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Lecture 37 26 Nov 2014

Energy Transport by EM waves & the Poynting Vector

EM waves can transport energy from one location to another (from Sun to our skin. This is also the basis for solar energy.)

Power transported by the wave is quantified by the Poynting vector (magnitude and direction)

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

Since $\vec{E}$ & $\vec{B}$ are perpendicular the magnitude $|\vec{S}| = \frac{1}{\mu_0} EB$

direction is always in the direction of wave propagation

For an EM wave,
we have that $\frac{E}{B} = c$

so we can write $|\vec{S}| = \frac{1}{\mu_0 c} E^2$

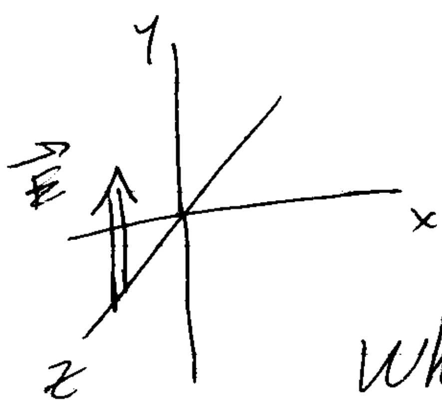
units of S are $\frac{\text{power}}{\text{unit area}}$

Since magnitudes of $\vec{E}$ + $\vec{B}$ change w/ time, so does $\vec{S}$.
It goes from a minimum of zero to a maximum value

So $|\vec{S}| = \frac{1}{\mu_0 c} E_m^2 \sin^2(kx - \omega t)$

QUESTION:

Suppose EM wave is propagating in $-z$ direction & picture of it @ a given time and instant is



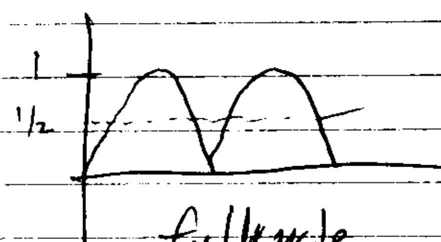
What is direction of magnetic field?

Since Poynting vector is changing w/ time, a better measure of energy in an EM wave is just to average it over one cycle of the wave - then we get intensity $I = \bar{S} = S_{avg}$ =

$$S_{avg} = \frac{1}{c\mu_0}\left[E_m^2 \sin^2(kx - \omega t)\right]_{avg}$$

~~average of~~ $\sin^2$ over 1 cycle is

just $1/2$



$$\Rightarrow \quad S_{avg} = \frac{1}{c\mu_0}\frac{E_m^2}{2}$$

energy density of E-field is

(energy per unit volume) $\frac{1}{2}\varepsilon_0 E^2$

that for B-field is

$$\frac{1}{2\mu_0}B^2$$

these are the same for an EM wave

Why? $E = cB \Rightarrow \frac{1}{2}\varepsilon_0 E^2 =$

$$\frac{1}{2}\varepsilon_0 (cB)^2$$

using $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$$\Rightarrow \quad 1/2\, \varepsilon_0 (c^2 B^2) = \frac{1}{2\mu_0} B^2$$

which is B-field energy density

Example: Solar Energy

Light from the Sun ~~measured~~ on Earth has an intensity

$\approx 1 kW/m^2$

What is the total power incident on a roof of dim. $8m \times 20m$?

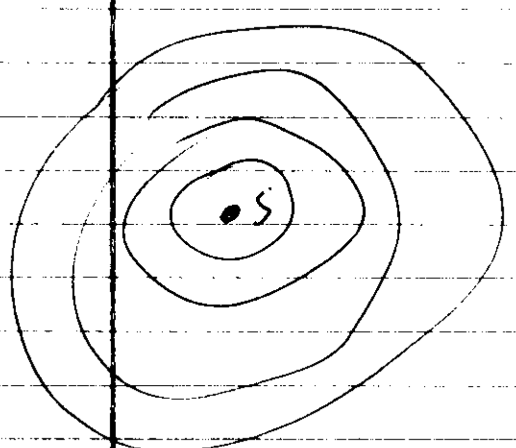
$$P = IA = (10^3 W/m^2) \cdot 8m \cdot 20m$$
$$= 0.16 \text{ MegaWatt}$$

solar panels @ the moment are about 10-20% efficient

Often it is the case that energy company pays you for generating electricity

Intensity varies w/ distance from the source of radiation.

This can be complicated, but for a point source, it is easy to model.



Assume that energy of EM waves is conserved going outward from the source.

power emitted goes through spheres of larger & larger radius.

Since intensity is power per unit area it should be the power of the source

divided by surface area of
of the spheres which is
@ distance r

$$\Rightarrow \quad I = \frac{P_s}{4\pi r^2}$$

$\frac{1}{r^2}$ law for intensity

QUESTION: radio station transmits
a 10kW signal @ frequency 100MHz.
@ distance 1km from antenna,
find electric & magnetic field
~~strengths~~ & energy incident
on a square plate of side 10cm
for 5min.

$$I = \frac{P_s}{4\pi r^2} = \frac{10\text{ kW}}{4\pi(1\text{ km})^2} = .8\,\frac{\text{mW}}{\text{m}^2}$$

$$I = \frac{1}{2c\mu_0} E_m^2 \Rightarrow E_m = \sqrt{2c\mu_0 I}$$

$$= 775\text{ V/m}$$

$B_m = E_m / c = 2.58\,nT$

Received energy $I = \frac{P}{A} = \frac{\Delta U / \Delta t}{A}$

$\Rightarrow \Delta U = I A \Delta t =$

$\left(.8 \frac{mW}{m^2}\right) \cdot (10cm)^2 (300s)$

$= 2.4\,mJ$