Lecture 20

12 OCT 2014

To make charges flow through a resistor, you need a potential difference across its terminals.

To do so, we need a "charge pump" which for historical reasons is called an "EMF device" where EMF is for "electromotive force"

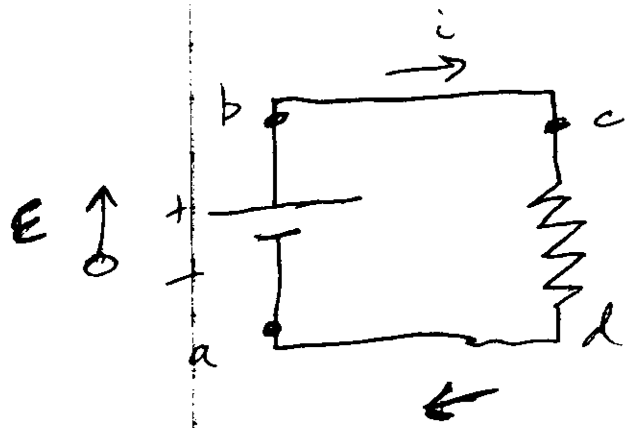
many examples of EMF devices are batteries, electric generators, solar cells, & fuel cells.

An EMF device does work on positive charge carriers to move them from its − terminal to + terminal

How to calculate the potential difference that an EMF device creates?

use the "loop rule" for a circuit: The sum of the changes in potential when going around a loop of a circuit is equal to zero. (AKA Kirchhoff voltage law)

similar to walking up a mountain, around, & then returning to the same point (change in potential is zero)



Say that potential @ point a is V_a

going from a to b, potential increase is $\mathcal{E}$ (EMF)

going from c to d, potential @ c is higher than @ d, so change is $-iR$

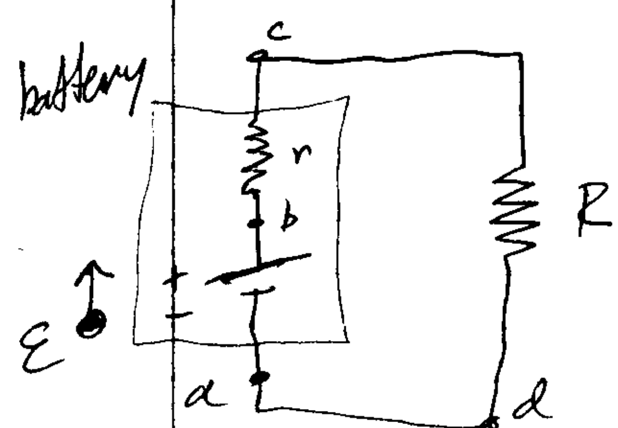
Finally, we end up where we started, so

$$V_a + \mathcal{E} - iR = V_a$$

$$\Rightarrow \quad \mathcal{E} = iR$$

Real batteries (EMF devices) have an internal resistance that is unavoidable.

We can model this by



QUESTION:

What is $\mathcal{E}$ here?

Just use the same reasoning!..

(remember that current is constant here due to conservation of charge)

from a to b $\quad \mathcal{E}$

b to c $\quad -ir$

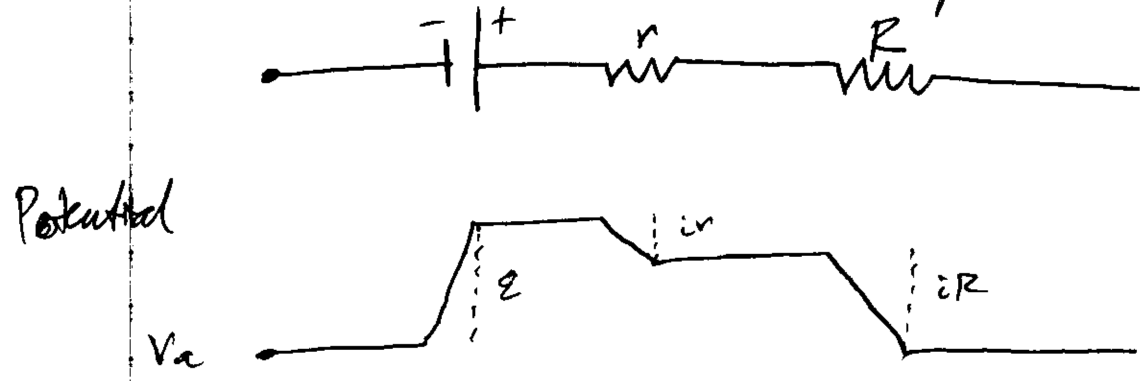
c to d $\quad -iR$

$\Rightarrow \quad \mathcal{E} - ir - iR = 0$

$\Rightarrow \quad \mathcal{E} = i(r+R) \Rightarrow \quad i = \dfrac{\mathcal{E}}{r+R}$

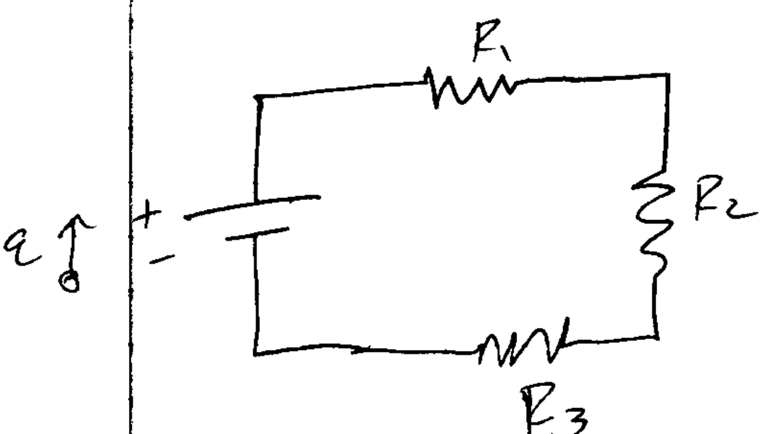
So the internal resistance causes the current to be lower than it should be.

can draw this kind of plot as well



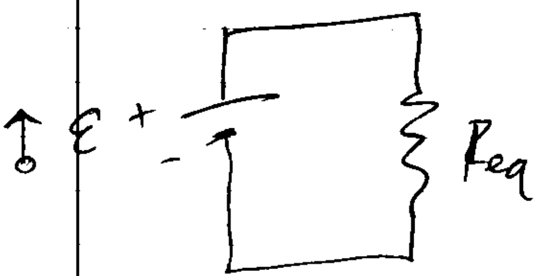
up & down a hill...

Resistors in Series



What is the equivalent resistance?

i.e., find Req such that



where potential across Req is the same as that across all three + current through is the same as well

1st, use what we did before

$$\mathcal{E} - iR_1 - iR_2 - iR_3 = 0$$

$$\Rightarrow \quad i = \frac{\mathcal{E}}{R_1 + R_2 + R_3}$$

$$\Rightarrow \quad \frac{\mathcal{E}}{i} = R_1 + R_2 + R_3$$

For equivalent circuit, $\mathcal{E}$ is unchanged + so is i

$$\Rightarrow \quad \mathcal{E} - iR_{eq} = 0$$

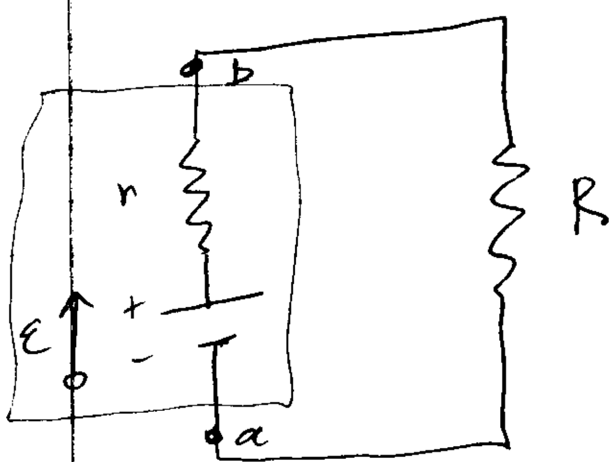
$$\frac{\mathcal{E}}{i} = R_{eq}$$

$$\Rightarrow \quad R_{eq} = R_1 + R_2 + R_3$$

so for resistors in series, just add them

Finding the Potential difference between two points in a circuit

Consider circuit from before:



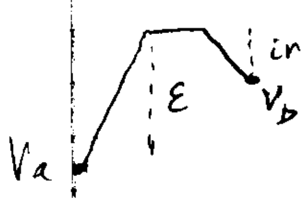
We might want to know the potential difference between a & b

QUESTION: What is $V_a - V_b$?

At a, say it is V_a & at b, V_b

then $V_a + \mathcal{E} - ir = V_b$

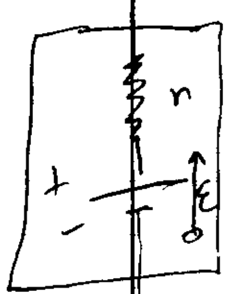
$\Rightarrow V_b - V_a = \mathcal{E} - ir$



we found i before to be

$i = \frac{\mathcal{E}}{R+r} \Rightarrow \Delta V = \mathcal{E} - \frac{\mathcal{E}}{R+r} r = \frac{\mathcal{E}}{R+r} R$

Power delivered by real batteries



$P = iV$ where V is potential difference across terminals

From before,

$$V = \mathcal{E} - ir$$

$$\Rightarrow P = i(\mathcal{E} - ir)$$

$$= i\mathcal{E} - i^2 r$$

$i\mathcal{E}$: power of EMF

$i^2 r$: power dissipated through internal resistance.

Multiloop circuits

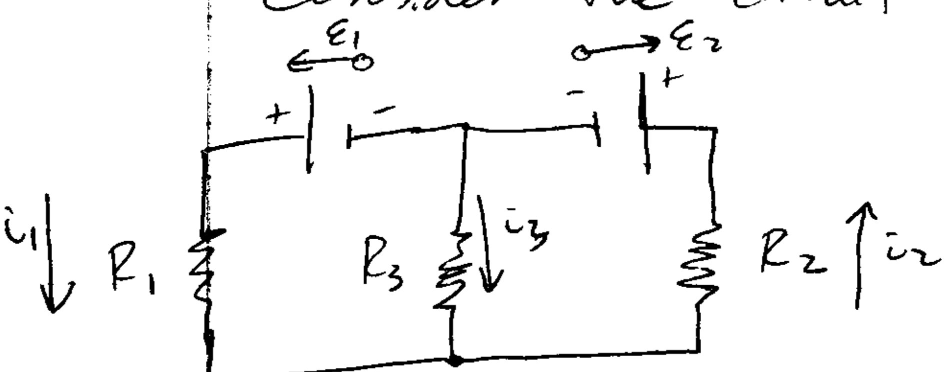
key thing to remember:

Kirchhoff current law –

the sum of currents entering a junction is equal to the sum of currents exiting it.

$i_1 = i_2 + i_3$

Consider the circuit



current law ⇒ $i_1 + i_3 = i_2$

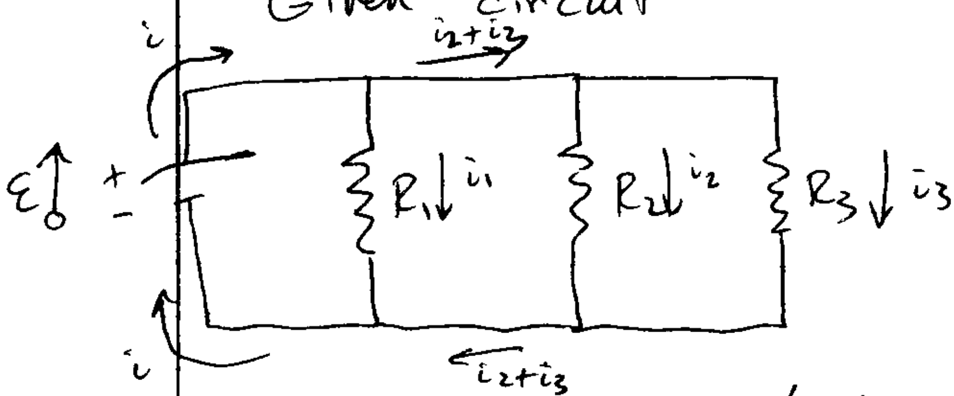
going around loop 1 $\mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0$

loop 2 $-i_3 R_3 - i_2 R_2 - \mathcal{E}_2 = 0$

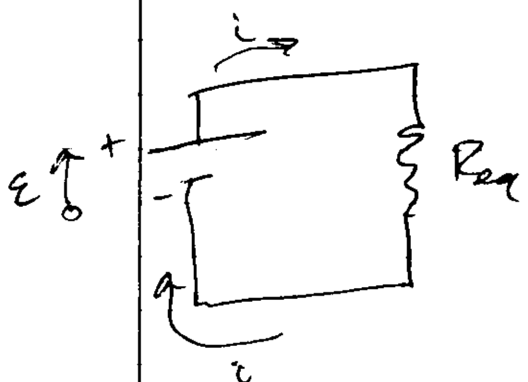
can solve for currents now...

Resistors in parallel

Given circuit



What is equivalent resistance?



$$\mathcal{E} - i_1 R_1 = 0$$

$$\mathcal{E} - i_2 R_2 = 0$$

$$\mathcal{E} - i_3 R_3 = 0$$

$$\Rightarrow \quad i_1 = \frac{\mathcal{E}}{R_1}$$

$$i_2 = \frac{\mathcal{E}}{R_2}$$

$$i_3 = \frac{\mathcal{E}}{R_3}$$

$$i = \frac{\mathcal{E}}{R_{eq}}$$

But $i = i_1 + i_2 + i_3 \Rightarrow$

$$\frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} + \frac{\mathcal{E}}{R_3} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$