13.3-13.5

#2: Gravity & Energy

Recall from Lecture 1:

$$m_e = 5.97\times10^{24}\,kg \qquad r_e = 6.37\times10^{6}\,m$$

$$\left|\vec{F}_b\right| = G\frac{m_e m_b}{r^2} = \left(6.67\times10^{-11}\,N\frac{m^2}{kg^2}\right)\frac{5.97\times10^{24}\,kg}{\left(6.37\times10^{6}\,m\right)^2}m_b = 9.8\frac{N}{kg}m_b$$

$$g = G\frac{m_e}{r_e^2}$$

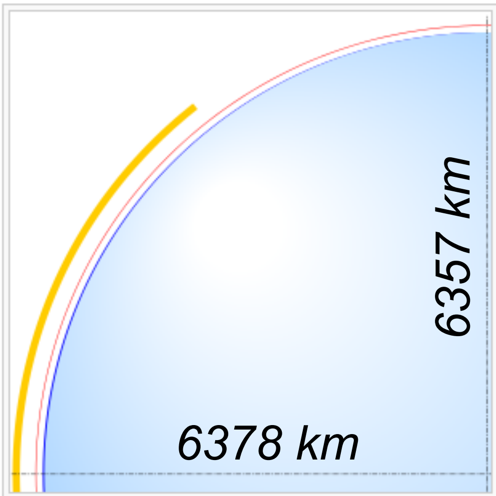
"Constant" g varies with distance from center of earth

Table 13-1 Variation of a_g with Altitude

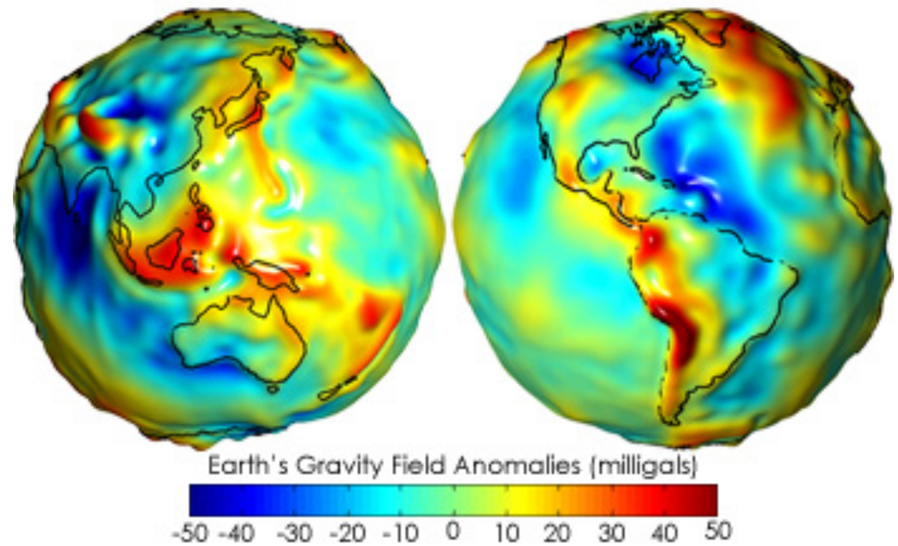
Altitude (km)	a_g (m/s^2)	Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest crewed balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

Reality is more complicated

1. Earth is not a perfect sphere



2. Earth's mass is not distributed uniformly

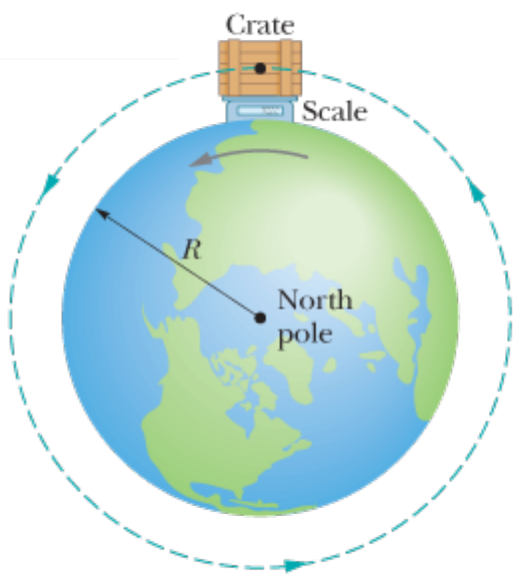


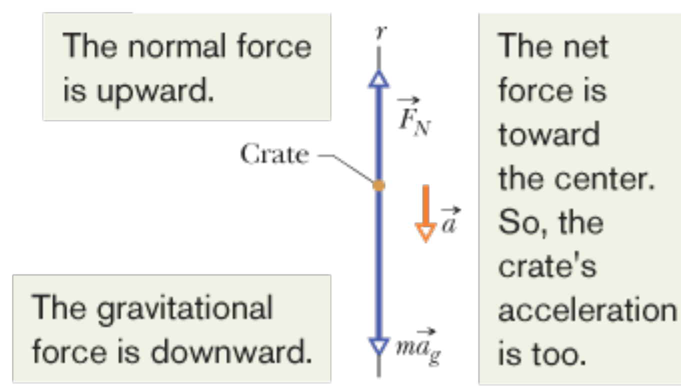
3. The Earth rotates

$$\vec{F}_{net} = \vec{F}_N - ma_g = -m\omega^2 r$$

$$\vec{F}_N = ma_g - m\omega^2 r$$

$$\omega^2 r = \left(\frac{2\pi}{86,400s}\right)^2 r_e = 0.03\frac{m}{s^2}$$





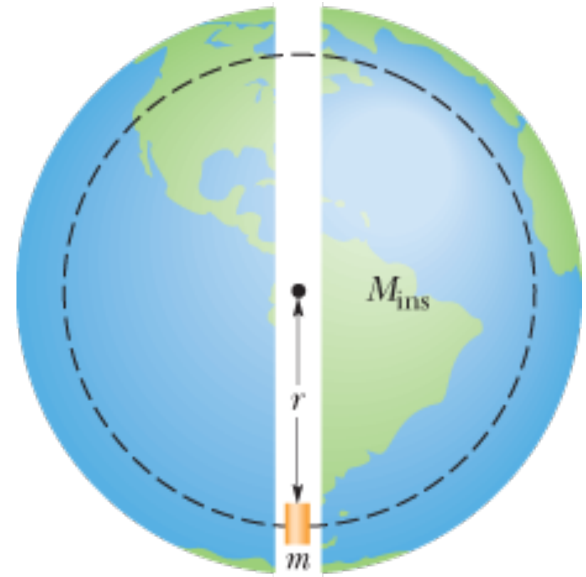
Let's take a journey through the earth.

As you go into earth, does the force that you feel from gravity:

A. Increase,

B. Decrease,

C. Stay the same?



At a radius r, only mass enclosed contributes

If the density of earth was uniform:

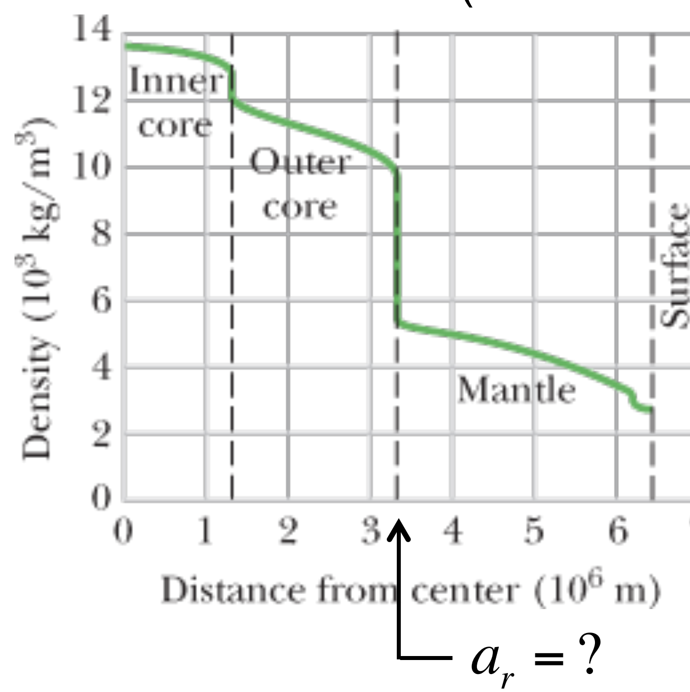
$$m_{enc} = \rho\left(\frac{4}{3}\pi r^3\right) = m_e \frac{r^3}{r_e^3}$$

$$a_r = G\frac{m_{enc}}{r^2} = G\rho\left(\frac{4}{3}\pi r\right)$$

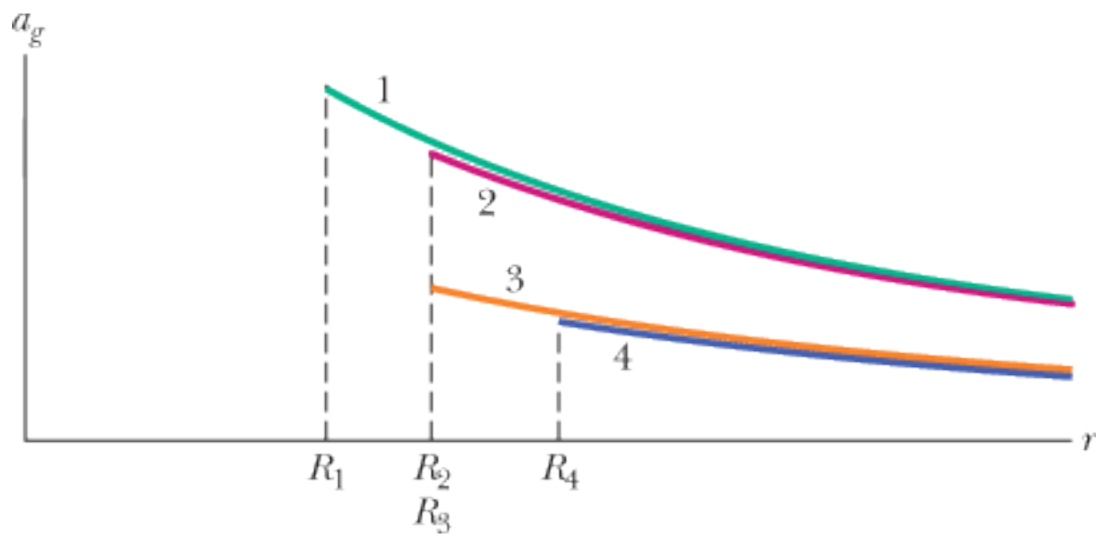
$$a_g = G\frac{m_e}{r_e^2} = G\rho\left(\frac{4}{3}\pi r_e\right)$$

$$\frac{a_r}{a_g} = \frac{r}{r_e}$$

But this is a bad assumption:
30% of mass in core (10% volume)

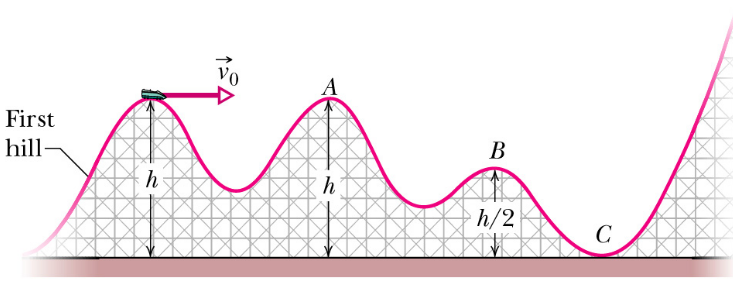


The figure gives the gravitational acceleration ag for four planets as a function of the radial distance r from the center of the planet, starting at the surface of the planet (at radius R1, R2, R3, or R4). Rank the four planets according to:
(a) mass and (b) mass per unit volume, greatest first.



Gravitational Potential Energy

Near the surface of the earth



$$\vec{F}_g = -mg\hat{j} = \text{constant}$$

$$W_g = \vec{F}_g \bullet \vec{d} = mg\left(h_i - h_f\right)$$

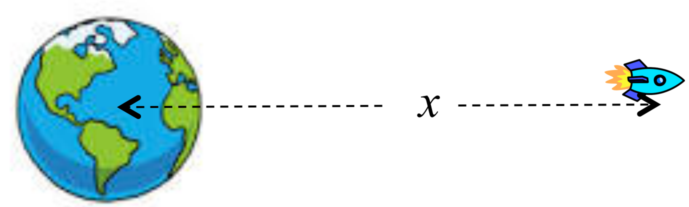
$$U_g = mgh \qquad W_g = -\Delta U$$

$E = K + U > 0$ for rocket to leave Earth

$$\frac{1}{2}mv^2 > G\frac{Mm}{r_i}$$

Escape speed $v > \sqrt{\frac{2GM}{r_i}} = (11 \text{ km/s})$

NOT Near the surface of the Earth



Let's send a rocket into space!

$$dW_g = \vec{F}_g \cdot d\vec{r} = -G\frac{Mm}{r^2}dr$$

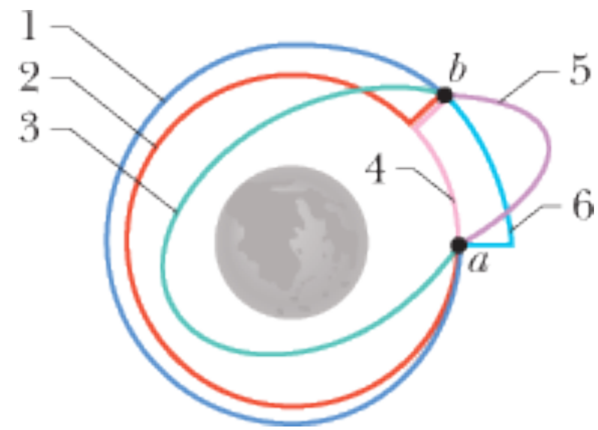
$$W_g = \int \vec{F}_g \cdot d\vec{r} = -GMm\int_{r_i}^{\infty} r^{-2}dr$$

$$W_g = GMmr^{-1}\Big|_{r_i}^{\infty} = -G\frac{Mm}{r_i}$$

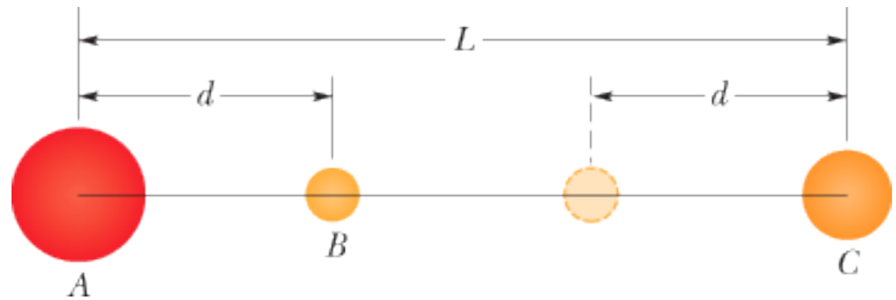
$$W_g = -\Delta U = U_i = -G\frac{Mm}{r_i}$$

$$U_g = -G\frac{Mm}{r}$$

The figure shows six paths by which a rocket orbiting a moon might move from point a to point b. Rank the paths according to
(a) the corresponding change in the gravitational potential energy of the rocket-moon system and (b) the net work done on the rocket by the gravitational force from the moon, greatest first.



The three spheres with masses m_A =80g, m_B =10g, and m_C =20g, have their centers on a common line, with L=12 cm and d=4.0 cm. You move sphere B along the line until its center-to-center separation from C is 4.0 cm. How much work is done on sphere B: (a) by you and (b) by the net gravitational force on B due to spheres A and C?



$$U_g = -G\frac{Mm}{r} \Rightarrow U_T = -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_2 m_3}{r_{23}} + \frac{m_1 m_3}{r_{13}}\right)$$ (3 particles)

The figure gives the potential energy function U(r) of a 25.0 kg projectile, plotted outward from the surface of a planet of radius R_s.

(a) What speed is required of a projectile launched at the surface if the projectile is to "escape" the planet? (b) If the particle is launched at half that speed, what will be its turning point relative to R_s?