Entanglement-Assisted Capacity of Quantum Multiple Access Channels

Min-Hsiu Hsieh, Igor Devetak, Andreas Winter

Presented by Constance Caramanolis to the class of Comp 598 Winter 2011

Background

- Applying generalized Pauli matrices to the maximally entangled state, this gives the maximally mixed state.

$$\frac{1}{d^2}\sum_{m=1}^{d^2}(U_m \otimes I)\Phi^{AB}(U_m^{\dagger} \otimes I) = \pi^A \otimes \pi^B$$

- Transpose Trick

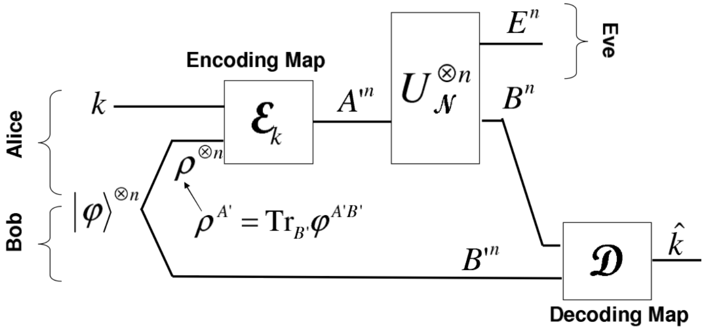
$$(I^A \otimes U^B)|\Phi\rangle^{AB} = (U^{tr\ A} \otimes I^B)|\Phi\rangle^{AB}$$

- Gentle Coherent Measurement

- Packing Lemma

EAC

- Prior to this paper, HSW was used to achieve the capacity $R = I(A;B)_\theta$
- The packing lemma and other tools achieve the same capacity.



Properties of the EAC

The new proof gives additional properties of the code

i) $\mathrm{Tr}\left\{[\mathcal{D}_k \circ ((\mathcal{N}^{\otimes n} \circ \mathcal{E}_k) \otimes I)](\varphi^{\otimes n})\right\} \geq 1 - \epsilon$;

ii) the encoded density operator satisfies $\mathcal{E}_k(\rho^{\otimes n}) = \rho^{\otimes n}$

iii) $\left\|[(\mathcal{D} \otimes I^{E^n}) \circ ((U_{\mathcal{N}}{}^{\otimes n} \circ \mathcal{E}_k) \otimes I) - (U_{\mathcal{N}}{}^{\otimes n} \otimes I)](\varphi^{\otimes n})\right\| \leq \epsilon$ where $\circ$ represents composition of two maps.

Theorem

Theorem

Consider a quantum multiple access channel $\mathcal{M} : A'B' \to C$. *For some states* $\rho_1^{A'}$ *and* $\rho_2^{B'}$ *define*

$$\theta^{ABC} = (I^{AB} \otimes \mathcal{M})(\varphi_1^{AA'} \otimes \varphi_2^{BB'})$$

where $|\varphi_1\rangle^{AA'}$ *and* $|\varphi_2\rangle^{BB'}$ *are purifications of* $\rho_1^{A'}$ *and* $\rho_2^{B'}$ *respectively. Define the two-dimensional region* $C_E(\mathcal{M}, \rho_1, \rho_2)$ *by the set of pairs of nonnegative rates* (R_1, R_2) *satisfying*

$$\begin{aligned} R_1 &\leq I(A; C|B)_\theta \\ R_2 &\leq I(B; C|A)_\theta \\ R_1 + R_2 &\leq I(AB; C)_\theta . \end{aligned}$$

Define $\tilde{C}_E(\mathcal{M})$ *as the union of the* $C_E(\mathcal{M}, \rho_1, \rho_2)$ *regions taken over all states* ρ_1, ρ_2.

Theorem continued

Theorem (cont.)

Then the entanglement-assisted capacity region $C_E(\mathcal{M})$ is given by the regularized expression

$$C_E(\mathcal{M}) = \overline{\bigcup_{n=1}^{\infty} \frac{1}{n} \tilde{C}_E(\mathcal{M}^{\otimes n})}$$

where the bar indicates taking closure. There is an additional single-letter upper bound on the sum rate

$$R_1 + R_2 \leq \max_{\rho_1,\rho_2} I(AB;C)_\theta.$$

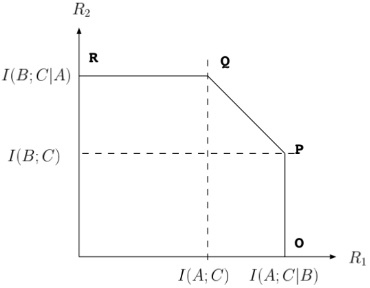
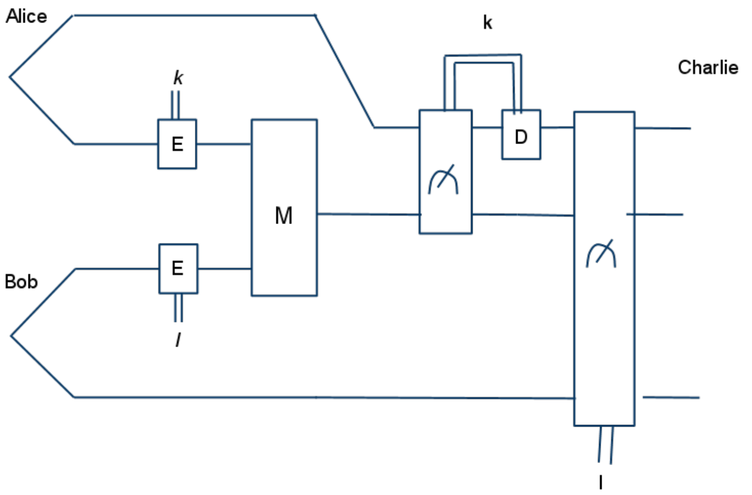


Figure: Capacity region of multiple access channel for fixed input states ρ_1 and ρ_2

The Channel

This is what it looks like



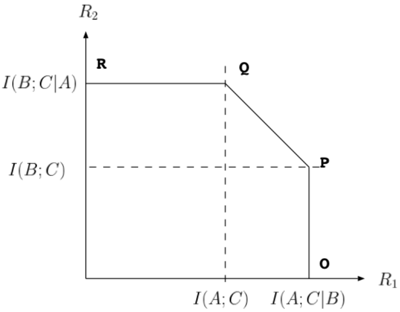
Successive Decoding

Means that Charlie will decode Alice's message then Bob's.

To the chalk board...

For the set of non negative rates (R_1, R_2), we get

$$R_1 \leq I(A; C\,|B)_\theta$$
$$R_2 \leq I(B; C\,|A)_\theta$$
$$R_1 + R_2 \leq I(A\,B; C)_\theta$$



The Collective Phase-Flip Example

The collective phase-flip channel is defined as

$$\mathcal{M}_p(\rho) = \sum_{k=0}^{d-1} p_k(\hat{Z}(k) \otimes \hat{Z}(k))\rho(\hat{Z}(k) \otimes \hat{Z}(k))^\dagger$$

When studying $\theta^{ABC} = (I^{AB} \otimes \mathcal{M}_p)(\Phi^{AA'} \otimes \Phi^{BB'})$, we get the following capacity for all pairs of nonnegative rates (R_1, R_2) which satisfy

$$\begin{aligned} R_1 &\leq 2 \log d \\ R_2 &\leq 2 \log d \\ R_1 + R_2 &\leq 4 \log d - H(p). \end{aligned}$$

Conclusion

- Can we single letterize the following

$$R_1 \leq I(A:C|B)_\theta \quad R_2 \leq I(B:C|A)_\theta$$

- Can we find a counter example to additivity to show that we aren't able to single letterize the above?
- Or are we able to do simultaneous decoding?
- If it is possible, to extend this to a multiway channel, s senders and r receivers?