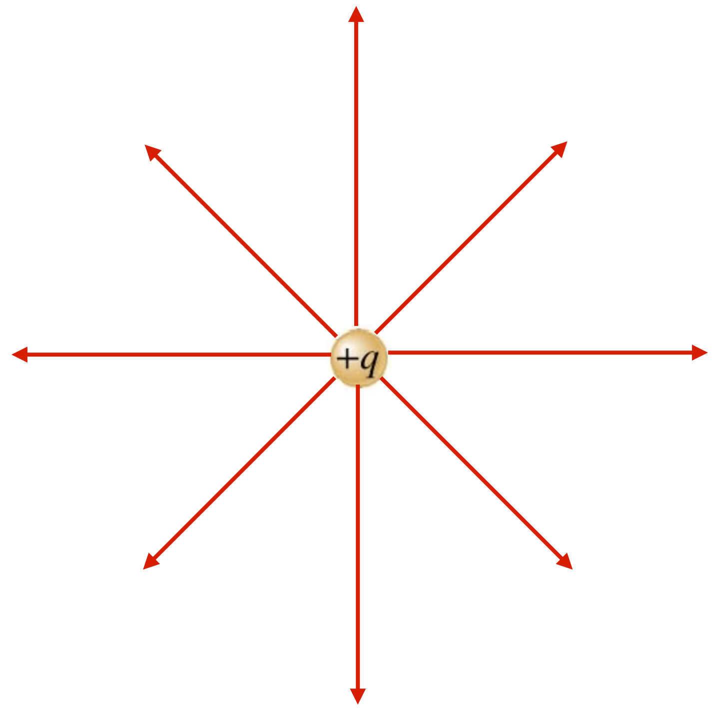
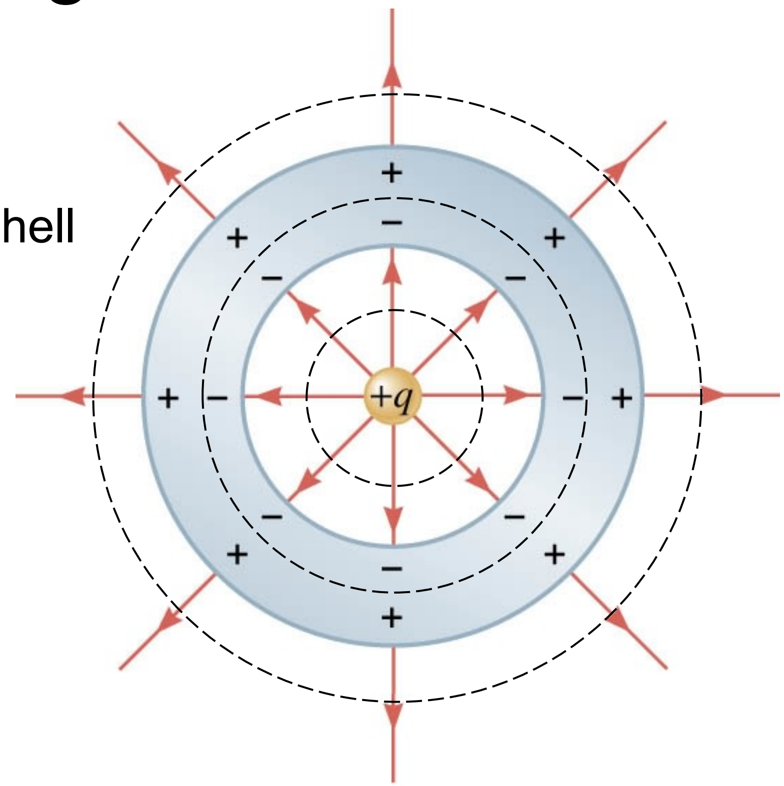
#10: Planar & Spherical

Imagine a point charge $\left|\vec{E}\right| = k\frac{q}{r^2}$



Hollow conducting shell

Imagine a point charge $\left|\vec{E}\right| = k\frac{q}{r^2}$



Now surrounded with a neutral conducting shell

What's the electric field?

There can be no electric field inside the conductor

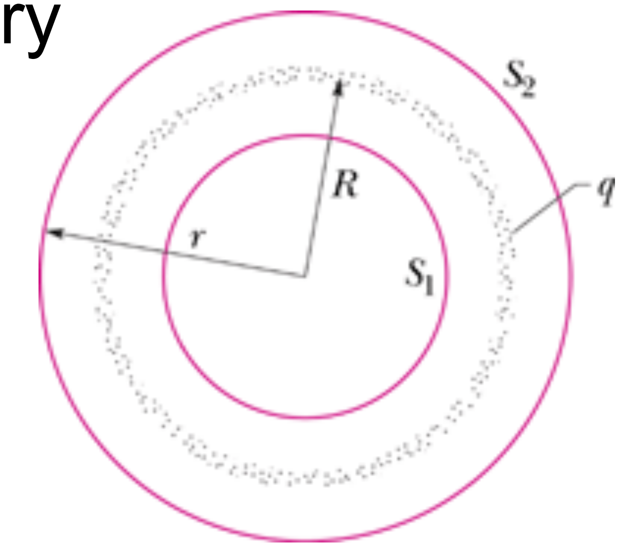
Field outside is the same

What if there are charges/fields outside?

The field inside the conductor is zero, but the conductor has no effect on the field outside the conductor!

A ***Faraday cage*** shield from fields originating from outside.
Fields originating inside a Faraday cage freely transmit through the cage
You cell phone needs an external antenna to receive, but not to transmit!

Spherical Symmetry



What if it is not a point charge, but a thin, uniformly-charged spherical shell (radius R)?

Same arguments for apply for r>R:

Spherical symmetry ⇨ $\vec{E} \bullet d\vec{A} = EdA$

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = \varepsilon_0 E\left(4\pi r^2\right) = q_{enc}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q_{enc}}{r^2}$$

Theorem 1: Any spherically symmetric charge distribution acts on particles outside the shell as a point charge at the center of the shell.

What about r<R?

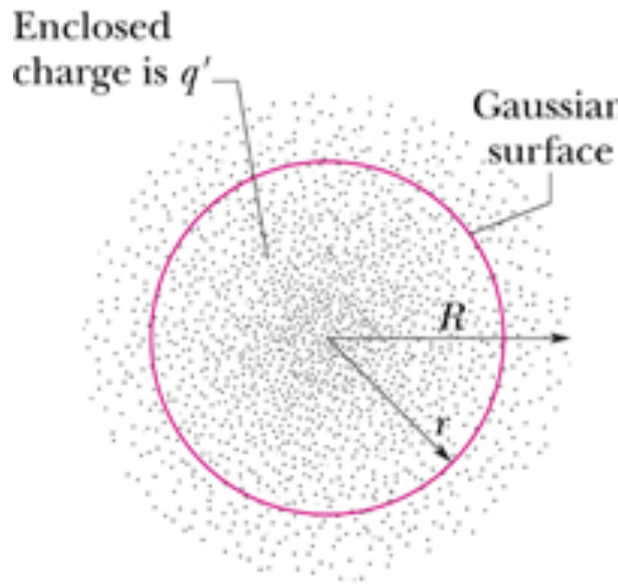
$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = \varepsilon_0 E\left(4\pi r^2\right) = q_{enc} = 0$$

$$E = 0$$

Theorem 2: Any spherically symmetric charge distribution exerts no force on a particle inside the shell.

Spherical Symmetry

Therefore, we can find the electric field within ***any*** sphericially symmetric charge distribution



$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^2}$$

For a uniform charge distribution:

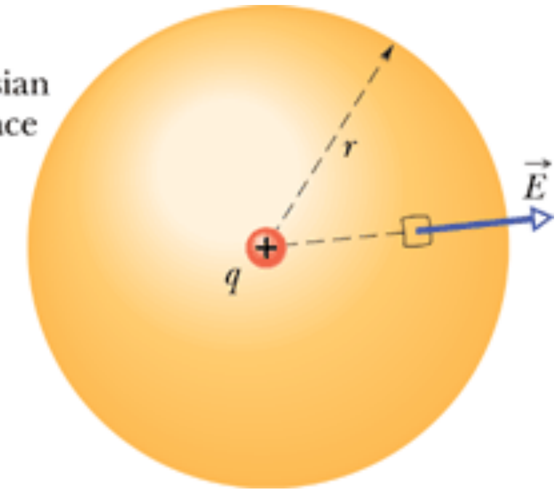
$$\rho = \frac{Q}{V} = Q\left(\frac{4}{3}\pi R^3\right)^{-1}$$

$$q_{enc} = \rho V = \rho\left(\frac{4}{3}\pi r^3\right) = Q\left(\frac{4}{3}\pi R^3\right)^{-1}\left(\frac{4}{3}\pi r^3\right) = Q\left(\frac{r}{R}\right)^3$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^2} = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3}$$

Coulomb's Law ⟺ Gauss' Law

Consider a spherical Gaussian surface of radius r about a point charge



$$\varepsilon_0 \Phi = \varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

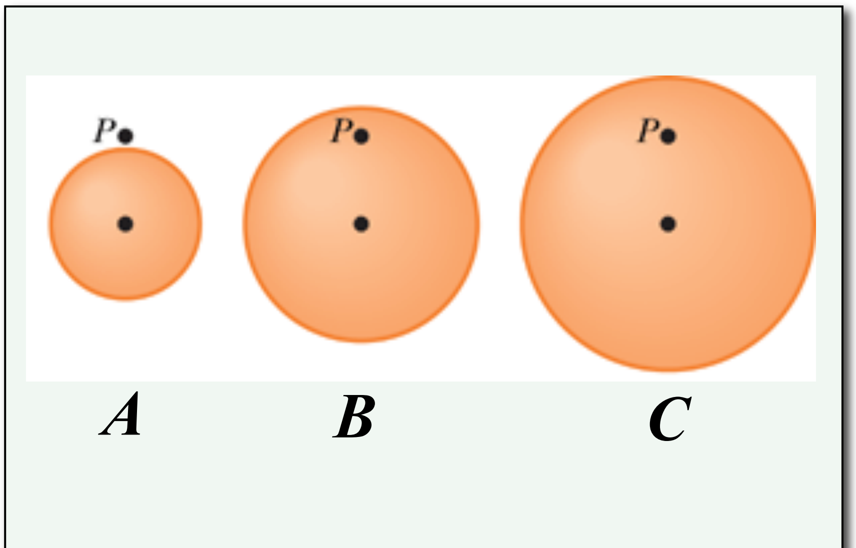
Spherical symmetry ⇨ $\vec{E} \bullet d\vec{A} = E dA$

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = \varepsilon_0 E \oint dA = q_{enc}$$

$$\varepsilon_0 E \left(4\pi r^2\right) = q_{enc}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q_{enc}}{r^2}$$

The figure below shows 3 solid ***uniform*** charge distributions each with total charge Q. Rank the electric field at Point P.



A. $E_A = E_B = E_C$

B. $E_A < E_B < E_C$

C. $E_C < E_B < E_A$

D. $E_B = E_C < E_A$

A solid sphere of radius ***a*** is concentric with a spherical conducting shell of inner radius ***2a*** and outer radius ***2.40a***. The sphere has a net uniform charge q_1; the shell has a net charge $q_2 = -q_1$. What is the magnitude of the electric field at radial distances (a) $r = 0$, (b) $r = 0.5\,a$, (c) $r = a$, (d) $r = 1.50a$, (e) $r = 2.30a$, and (f) $r = 3.50a$? What is the net charge on the (g) inner and (h) outer surface of the shell? (i) Finally, sketch the electric field as a function of radius.

A solid sphere of radius ***a*** is concentric with a spherical conducting shell of inner radius ***2a*** and outer radius ***2.40a***. The sphere has a net uniform charge q_1; the shell has a net charge $q_2 = -2q_1$. What is the magnitude of the electric field at radial distances (a) $r = 0$, (b) $r = 0.5\,a$, (c) $r = a$, (d) $r = 1.50a$, (e) $r = 2.30a$, and (f) $r = 3.50a$? What is the net charge on the (g) inner and (h) outer surface of the shell? (i) Finally, sketch the electric field as a function of radius.

The figure below shows a cross section through a very large nonconducting slab of thickness $d = 9.4\ mm$ and uniform volume charge density $\rho = 5.8\ fC/m^3$. The origin of the x axis is at the slab's center. What is the magnitude of the slab's electric field at $x = 0$, ***2.0 mm, 4.7 mm and 20 mm?***