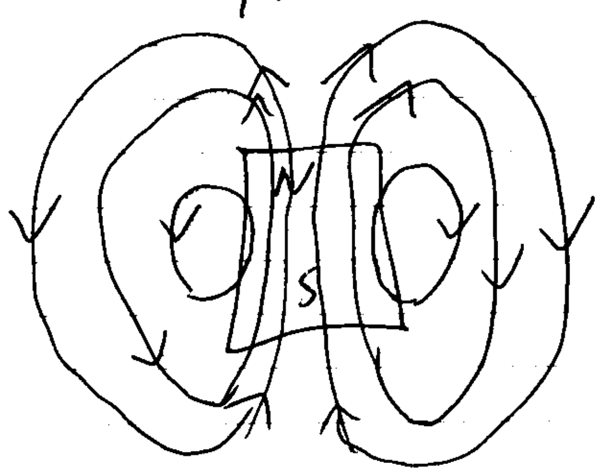
Lecture 34

17 Nov 2014

We can summarize ~~everything~~ much of what we have learned so far w/ Maxwell's equations. Four equations that govern electromagnetism

1) Gauss' law for magnetic fields

There are no sources or sinks for magnetic fields. No "magnetic charges" or "magnetic monopoles"

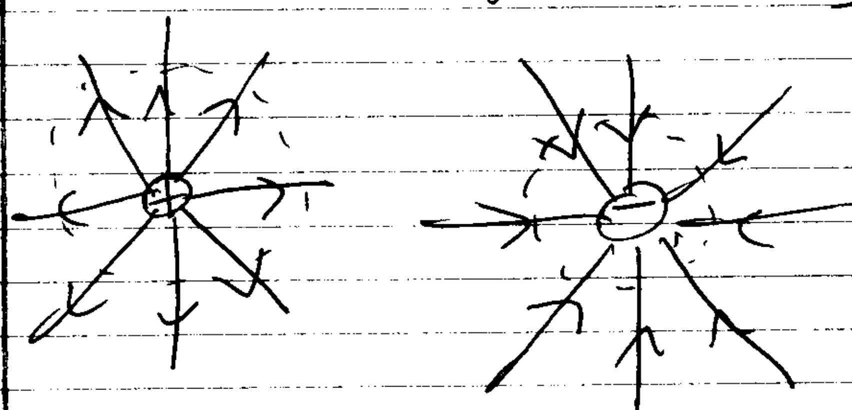


Pick any Gaussian surface + magnetic flux through it is equal to zero

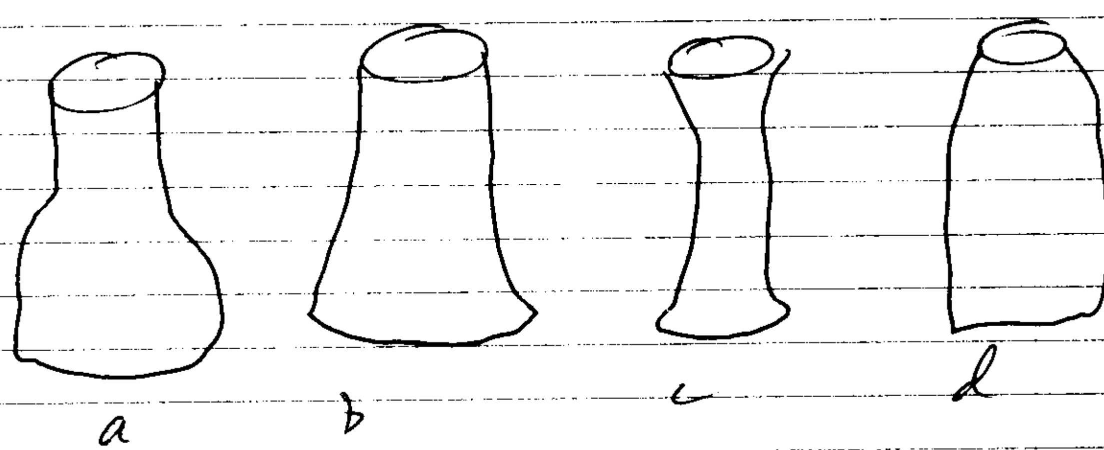
$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

can contrast this w/
Gauss' law for electric fields

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$



QUESTION: Given four surfaces



+ information

	A_{top}	B_{top}	A_{bot}	B_{bot}
a	2	6, out	4	3, in
b	2	1, in	4	2, in
c	2	6, in	2	8, out
d	2	3, out	3	2, out

Rank according to magnetic flux through curved part

$12 - 12 = 0$

$-2 - 8 = -10$

$-12 + 16 = 4$

$6 + 6 = 12$

Faraday's law of induction:

changing magnetic flux produces an EMF:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

~~equivalent to~~

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

↑ electric field along a closed loop

↑ changing flux encircled by the loop

same as changing magnetic field ~~[illegible]~~ induces an electric field

Can the opposite happen? yes!

That is, a changing electric field induces a magnetic field.

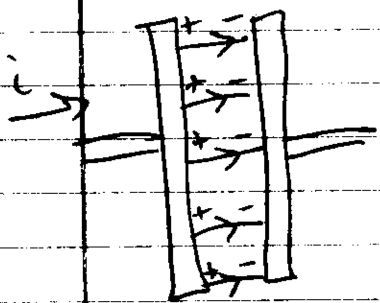
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

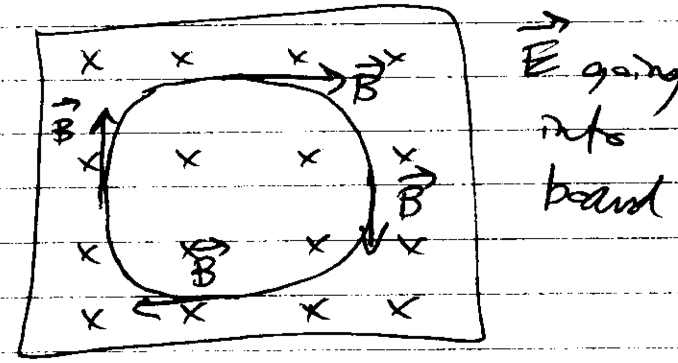
(Maxwell's law of induction — has similar form to Faraday law)

magnetic field along a closed loop

electric flux encircled by loop

Example of this: consider a parallel plate capacitor that is changing other view





induced $\vec{B}$ - field as

E-field is changing (as capacitor is changing)

Since there is no sign, induced B-field follows right hand rule.

However, recall before the Ampere law

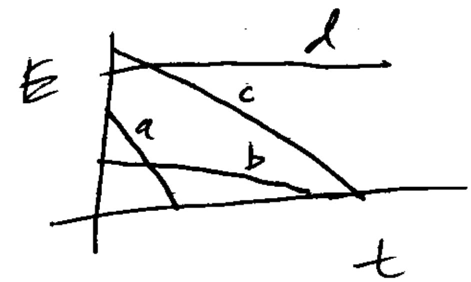
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

this is still valid. There is a more general law of which both of these are a special case:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

QUESTION:

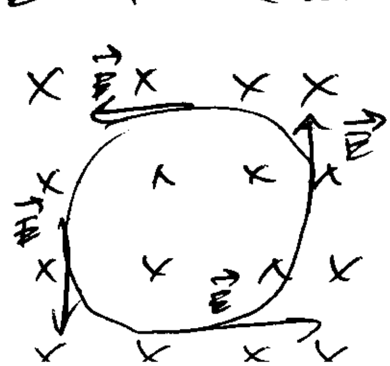
Figure shows graph of four changing, uniform electric fields all contained w/in an identical region.
Rank according to induced magnetic field



a > c > b > d

(highest to smallest slopes)

If we have changing B-field, what are directions of induced E-field around a loop?



increasing B-field into page

QUESTION: When a parallel plate capacitor is changing, there is both an E-field & B-field between plates. After some time, charging stops, & what can we say about magnetic & electric fields?

B-field is zero & E-field is constant

Considering equation

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \left[\varepsilon_0 \frac{d\Phi_E}{dt} + i_{enc} \right]$$

↑ this has dimensions of current

this quantity is called "displacement current"

can say something about this for parallel plate capacitor

we can relate real current in circuit to displacement current associated w/ changing E field.

change q on plates is given by

$$q = \varepsilon_0 A E$$

⇒ real current is

$$i = \frac{dq}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

we can use $i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$

to get displacement current.

$\Phi_E = EA$

$\Rightarrow i_d = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt}$

so they are equal.

can figure out induced magnetic field