Lecture 14

26 SEP 2014

Most important results from last time:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = -\frac{W}{q}$$

electric potential difference between ~~initial~~ final & initial locations is equal to $-\frac{W}{q}$

electric potential @ a distance R from charge q is

$$V = \frac{kq}{R}$$

using principle of superposition, we showed that net potential due to a group of charges is the algebraic sum of potentials due to individual charges

$$V_{net} = \sum_j V_j = k \sum_j \frac{q_j}{r_j}$$

potentials due to a continuous
charge distribution
(need to use calculus)

Recipe: 1) break the charge distribution into small elements, each of which we model as point charges (tiny amount of charge dq)

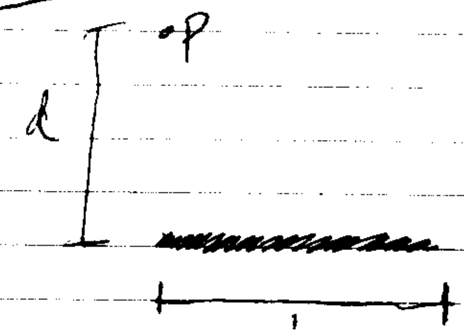
2) figure out potential due to each of them $dV = \frac{k \cdot dq}{r}$

3) Sum them up (integrate)

$$V = \int dV = k \int \frac{dq}{r}$$

Example: Suppose a line of charge

calculate potential @ point P

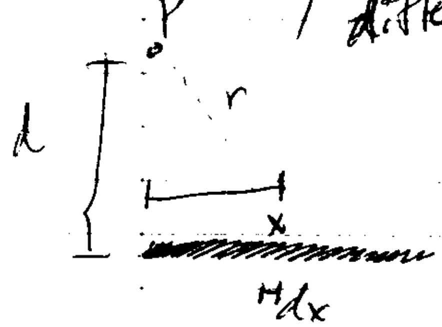


charge q, length L
uniformly distributed charge

<u>Question</u>: How to approach problem?

1) Consider a differential element dx along the line (think of it as a point charge)

differential charge due to it is $dq = \lambda\, dx$ where $\lambda = \frac{q}{L}$



2) differential Potential at location x is

$$dV = \frac{k\,dq}{r} = \frac{k \cdot \lambda \cdot dx}{(d^2+x^2)^{1/2}}$$

(this dV is positive b/c charge on rod is positive)

3) Integrate over the length of the line of charge from $x=0$ to L

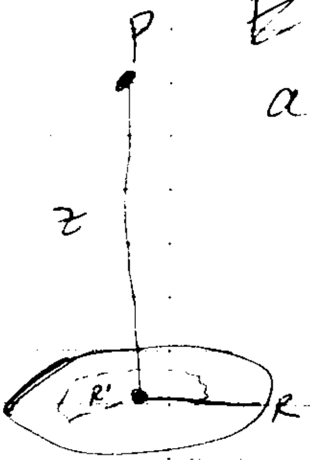
$$V = \int dV = \int_0^L \frac{k \cdot \lambda \cdot dx}{(d^2+x^2)^{1/2}}$$

$$= k\cdot\lambda \int_0^L \frac{dx}{(d^2+x^2)^{1/2}}$$

$$= k\cdot\lambda \left[\log\left(x+(x^2+d^2)^{1/2}\right)\Big|_0^L\right]$$

$$= k\cdot\lambda \left[\log\left(L+(L^2+d^2)^{1/2}\right)-\log d\right]$$

$$= k\cdot\lambda \log\left(\frac{L+(L^2+d^2)^{1/2}}{d}\right)$$



Electric Potential away from a changed disk w/ radius R

Assume surface change density is a constant σ

Consider change element consisting of a ring around center of radius R' + radial width dR'

then differential change is

$$dq = \sigma\,\underline{(2\pi R')(dR')}$$

← area of ring

Now get potential.

Since all points on the differential ring are @ distance $\sqrt{z^2+(R')^2}$ away, ditto potential due to ring is

$$dV = k\,\frac{dq}{r} = k \cdot \frac{\sigma\, 2\pi R'\, dR'}{\sqrt{z^2+(R')^2}}$$

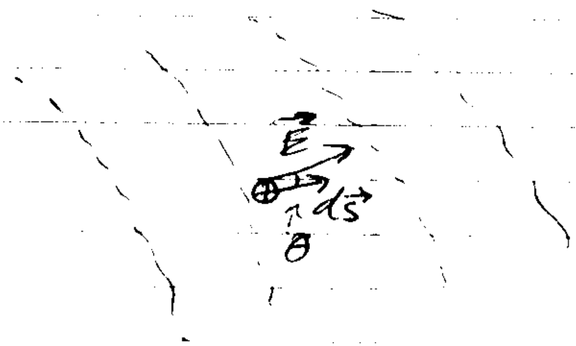
then integrate

$$V = \int dV = k \cdot \sigma \cdot 2\pi \int_0^R \frac{R'\,dR'}{\sqrt{z^2+R'^2}}$$

$$= k \cdot \sigma \cdot 2\pi \left[\sqrt{z^2+R'^2}\,\Big|_0^R \right]$$

$$= k \cdot \sigma \cdot 2\pi \left[\sqrt{z^2+R^2} - z \right]$$

Calculating E-field from Potential



QUESTION:

What is the work done in moving a test charge q_0 a tiny distance $d\vec{s}$?

equipotential lines
suppose they are very close so that the potential difference between them is dV

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

But here $V_f - V_i = dV$

& $\vec{E} \cdot d\vec{s}$ is equal to $-dV$

So $-dV = \vec{E} \cdot d\vec{s} = E\cos\theta \, ds$

$$\Rightarrow \frac{-dV}{ds} = E\cos\theta$$

or equivalently,

$$E_s = -\frac{\partial V}{\partial s}$$

where E_s is component of E-field in the direction of s.

Since we have 3 directions in space, this leads to

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

We can use this to recover formula for E-field due to a charged disk

We just calculated

$$V = k \cdot \sigma \cdot 2\pi \left[\sqrt{z^2+R^2} - z\right]$$

so $$E_z = -\frac{\partial V}{\partial z} = k \cdot \sigma \cdot 2\pi \cdot \left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

which is what we calculated

Potential Energy of a system of point charges — equal to the work you need to do to assemble the system, bringing in charges one @ a time from ∞.

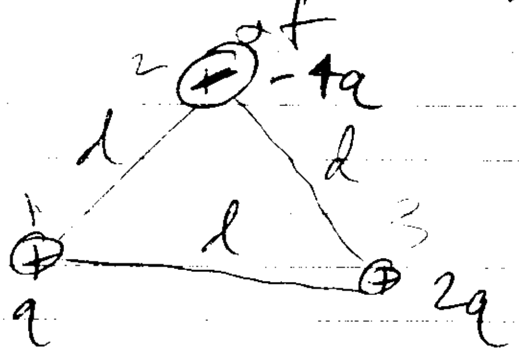
~~So Question, What is~~

potential energy is $= U = q_2 V$

where V is potential set up by a single charge q_1, $V = \frac{kq_1}{r}$

so $U = \frac{q_2 q_1 k}{r}$

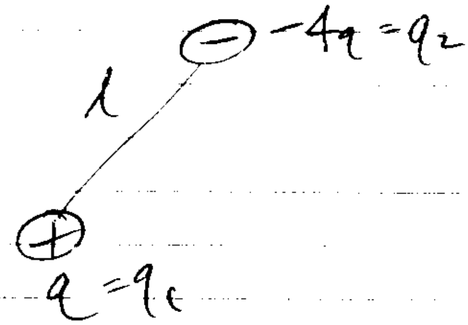
QUESTION: What is potential energy of



1st bring in q_1, no work needed to do so since nothing else is there.

Now bring in 2nd one, work to do so is

$$\frac{k \cdot q_1 \cdot q_2}{d}$$



Now bring in 3rd one, & work needed is

$$\frac{kq_1q_3}{d} + \frac{kq_2q_3}{d}$$

So the total is

$$k\left[\frac{q_1 q_2}{d} + \frac{q_2 q_3}{d} + \frac{q_1 q_3}{d}\right]$$

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potential of a charged conductor

recall that the E-field inside a conductor is zero (otherwise the electrons would be moving + this is not observed)

QUESTION: What does this imply about potential at various locations inside?

Recall $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$

$\Rightarrow V_f - V_i = 0$ potential is the same at different locations inside