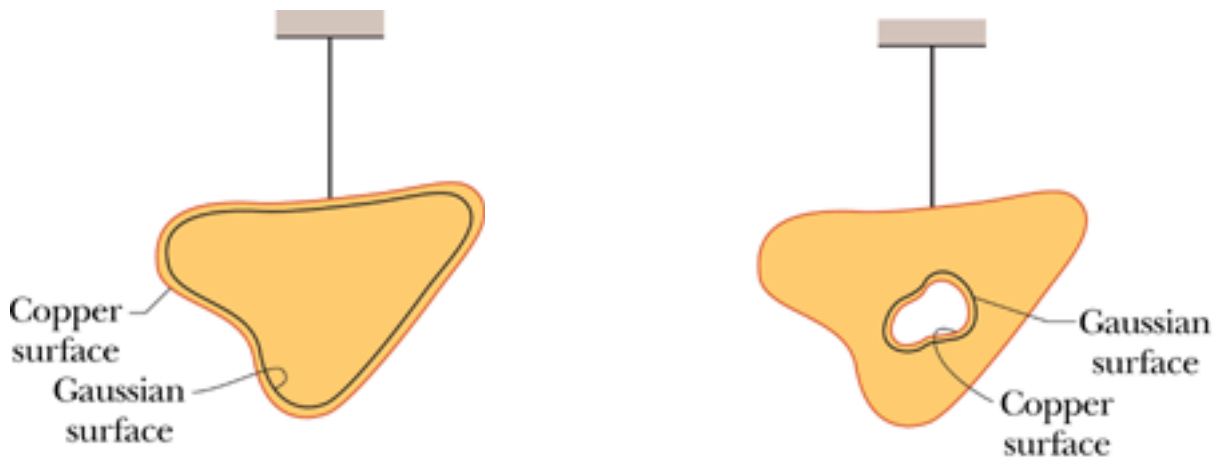
A. Positive charge
B. Negative charge
C. No charge

#9: Conductors & Cylindrical Symmetry

The electric field inside a conductor is zero ***if there is no current***.

Excess charge on a conductor is always found on the outer surface.

Imagine a Gaussian surface anywhere inside a conductor:



$\vec{E} = 0$ everywhere on the Gaussian surface

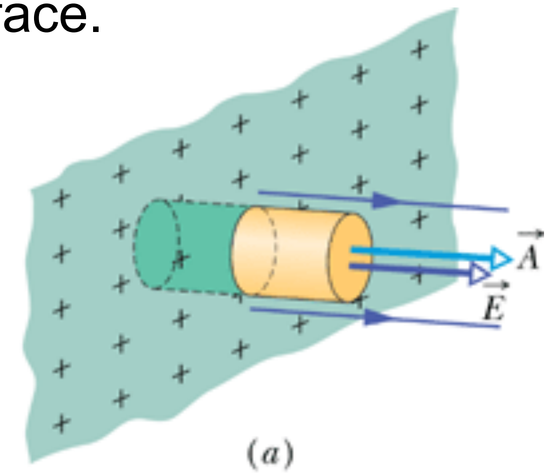
$$\Rightarrow \quad \varepsilon_0 \oint \vec{E} \bullet d\vec{A} = 0 = q_{enc}$$

There can be no net charge anywhere inside the conductor

Charges on a conductor are distributed on the surface.

The distribution of charges is generally complex - arranged so as to make the electric field perpendicular to the surface.

If the electric field were not perpendicular, then the charges would move along the surface.

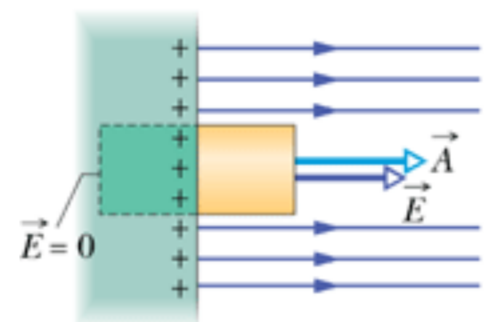


(a)

Imagine a cylindrical Gaussian surface that is half inside and half outside a conductor

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = \varepsilon_0 EA = q_{enc} = \sigma A$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$



(b)

Electric field just out side a conductor is perpendicular to the surface and proportional to the surface charge density.