Lecture 8

12 SEP 2014

Calculating Electric Field due to a continuous charge distribution:

uniform charge density – λ, σ, ρ

differential form of Coulomb law – $dE = \frac{k\,dq}{r^2}$

1) Divide charge distribution into infinitesimally small differential elements

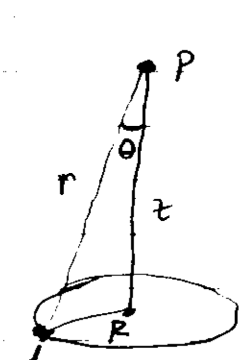
2) Treat each of these as a point charge & compute electric field

3) Apply principle of superposition: sum all of the contributions (need to integrate)

Another example: ring of charge
total charge is Q
radius is R what is λ?

$$\lambda = \frac{Q}{2\pi R}$$

QUESTION: What is direction of electric field at P?

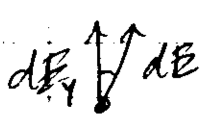


ds - little bit of arclength

use symmetry,
points up
suffices to calculate only the
y component of E-field

$$dE = \frac{k \cdot dq}{r^2} = \frac{k \cdot \lambda \cdot ds}{r^2}$$

$$r^2 = z^2 + R^2 \quad \Rightarrow \quad = \frac{k \cdot \lambda \cdot ds}{z^2 + R^2}$$



$$dE_y = dE \cdot \cos\theta = dE \cdot \frac{z}{r} = \frac{dE \cdot z}{\sqrt{z^2+R^2}}$$

$$dE_y = \frac{k \cdot \lambda \cdot z \cdot ds}{(z^2+R^2)^{3/2}}$$

Integrate along ring from 0 to $2\pi R$

$$E = \int dE \cos\theta = \int_0^{2\pi R} \frac{k \cdot \lambda \cdot z}{(z^2+R^2)^{3/2}} ds$$

$$= \frac{k \cdot \lambda \cdot z}{(z^2+R^2)^{3/2}} \cdot 2\pi R$$

but $\lambda \cdot 2\pi R = q$

So

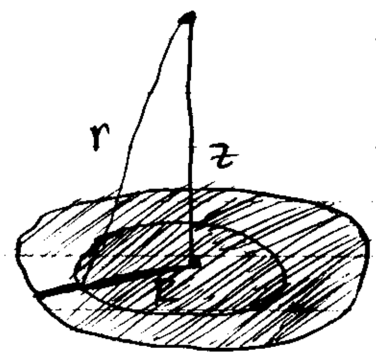
$$E = \frac{k \cdot z \cdot q}{(z^2+R^2)^{3/2}}$$

<u>test cases:</u>

$z \gg R \Rightarrow E = \frac{kq}{z^2}$

$z = 0 \Rightarrow E = 0$

<u>Next example</u>: Changed disk of radius R



What is electric field due to changed disk?

total change Q

QUESTION: What is surface change density? How to solve problem?

Area $-$ πR^2 so

$$\sigma = \frac{Q}{\pi R^2} \quad \left(\text{surface change density}\right)$$

We can use what we already solved to solve this problem!

disk consists of concentric rings

$$dq = \sigma \cdot dA = \sigma \cdot 2\pi r \cdot dr$$

take differential area to be that for a ring

We already calculated the E-field due to a ring of radius R to be

$$E = \frac{k \cdot z \cdot q}{(z^2 + R^2)^{3/2}}$$

So now differential E-field due to a ring of radius r is

$$dE = \frac{k \cdot z \cdot dq}{(z^2 + r^2)^{3/2}}$$

$$= \frac{k \cdot z \cdot \sigma \cdot 2\pi r \cdot dr}{(z^2 + r^2)^{3/2}}$$

now we can integrate over the rings to get

$$\int dE = \int_0^R \frac{k \cdot z \cdot \sigma \cdot 2\pi r \cdot dr}{(z^2 + r^2)^{3/2}}$$

$$= k \cdot z \cdot \sigma \cdot \pi \int_0^R \frac{2r \, dr}{(z^2 + r^2)^{3/2}}$$

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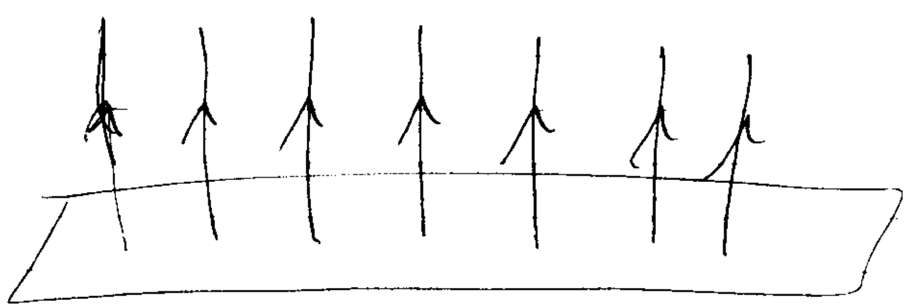
$$= k \cdot \sigma \cdot z \cdot \pi \left\{ \left[z^2 + r^2 \right]^{-1/2} \cdot (-2) \right\} \Big|_0^R$$

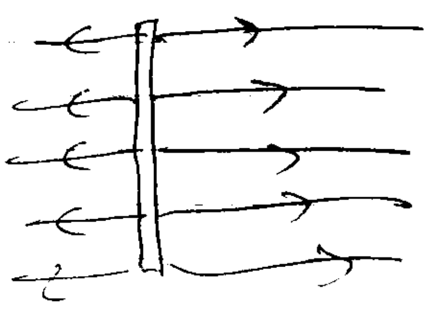
$$= k \cdot \pi \cdot \sigma \left[1 - \frac{z}{\sqrt{z^2 + R^2}} \right]$$

~~Sp~~ Interesting limits

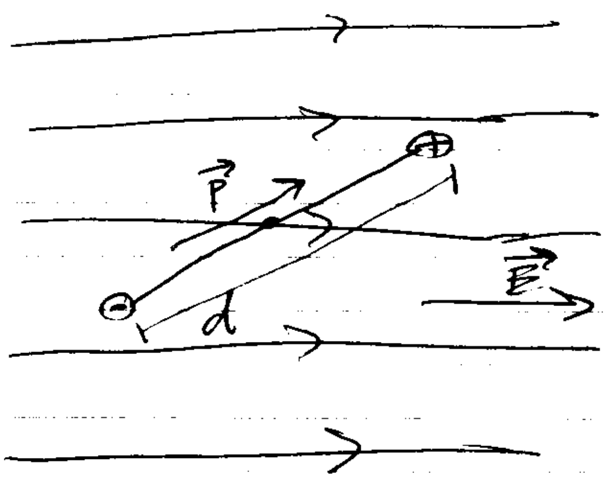
$R >> z$

$\Rightarrow k \cdot \pi \cdot \sigma$ electric field ~~surface charge density~~ for an infinite plane





Return to dipoles, but now consider them in the presence of an electric field
external uniform



Two concepts: torque + potential energy

analogous to statics (notions of torque + moment there)

dipole moment vector from ⊖ to ⊕

electric field will act on dipole

QUESTION: In which direction do you expect it, to ...?

Torque characterizes tendency of of a force to rotate an object

defined as cross product of + force vector distance vector.

$$\vec{\tau} = \vec{d} \times \vec{F}$$

using the fact that

$$\vec{p} = q\vec{d} \quad + \quad \vec{E} = \frac{\vec{F}}{q}$$

we can write

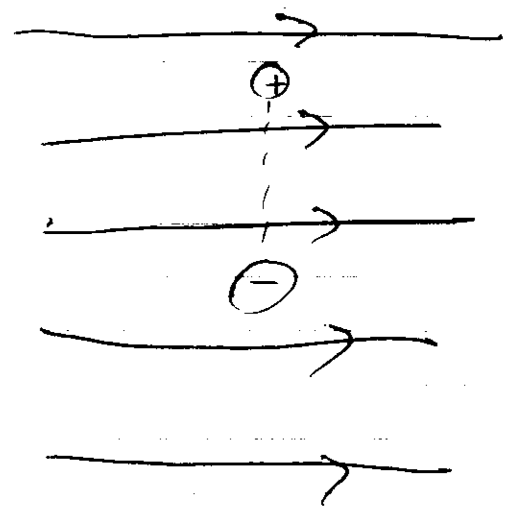
$$\vec{\tau} = \vec{p} \times \vec{E}$$

For our case, magnitude works out to

$$\tau = pE\sin\theta$$

potential energy

QUESTION: If situation is



is it easier for the dipole to rotate right or left? I.e., which one requires energy?

convention is to **define** the above configuration as the zero of potential energy,

rotating left is like going "uphill," so has higher potential energy

rotating right is like going "downhill," & has lower potential energy

defined as
$U = pE\cos\theta$
where
θ is
angle
wrt field