Lecture 36 — 24 NOV 2014

Ch. 33 - Electromagnetic Waves

- biggest achievement of Maxwell was to prove mathematically that a beam of light is a traveling wave of electric + magnetic fields
- Heinrich Hertz proved experimentally that electromagnetic waves exist, validating the theory of Maxwell (thinking about the question on + off for a decade) 1879 - 1889
- Guglielmo Marconi was the first to use EM waves for communication.

  (Italian gov't officials referred him to an insane asylum for claiming this, at which point he moved to England) "to the Longara"

EM waves include

visible light, infrared, ultraviolet, radio waves, X-rays, gamma rays.

EM waves have a wavelength & frequency

wavelength (m)	10^{8}	10^{2}	10^{-3}	10^{-4}	10^{-5}
	long waves	radio waves	infrared		
frequency (Hz)	10^{2}	10^{6}	10^{12}	10^{13}	

(m)	10^{-6}	10^{-7}	10^{-8}	10^{-10}	
	visible		Ultraviolet	X rays	
(Hz)	10^{14}	10^{15}	10^{16}	10^{18}	

All EM waves travel @ same speed $c \approx 3.0 \times 10^8$ m/s

- nothing travels faster than c
- c is independent of frame of reference (principle of relativity)

Maxwell came to his conclusions about EM waves and light by purely mathematical considerations

Consider Maxwell equations in free space w/ no charges or currents

Then

$$\oint \vec{E} \cdot dA = 0 \ , \quad \oint \vec{B} \cdot d\vec{A} = 0 \ ,$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A} \ ,$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} \ ,$$

changing E-field gives B-field +
changing B-field gives E-field

equivalent form of the equations
in this case are

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

these two are coupled

take curl of 1st to get

$$\nabla \times \nabla \times \vec{E} = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t} \right)$$

$$= -\frac{\partial \left[\nabla \times \vec{B} \right]}{\partial t}$$

$$= -\frac{\partial}{\partial t} \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$= -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

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$$\nabla \times \nabla \times \vec{B} = \nabla \times \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$= \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left[\nabla \times \vec{E}\right]$$

$$= -\mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left[\frac{\partial \vec{B}}{\partial t}\right]$$

$$= -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

can then use identity
$\nabla \times \nabla \times \vec{B} = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B}$
to get
$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$
$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$

simple versions of these are

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

solutions are of the form of propagating waves

$$E = E_m \sin(kx - \omega t)$$

$$B = B_m \sin(kx - \omega t)$$

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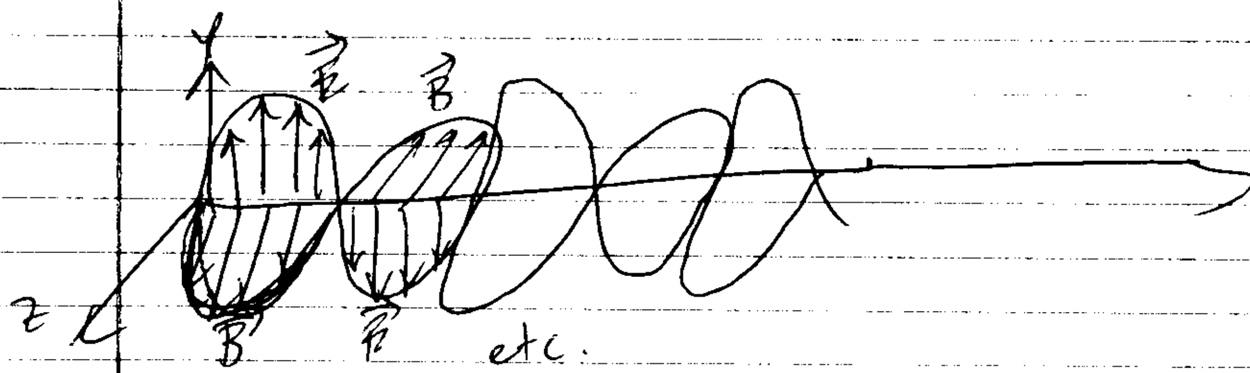
picture of a propagating EM wave is



$\dfrac{\omega}{k} = \dfrac{\text{time frequency}}{\text{wave number}}$ = speed of propagation (call this c for now)

take the x derivative twice to get

$-E_m k^2 \sin(kx - \omega t)$

take the t derivative twice to get

$-E_m \omega^2 \sin(kx - \omega t)$

plug in to get

$-E_m k^2 \sin(kx - \omega t) = \mu_0 \varepsilon_0 [-E_m \omega^2 \sin(kx - \omega t)]$

$\Rightarrow k^2 = \mu_0 \varepsilon_0 \omega^2$

$$\Rightarrow \quad \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 3.0 \times 10^8 \text{ m/s}$$

Maxwell then suggested that since the speed of propagation matches the speed of light that light itself should be an EM wave. (he thought it was much more than just a coincidence.)

proven correct much later...

key facts about EM waves:

1. E- & B-fields are always perpendicular to the direction in which the wave is traveling. Transverse wave
2. E-field is always perpendicular to B-field
3. cross product $\vec{E} \times \vec{B}$ gives direction of travel

4. fields vary sinusoidally +
are in phase w/ each other.