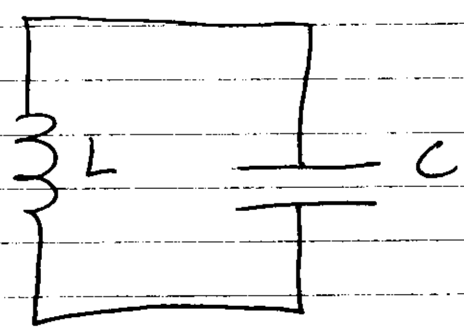
Lecture 31

10 NOV 2014

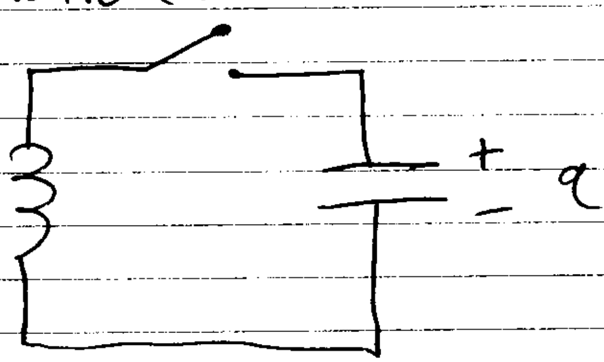
we've seen RC circuits &
RL circuits.

What about LC circuits?

Consider the following

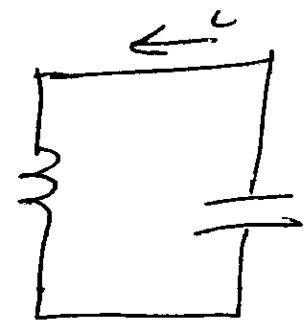


Suppose capacitor is initially changed before closing a switch:

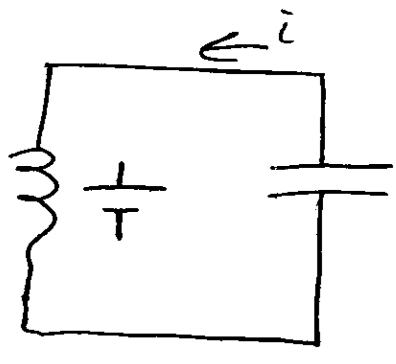


energy stored in electric field of capacitor is $\frac{q^2}{2C}$

After closing switch,
increasing current goes from positive to negative



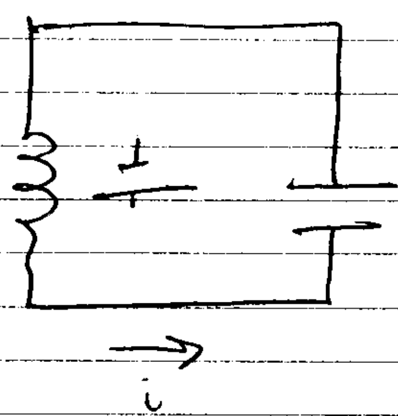
But the inductor sets up an EMF to oppose the direction of increasing current
(it resists a changing current)



electrical energy stored in capacitor is being converted to magnetic energy stored in magnetic field of the inductor

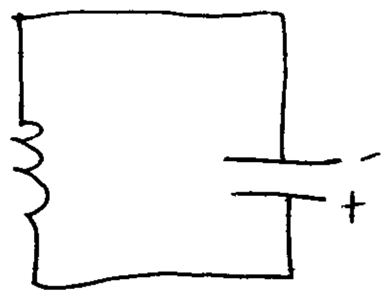
Eventually the capacitor loses all of its charge & current through the inductor is at its maximum. (All electrical energy has been converted to magnetic energy)

The current through the inductor starts decreasing, but then the inductor sets up an EMF in the direction of decreasing current, so picture looks like



The energy needed to do this is coming from stored magnetic energy

So this current transfers positive charge to the bottom plate of capacitor and eventually the picture becomes



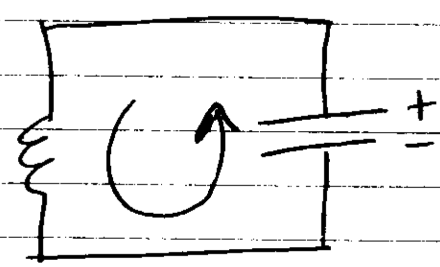
so that all magnetic energy goes to electrical energy stored in capacitor.

things continue as in beginning but in opposite way

behavior is like that of a harmonic oscillator.

How to describe this mathematically?, use the loop rule for beginning scenario

$$q = VC \quad \& \quad \mathcal{E}_L = -L\frac{di}{dt}$$



<u>Question</u>: What is loop rule?

$$-\frac{q}{C} - L\frac{di}{dt} = 0$$

but $i = \frac{dq}{dt} \Rightarrow \frac{di}{dt} = \frac{d^2q}{dt^2}$

$$\Rightarrow \quad \frac{q(t)}{C} + L\frac{d^2q(t)}{dt^2} = 0$$

This is a differential equation describing dynamics of current & charge in circuit.

This is exactly analogous to a mass on a frictionless spring.

guess a solution of the form

$$q(t) = Q\cos(\omega t + \phi)$$

where Q is amplitude, ω is angular frequency, & ϕ is phase.

then $$i = \frac{dq}{dt} = -\omega Q \sin(\omega t + \phi)$$

so amplitude for current is

$$I = \omega Q \quad \text{so}$$

$$i = -I\sin(\omega t + \phi)$$

$$\frac{d^2 q}{dt^2} = -\omega^2 Q \cos(\omega t + \phi)$$

$\Rightarrow$ substituting back in gives

$$\frac{Q}{C} \cos(\omega t + \phi)$$

$$- L\omega^2 Q \cos(\omega t + \phi) = 0$$

solution occurs when

$$\omega = \frac{1}{\sqrt{LC}}$$

initial conditions give the phase ϕ.

Question:
What is electrical energy stored in capacitor at any given time?

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C} \cos^2(\omega t + \phi)$$

magnetic energy in inductor is

$$U_B = \frac{1}{2} L i^2 = \frac{1}{2} L \omega^2 Q^2 \sin^2(\omega t + \phi)$$

energy oscillates back & forth
b/c sin & cos are out of
phase by 90°.

this is all analogous to a
mass moving on a spring