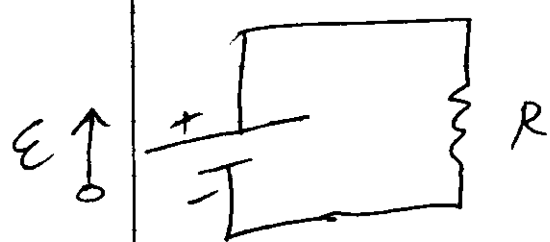
T

Lecture 21

15 OCT 2014

Review of last time:

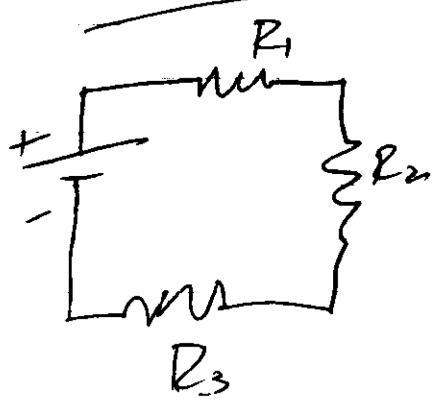


EMF device w/ ~~EMF~~ EMF $\mathcal{E}$ is like a battery.

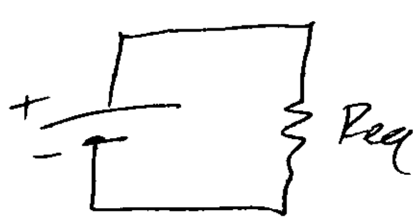
$$V_a + \mathcal{E} - iR = V_a$$

$$\Rightarrow \quad \mathcal{E} = iR$$

Resistors in Series:

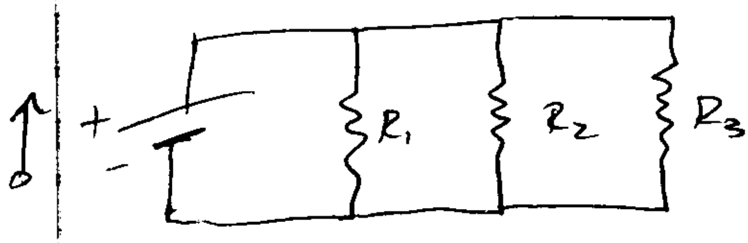


What is equivalent circuit?

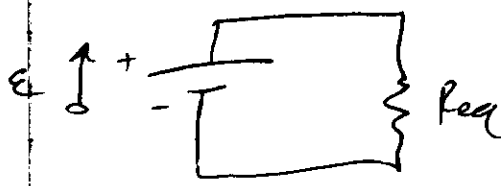


$$R_{eq} = R_1 + R_2 + R_3$$

Resistors in Parallel:

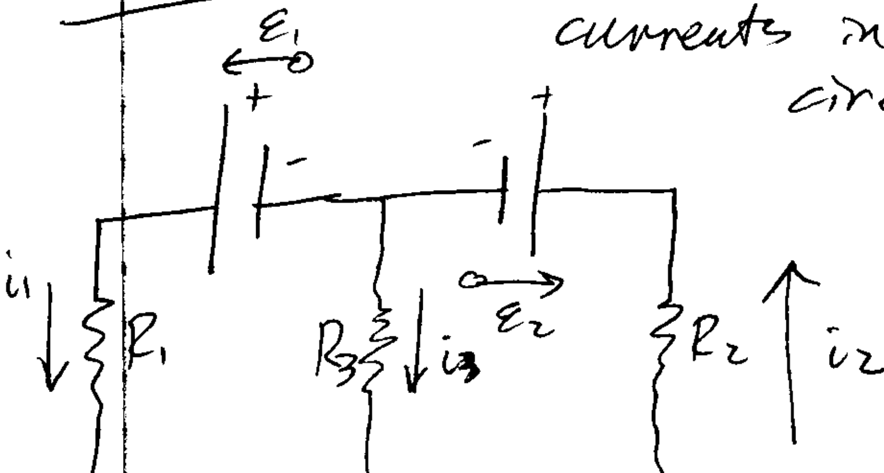


What is equivalent circuit?



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

QUESTION: What are the three currents in the following circuit?



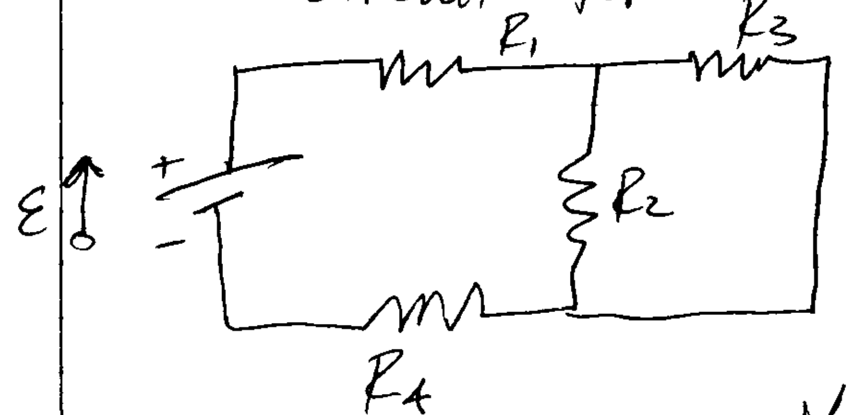
$$i_1 + i_3 = i_2$$

$$\mathcal{E}_2 + i_2 R_2 + i_3 R_3 = 0$$

$$\mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0$$

QUESTION:

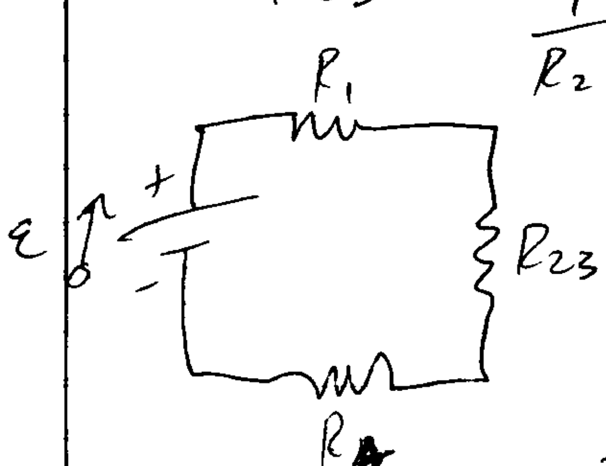
How to find equivalent circuit for



can then use this to figure out currents

1st step: $R_2 + R_3$ in parallel so

$$R_{23} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = \frac{R_2 R_3}{R_2 + R_3}$$



So current equals

$$\frac{\mathcal{E}}{R_1 + R_{23} + R_4} = i$$

$\Rightarrow$ potential drop across R_1

is iR_1

& across R_{23} is iR_{23}

& across R_4 is iR_4

but voltage across R_2 & R_3

is the same, so the

current through R_2

is $\frac{iR_{23}}{R_2}$ & that

through R_3 is $\frac{iR_{23}}{R_3}$

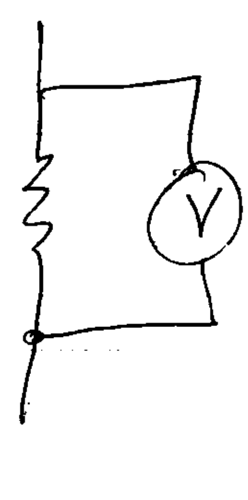
Ammeter & Voltmeter

devices used to measure current & voltage, respectively

to measure current through a wire, "break it" & connect ammeter in •

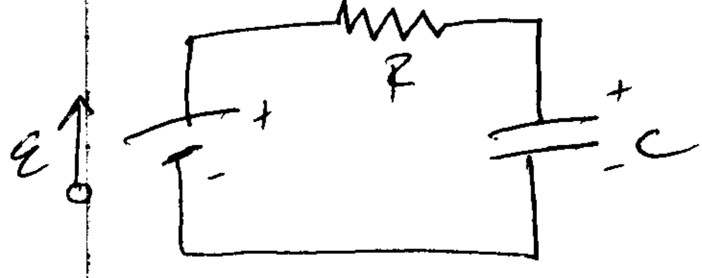
important that resistance of ammeter is very small so that it does not alter current.

to measure voltage across a resistor use voltmeter as



important that resistance of voltmeter be much larger than that of resistor.

RC circuits



We know that when we connect the circuit, the capacitor charges up to a point where it is fully charged, & then current no longer flows.

QUESTION: How can we use the loop rule to relate $\mathcal{E}$, i, R, q on capacitor & C?

$$\mathcal{E} - iR - \frac{q}{C} = 0$$

↑ negative b/c of potential difference across capacitor cannot solve b/c two independent variables q & i, but

$$i = \frac{dq}{dt} \Rightarrow$$

$$\mathcal{E} = R\frac{dq}{dt} + \frac{q}{C}$$

more explicitly, $\mathcal{E} = R\frac{dq(t)}{dt} + \frac{q(t)}{C}$

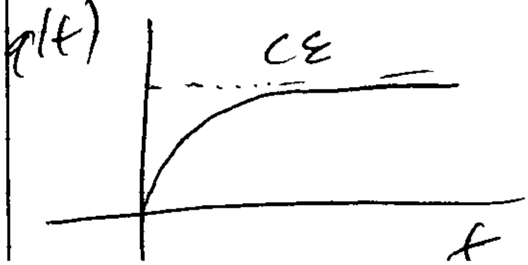
This is a differential equation that we need to solve.

Initial condition is that $q = 0$ @ $t = 0$.

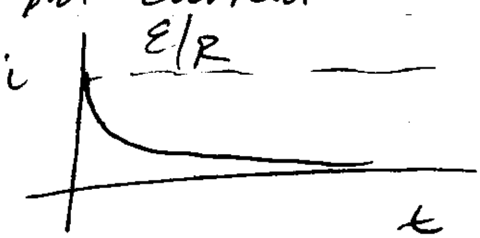
solution will be

$$q(t) = C\mathcal{E}\left(1 - \exp\{-t/RC\}\right)$$

So charge will look like



but current



$$\frac{dq(t)}{dt} = C\mathcal{E} \exp\{-t/RC\} \frac{1}{RC}$$

$$= \frac{\mathcal{E}}{R} \exp\{-t/RC\}$$

$\Rightarrow$ $V(t)$ for capacitor is

$$V(t) = \frac{q(t)}{C} = \mathcal{E}\left(1 - \exp\{-t/RC\}\right)$$

Why is $q(t)$ a solution?

b/c $\displaystyle \mathcal{E} = R\frac{dq(t)}{dt} + \frac{q(t)}{C}$

$$= R \cdot \frac{\mathcal{E}}{R} \exp\{-t/RC\} + \mathcal{E}\left(1 - \exp\{-t/RC\}\right)$$

$$= \mathcal{E}$$

So it satisfies.

Furthermore, initial condition $q(t) = 0$ is satisfied

RC is called the <u>time constant</u>

(it has dimensions of time
$1\Omega \times 1F = 1s$)

~~During~~ After a time $t = RC$,

charge is $C\mathcal{E}(1-e^{-1})$
$\approx (0.63)\, C\mathcal{E}$

so after this time the ~~[illegible]~~ capacitor is 63% charged.

<u>Discharging a capacitor</u>

What if we charge it + then disconnect battery so that

Then the same loop rule gives

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

& solution is

$$q = q_0 \exp\{-t/RC\}$$

where q_0 is initial charge $= C\mathcal{E}$

so charge decreases exponentially fast with time.

What about current?

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)\exp\{-t/RC\}$$

& current does as well. ..