Lecture 30 — 7 NOV 2014

Consider a solenoid as the basic kind of inductor

establishing a current through the coils induces a magnetic flux $\Phi_B$

the induced flux is proportional to the current (follows from discussion on solenoids)

$$i \propto \Phi_B$$

proportionality constant is called inductance $L$ & we also need # of turns & define $L$ by

$$i L = N \Phi_B$$

where $N$ is # of turns

$$\Rightarrow \quad L = \frac{N\Phi_B}{i}$$

unit of inductance is the

Henry $= \left[\frac{T \cdot m^2}{A}\right]$ b/c $\frac{\text{flux}}{\text{current}}$

What is inductance for a solenoid?

Let $n$ be # of turns per unit length & consider a length $\ell$

then for this length,

$$N\Phi_B = n\ell BA$$

where $A$ is cross-sectional area

But before we found for a solenoid that $B = \mu_0 i n$

$$\Rightarrow \quad L = \frac{N\Phi_B}{i} = \frac{n\ell BA}{\frac{B}{\mu_0 n}} = \mu_0 n^2 \ell A$$

$$\Rightarrow \quad \frac{L}{\ell} = \mu_0 n^2 A$$

So inductance per unit length only depends on geometry (# of turns per unit length & cross-sectional area)

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Self-induction

a current in one coil produces magnetic flux in another

- changing the current induces an EMF according to Faraday's law $\left(\mathcal{E} = -\frac{d\Phi_B}{dt}\right)$

For a solenoid inductor, we have

$$N\Phi_B = Li$$

Faraday's law gives that

$$\mathcal{E}_L = \frac{-d(N\Phi_B)}{dt}$$

(induced EMF for inductor)

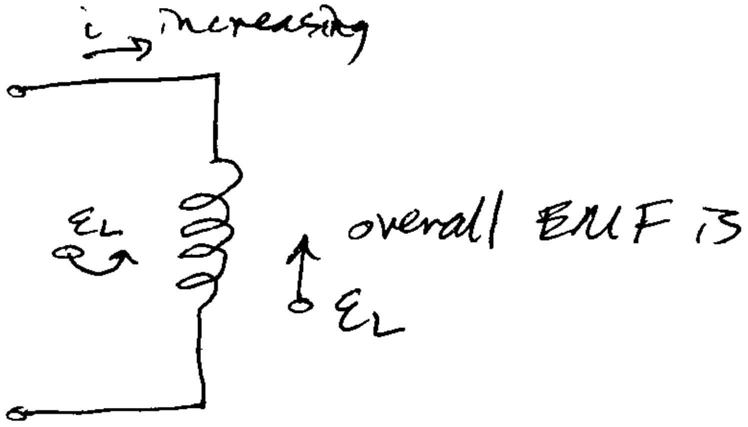
Combining these equations gives

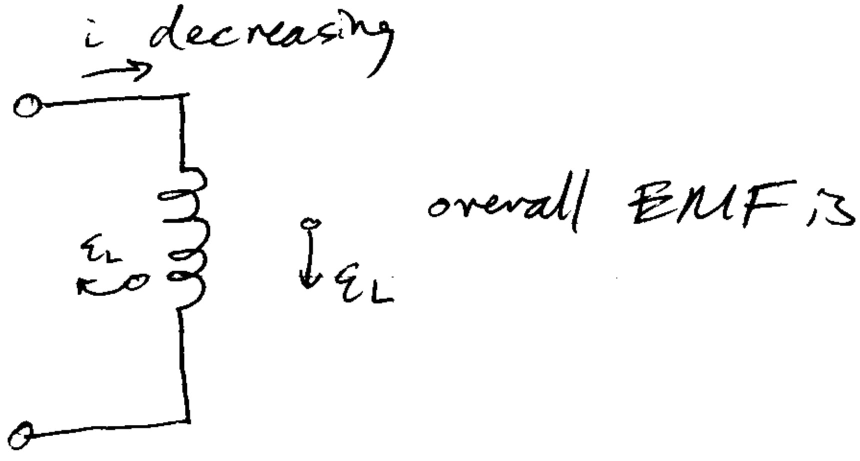
$$\boxed{\mathcal{E}_L = -L\frac{di}{dt}}$$

direction of induced EMF

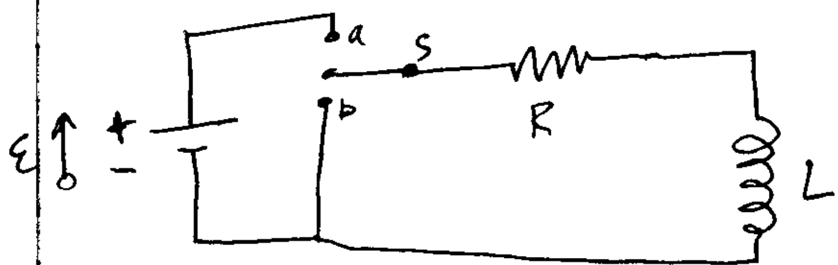
<u>opposes</u> direction of increasing current

For a picture, we have



## RL circuits




When the switch is flipped to a, there is a rapid increase in current

→ through resistor.

Since the inductor is present, + current it increases ~~acts to~~ has an induced EMF that opposes direction of current.

So the current through the resistor is initially less than $\mathcal{E}/R$ (which is what it would be if the inductor were not there)

After some time, the ~~cur~~ change in current gets smaller & so the induced EMF of inductor gets smaller as well

$$\left( \mathcal{E}_L = -L\frac{di}{dt} \right)$$

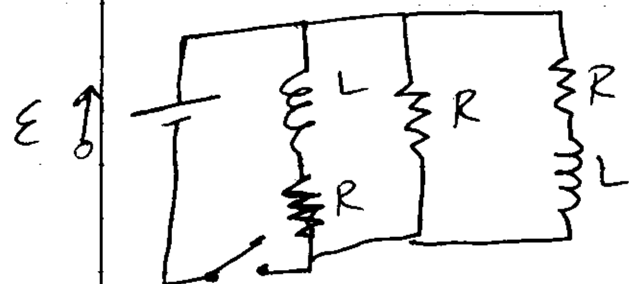
Rule of thumb: After switch is closed,

At $t=0$ inductor acts like a broken wire

At $t=\infty$ inductor acts like an ordinary wire

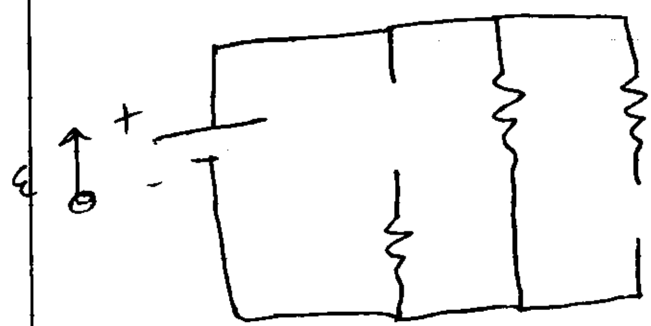
Simple example to apply this rule of thumb

QUESTION:

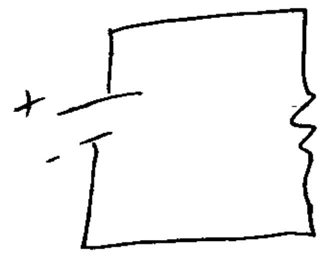



What is current through battery just after switch is closed?

can picture as

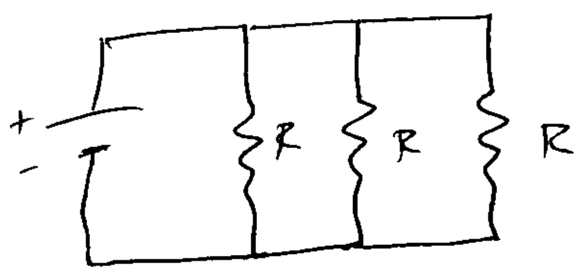



so circuit is just




& $i = \frac{\varepsilon}{R}$

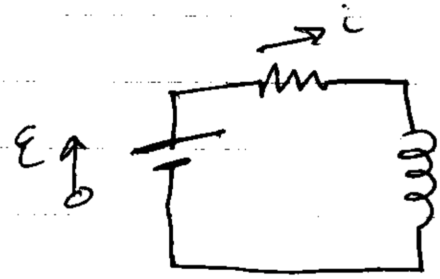
After a long time




these are in parallel, so


& $i = \frac{\varepsilon}{R/3}$

Solve the RL circuit equation




loop rule

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

minus sign because direction of EMF opposes direction of increasing current

$$\mathcal{E} = iR + L\frac{di}{dt}$$

differential equation w/ initial condition $i(0) = 0$

Solution is

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

Why solution? $i(0) = 0$

&

$$\frac{di(t)}{dt} = \frac{\mathcal{E}}{L} e^{-Rt/L}$$

$$iR = \mathcal{E}\left(1 - e^{-Rt/L}\right)$$

$$L\frac{di(t)}{dt} = \mathcal{E} e^{-Rt/L}$$

summing them gives $\mathcal{E}$

Substitute in to match w/ qualitative behavior: $i(0) = 0$ so no current going through resistor initially & $i(\delta)$ is small b/c of induced opposing EMF

time constant for
circuit is $L/R$
this indicates how long it takes

for current to reach certain
fractions of its final value.

If switch is thrown to (b)
(disconnecting battery) then the
circuit equation becomes

$$L\frac{di}{dt} + iR = 0$$

& solution is

$$i(t) = \frac{\mathcal{E}}{R} e^{-tR/L}$$

so current doesn't immediately drop
off but decays exponentially
to zero.