Lecture 3

29 AUG 2014

Escape speed - What is the speed which a point mass would require in order to escape from the Earth's gravitational field in order to reach the point @ ∞?

- Assume that there is only the point mass m + the Earth w/ mass M.
- Assume also that only energies involved are potential energy of the point-mass-Earth system + the kinetic energy of the point mass.

Can reason about this by thinking of the "end scenario" if the point mass escapes + by applying conservation of energy

At the end, the potential energy is zero (remember, by convention, potential energy between two masses is zero if they are infinitely far apart). Also we only require that the speed of the point mass is zero when it "reaches ∞", so that its kinetic energy is zero.

So the total energy at the end is zero. By conservation of energy, the final energy is equal to the initial energy needed to reach ∞.

But the initial energy is

$$K + U = \tfrac{1}{2}mv^2 - \frac{GmM}{R} = 0$$

So, solving for v gives the escape speed.

$$v = \sqrt{\frac{2GM}{R}}$$

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to escape from gravitational field of Earth, we require speed of at least 11.2 km/s

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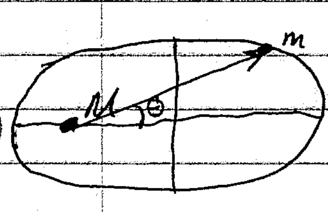
13-7 Kepler's laws for planetary motion

can be derived from Newton's law of universal gravitation

~~We will just discuss them~~.

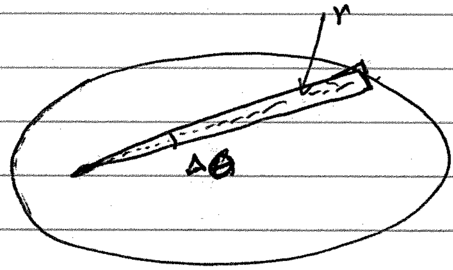
1. Law of orbits – planets move in elliptical orbits, w/ the Sun at one focus.



2. <u>Law of Areas</u> during the planet's orbit, it sweeps out equal areas in equal times

(i.e., planet moves most slowly when farthest from the sun & most rapidly when nearest to the sun)

can show that this is equivalent to conservation of angular momentum



$\Delta\theta$ is small angle swept out in small time Δt

area of this wedge is $\approx$ the area of a triangle w/base $r\Delta\theta$ & height r, so area is $\Delta A = \frac{1}{2} r\Delta\theta \, r$
$= \frac{1}{2} r^2 \Delta\theta$

(5)

divide both sides by Δt to get a rate

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{\Delta \theta}{\Delta t}$$ & then

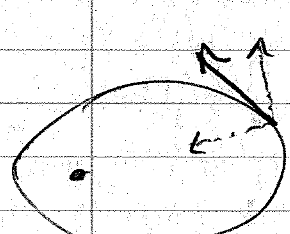
take the limit as $\Delta t \to 0$ to arrive at the instantaneous rate at which area is swept out

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

Abbreviating $\frac{d\theta}{dt} = \omega$ as angular speed, we can write as

$$\frac{dA}{dt} = \frac{1}{2} r^2 \omega$$

The orbiting planet has a momentum, w/ a radial + perpendicular component



The magnitude of angular momentum in this case is given by

$$L = r \cdot p_\perp$$

where $p_\perp$ is the magnitude of momentum perpendicular to r

$$p_\perp = m \cdot v_\perp$$

Using that $v_\perp = \omega r$,

we get $L = r^2 m \omega$

Substituting back, we can write

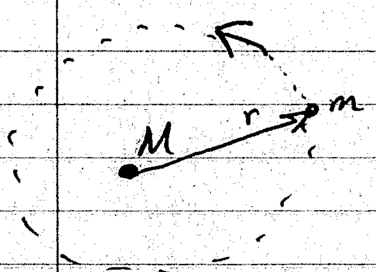
$$\frac{dA}{dt} = \frac{L}{2m}$$

So if $\frac{dA}{dt}$ is constant, this means angular momentum is constant & thus conserved. The opposite is true as well

Kepler's 3rd law:

square of the period of a planet's orbit is proportional to the cube of the semimajor axis of its orbit.

Consider simplified scenario w/ circular orbit.



Using Newton's 2nd law ($F=ma$), we get that

$$\frac{GMm}{r^2} = m \cdot \omega^2 r$$

where $\omega^2 r$ is due to centripetal acceleration

then using that angular frequency $\omega = \frac{2\pi}{T}$, we find

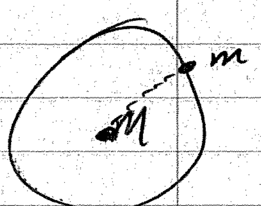
$$\frac{GM}{r^2} = \left(\frac{2\pi}{T}\right)^2 r$$

$$\Rightarrow T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

13-8 Energy of Orbits of Satellites

total mechanical energy = kinetic + potential energy

Consider a satellite orbiting earth w/ a circular path



potential energy of system is

$$U = -\frac{GMm}{r}$$

can write Newton's 2nd law as

$$\frac{G \cdot M \cdot m}{r^2} = m \frac{v^2}{r}$$

where $\frac{v^2}{r}$ is centripetal acceleration

By manipulating this, we get that the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

So for a satellite in circular orbit

$$K = -\frac{U}{2}$$

& the total mechanical energy is

$$E = K + U = \frac{U}{2} = -\frac{GMm}{2r}$$