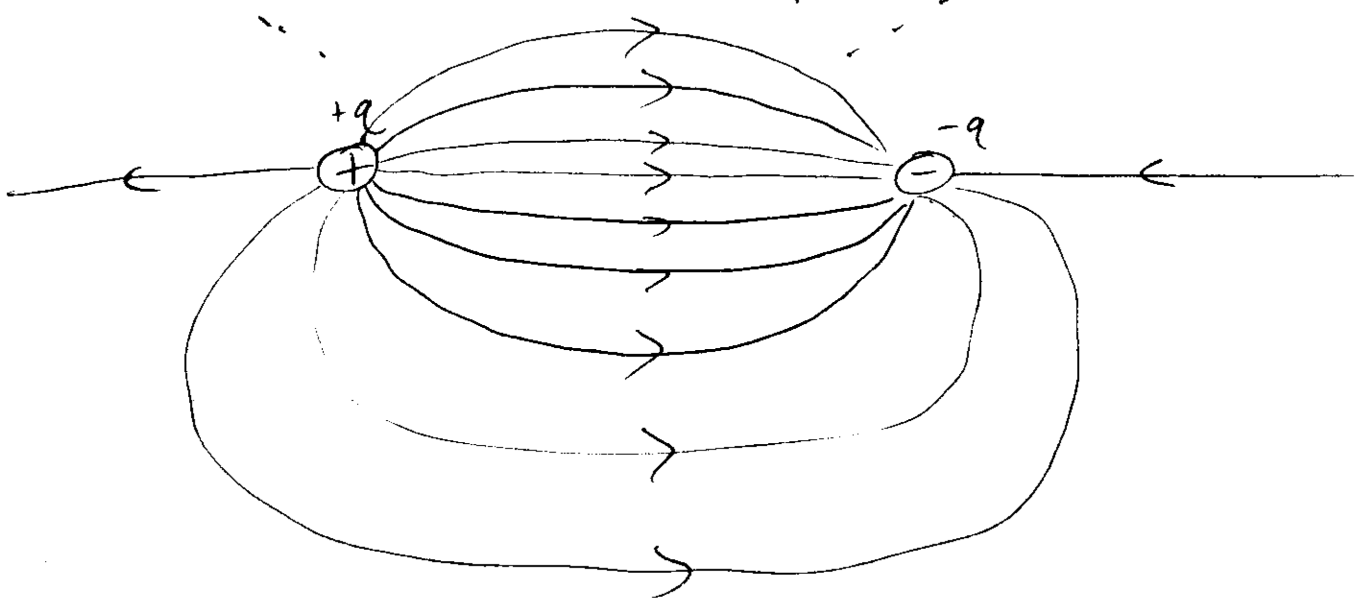
Lecture 7

10 SEP 2014

Recall electric dipole



Let d be the distance between charges

Then electric field along dipole axis goes as

$$2k \frac{qd}{z^3}$$

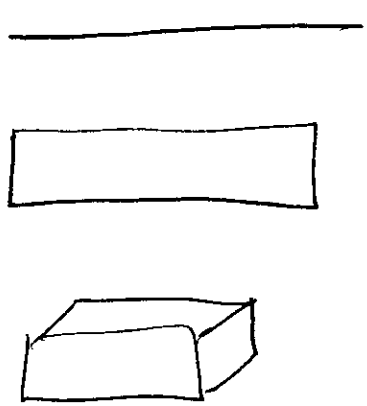
where z is distance from dipole center the product qd is the magnitude of a vector called the <u>electric dipole moment</u> $\vec{p}$

convention is that this vector points from negative to positive charge

direction of E-field along dipole axis is always in the direction of dipole moment vector

So far, we've only dealt w/ point charges

- objects that we deal w/ in reality can be approximated as continuous
- so we will be interested in computing electric field due to a line, area, & volume



we will require calculus to figure this out

- we will assume for simplicity that charge is uniformly spread over these objects.
- So, concept of charge <u>density</u> is useful

for lines, charge per unit length

$$\lambda = \frac{\text{total charge}}{\text{length of line}} = \frac{q}{L}$$

for areas, charge per unit area

$$\sigma = \frac{q}{A}$$

for volumes, charge per unit volume

$$\rho = \frac{q}{V}$$

To compute electric field due to a continuous charge distribution

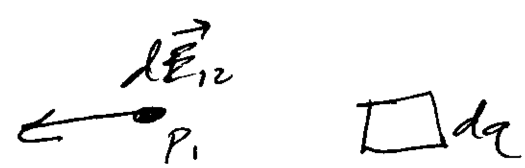
1) Divide the charge distribution into infinitesmally small differential elements

2) Treat each of these as a point charge & compute electric field

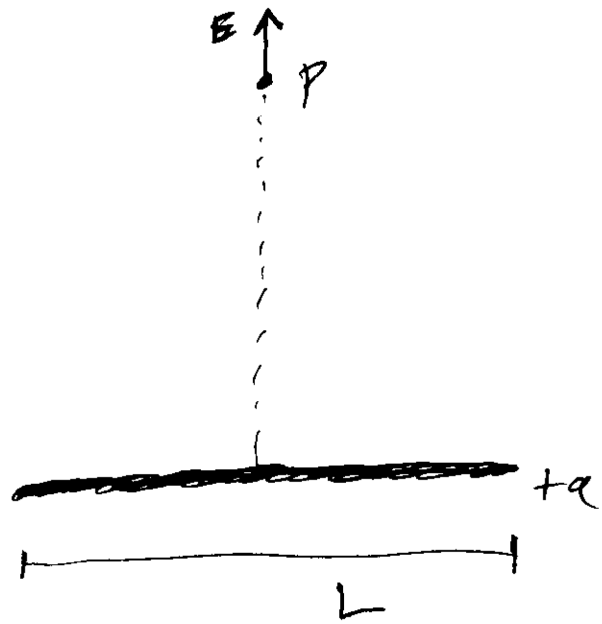
3) Integrate over these elements (sum)

use differential form of the Coulomb law for electric field

$$d\vec{E}_{12} = \frac{k\,dq}{r_{12}^2}\,\vec{r}_u$$



Example: uniform line of charge +q spread over length L



$\lambda = \frac{q}{L}$

Question: What is direction of E-field at point P?

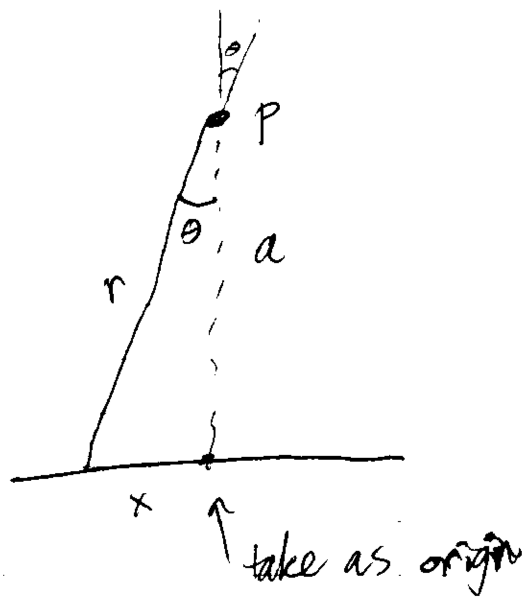
↑ b/c x componants cancel

So it suffices to calculate only the y components of the E-field due to different points along the line.

The differential element of charge is

$dq = \lambda\, dx$ (assumption of uniform charge)

$r = (a^2+x^2)^{1/2}$



$$dE = k \cdot \frac{dq}{r^2}$$

$$dE_y = dE \cos\theta$$

$$\cos\theta = \frac{a}{r} = \frac{a}{(a^2+x^2)^{1/2}}$$

$$\Rightarrow dE_y = \frac{k \cdot \lambda \cdot dx}{(a^2+x^2)} \cdot \frac{a}{(a^2+x^2)^{1/2}}$$

$$= \frac{k \cdot \lambda \cdot a \cdot dx}{(a^2+x^2)^{3/2}}$$

Apply principle of superposition & integrate over the whole line

$$E_y = \int_{-\frac{L}{2}}^{L/2} \frac{k \cdot \lambda \cdot a \cdot dx}{(a^2+x^2)^{3/2}}$$

$$= k \cdot \lambda \cdot a \int_{-\frac{L}{2}}^{L/2} \frac{dx}{(a^2+x^2)^{3/2}}$$

$$= k \cdot \lambda \cdot a \left[\frac{x}{a^2\sqrt{a^2+x^2}} \Bigg|_{-L/2}^{L/2} \right]$$

$$= \frac{2 \cdot k \cdot \lambda \cdot L}{a\sqrt{4a^2+L^2}}$$

Question: In the limit as $L \ll a$, what should the electric field look like? (point charge)

$$\rightarrow \quad \frac{k\lambda L}{a^2} = \frac{kq}{a^2}$$

What about when $L \gg a$?

Line charge limit

$$\rightarrow \frac{2k\lambda}{a}$$