Test 2 - Physics 2113 - Fall 2016

October 12, 2016

Last Name:______________________ First Name: KEY

TTH	12:0 pm Wilde Sec. 6	1:30 pm Zuniga-Hansen Sec. 7			
MWF	8:30am Rupnik Sec. 1	10:30am Stadler Sec. 2	12:30pm Blackmon Sec. 3	1:30pm Launey Sec. 4	2:30 pm Zuniga-Hansen Sec. 5

Be sure to write your name and circle your section.

Answer all 3 problems (25 points each) and 5 questions (6 points each).

Please read the questions carefully.

You may detach and use formula sheet provided at the back of this test. No other reference materials are allowed.

You are strictly forbidden from having access to any electronic communications device during a test. This includes cell phones, pagers, smartphones and tablet or notebook computers. You may not use calculator software on such a device during the test. Any student found with such a device will be assumed to be using it to cheat, and will be reported to the Dean of Students for disciplinary action. Any student who observes another student using such a device during the test should notify the instructor or proctor immediately.

Please use clear, complete sentences if explanations are asked for.

Some questions are multiple choice. You should work these problems starting with the basic equation listed on the formula sheet and write down the steps. Although the work will not be graded, this will help you make the correct choice and be able to determine if your thinking is correct. ***Be sure to mark your final answer clearly.***

On problems that are not multiple choice, you ***must show all of your work***. No credit will be given for an answer without explanation or work. These will be graded in full, and you are expected to show all relevant steps that lead to your answer.

YOU GET 60 min (1 hour)

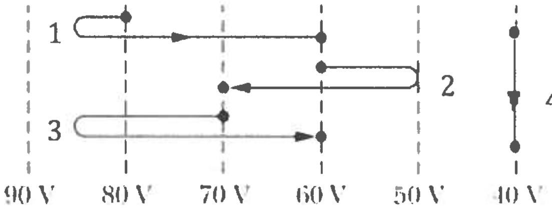
Question 1: [6 pts] The electric potential at a certain point is given by **$V = -7.5x^2 + 3y$**, where V is in volts and x and y are in meters. What is the **x component** of the electric field at point (x, y)?

[✓ A)] $E_x = 15x$
B) $E_x = -15x$
C) $E_x = -2.5x^3$
D) $E_x = 15x + 3$

$V = -7{,}5x^2 + 3y$

$E_x = -\frac{\partial V}{\partial x} = 15x$

Question 2: [6 pts] You move a proton from one equipotential surface to another along one of the four paths shown in the figure to the right. Rank the paths according to the work you need to do, **greatest** first (taking signs in account).



	ΔV(
1	-20
2	10
3	-10
4	0

A) 1 > 2 = 3 > 4
[✓ B)] 2 > 4 > 3 > 1
C) 4 > 1 > 2 > 3
D) 1 > 3 > 4 > 2

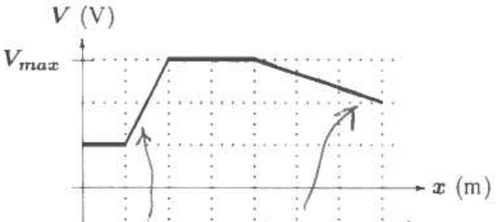
$W_{you} = \Delta U = q\Delta V \propto \Delta V$

$q = e > 0$

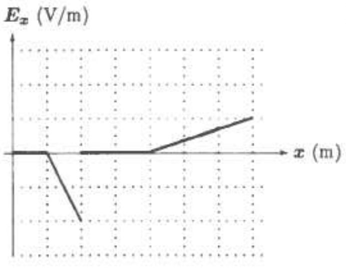
$\Delta V = V_f - V_i$

$\Rightarrow 2 > 4 > 3 > 1$

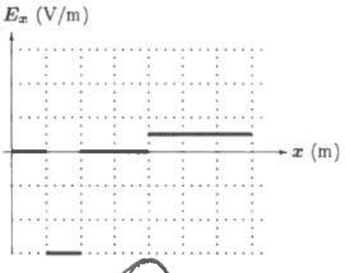
Question 3: [6 pts] The figure to the right shows electric potential ***V*** along an *x* axis. Choose the appropriate plot of the component of the electric field along the x axis, from the figure bellow and circle the best answer.



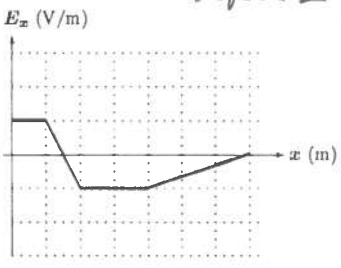
slope $\propto 2$ slope $\propto -\frac{1}{3}$



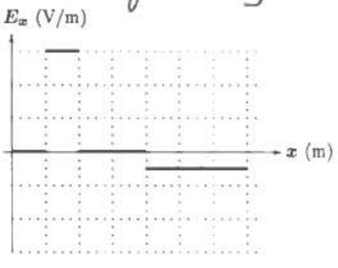
(a)



[✓ (b)]



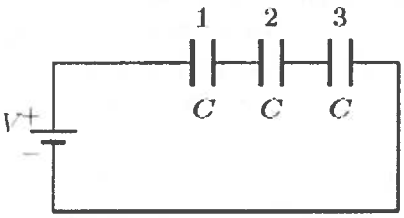
(c)



(d)

E_x … represented by the negative slope in V-x plot
(for V=const, $E_x = 0$)

Question 4: [6 pts] Figure to the right shows a battery of potential difference V, and three identical capacitors of capacitance C.
Circle the correct answer bellow!



Rank

(a) the potential difference across each capacitor.

○ $V_1 > V_2 > V_3$
○ $V_1 = V_3, V_2 = 0$
○ $V_1 < V_2 < V_3$
● $V_1 = V_2 = V_3$

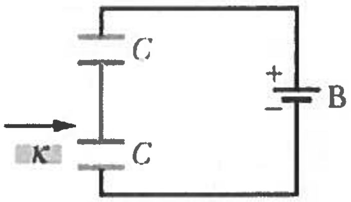
(b) the magnitude of the charge on each capacitor.

○ $q_1 < q_2 < q_3$
○ $q_1 = q_3,\ q_2 = 0$
○ $q_1 > q_2 > q_3$
● $q_1 = q_2 = q_3$

capacitors: connected in series: $q_1 = q_2 = q_3 = q_{eq}$, $V_1 + V_2 + V_3 = V$

$V_i = \frac{q_i}{C_i} = \frac{q_{eq}}{C_i} \propto \frac{1}{C_i}$ but all C_i are the same ⇒ $V_1 = V_2 = V_3 = V$

Question 5: [6 pts] A dielectric slab is inserted between the plates of one of the two identical capacitors in the figure. The battery is connected to the capacitors before, during the insertion, and after.
Circle **T** if the statement is correct and **F** if it is false.



A. The capacitance of the capacitor with the dielectric increases. [✓ T] F

B. The charge of the capacitor with the dielectric decreases. T [✓ F]

C. The potential of the capacitor with the dielectric remains the same. T [✓ F]

A: $C_K = K C_{air}$, $K > 1$ always ⇒ C_K increased

B: after: $q = q_K = q_{eq}$ ... because of the series connection

$q_{eq} = C_{eq} V$

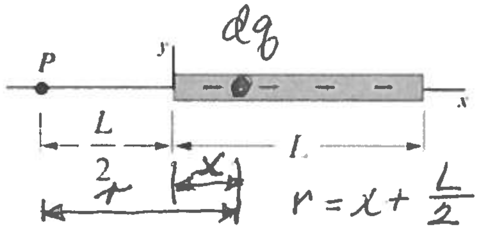
$C_{eq,\,before} = \frac{C}{2}$, $C_{eq,\,after} = \frac{C}{1+\frac{1}{K}} > \frac{C}{2}$ ... increased

⇒ q_{eq} increased ⇒ both q and q_K increased

C: $V = \frac{q}{C}$, C is the same, q increased ⇒ V of the C with air increased

$V_K = V_{battery} - V$ ⇒ V_K must decrease

(which is also true if there would be only one capacitor connected to the battery).



Problem 1 **(show your work)** [25 pts]: A thin charged rod of length ***L*** lies on the x-axis as shown in the figure. The linear charge density of the rod is ***λ = -c(x+L/2)***, where c is a constant in C/m^2. Point ***P*** is located at distance ***L/2***, to the left of the rod.

(a) [4 pts] Write down an expression for the infinitesimal charge ***dq*** at a distance ***x*** from the origin, in terms of ***x*** and the constants.

$$dq = \lambda\, dx = -c\left(x+\frac{L}{2}\right)dx$$

(b) [6 pts] Write down the potential ***dV*** at point **P** due to the infinitesimal charge ***dq*** at the distance *x* from the origin, in terms of ***x*** and the constants.

$$dV = k\frac{dq}{r} = k\frac{-c\cancel{\left(x+\frac{L}{2}\right)}dx}{\cancel{x+\frac{L}{2}}}$$

$$dV = -ck\,dx$$

(c) [10 pts] **Write down** the integral, with correct limits, to find the potential at **P** and **evaluate** it. Express your answer in terms of ***c, L*** and fundamental constants.

$$V = \int dV = -ck\int_0^L dx = -ckL$$

$$V = -ckL = -\frac{cL}{4\pi\varepsilon_0}$$

OR

(d) [5 pts] How much work does the electric field do to bring a point charge ***q₀*** from infinity, where the potential due to the rod is zero, to point ***P***? Express your answer in terms of ***c, L, q₀*** and fundamental constants.

$$W_{field} = -\Delta U = -q_0\,\Delta V = -q_0(V_f - V_i)$$

$$V_i = 0$$

$$W_{field} = -q_0\left(-\frac{cL}{4\pi\varepsilon_0}\right) = \frac{q_0 cL}{4\pi\varepsilon_0} = ckLq_0$$

OR

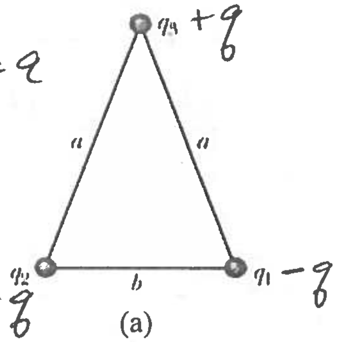
Problem 2 (show your work) [25 pts]: Three particles, having charges $q_1 = -q$, $q_2 = -q$, and $q_3 = +q$ are positioned at the vertices of an isosceles triangle as shown in figure (a).

A. (8 pts) What is the electric potential energy of the three charged particles system in figure (a)? Express your answer in terms of a, b, q, and fundamental constants, as needed.

$$U_{sys} = k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}} \qquad r_{12}=b,\ r_{13}=a,\ r_{23}=a$$

$$U_{sys} = kq^2\left(\frac{1}{b} - \frac{1}{a} - \frac{1}{a}\right)$$

$$U_{sys} = kq^2\left(\frac{1}{b} - \frac{2}{a}\right) = U_{sys,i} \underset{OR}{=} \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{2}{a}\right)$$



(a)

B. (8 pts) How much work do **you** need to do to exchange the positions of charges q_1 and q_3? The new positions are shown in figure (b). Express your answer in terms of a, b, q, and fundamental constants.

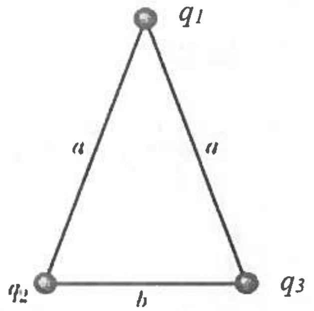
$$W_{you} = \Delta U = U_{sys,f} - U_{sys,i}$$

• term due to q_1 and q_3 is the same in U_i and U_f, so it cancels out;

$$W_{you} = \left(k\frac{q_1 q_2}{a} + k\frac{q_2 q_3}{b}\right) - \left(k\frac{q_1 q_2}{b} + k\frac{q_2 q_3}{a}\right)$$

$$= kq^2\left(\frac{1}{a} - \frac{1}{b} - \frac{1}{b} + \frac{1}{a}\right)$$

$$W_{you} = kq^2 2\left(\frac{1}{a} - \frac{1}{b}\right) \underset{OR}{=} \frac{q^2}{2\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$



(b)

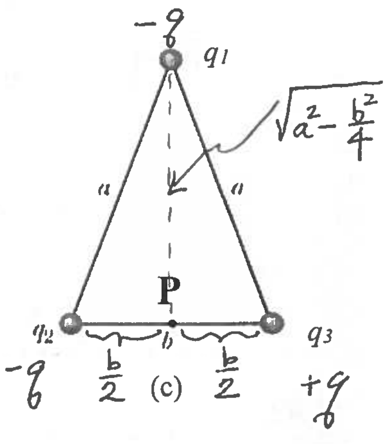
$r_{12}=a,\ r_{23}=b$
$(r_{13}=a)$

C. (9 pts) Find the electric potential at point **P** (in the middle of side b) due to the three charges, using the configuration shown in figure (c). Express your answer in terms of a, b, q, and fundamental constants.

$$V(P) = \sum V_i = k\frac{q_2}{b/2} + k\frac{q_3}{b/2} + k\frac{q_1}{\sqrt{a^2-\frac{b^2}{4}}}$$

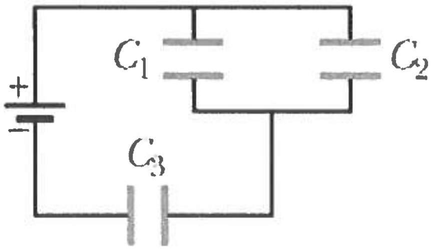
$$V(P) = kq\left(-\cancel{\frac{2}{b}} + \cancel{\frac{2}{b}} - \frac{1}{\sqrt{a^2-\frac{b^2}{4}}}\right)$$

$$V(P) = -\frac{kq}{\sqrt{a^2-\frac{b^2}{4}}} \underset{OR}{=} -\frac{q}{4\pi\varepsilon_0\sqrt{a^2-\frac{b^2}{4}}}$$



(c)

Problem 3 **(show your work)** (25 pts): In the circuit $C_1 = C_3 = 6\ \mu F$ and $C_2 = 4\ \mu F$. The battery supplies $V = 20$ V.



(a) [8 pts] What is the equivalent capacitance of the circuit?

C_1, C_2 in parallel: $C_{12} = C_1 + C_2$

C_{12} and C_3 in series:

$$C_{123} = \frac{C_{12}C_3}{C_{12}+C_3} = \frac{(C_1+C_2)C_3}{C_1+C_2+C_3}$$

$$C_{123} = C_{eq} = \frac{(6\mu F+4\mu F)(6\mu F)}{(6+6+4)\mu F} = \underline{3.75\mu F}$$

(b) [8 pts] What is the total charge on the equivalent capacitance?

$$q_{eq} = C_{eq}V = (3.75\mu F)(20\,V) = \underline{75\mu C = 7.5\times10^{-5}C}$$

(c) [9 pts] What is the energy stored by capacitor C_1?

$$U_1 = \frac{q_1V_1}{2} = \frac{C_1V_1^2}{2} = \frac{q_1^2}{2C_1}$$

- because C_3 is in series with C_{12}: $q_3 = q_{eq} = 75\mu C$

$$\Rightarrow V_3 = \frac{q_3}{C_3} = \frac{75\mu C}{6\mu F} \doteq 12.5V$$

- $V_1 = V - V_3 = (20-12.5)V = 7.5V$

$$U_1 = \frac{C_1V_1^2}{2} = \frac{(6\times10^{-6}F)(7.5V)^2}{2} = 1.6875\times10^{-4} \simeq \underline{1.7\ J}$$

OR: $q_1 = C_1V_1 = (6\mu F)(7.5)V = 45\mu C \Rightarrow U_1 = \frac{q_1V_1}{2} = \frac{(45\times10^{-6})(7.5)}{2}$

$$\underline{U_1 = 1.6875\ J(\times10^{-4})}$$