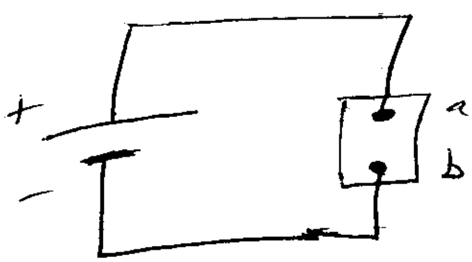
Lecture 19

10 OCT 2014

Power in Electric Circuits

Given is a circuit w/ a battery connected to some unknown device through which current flows.



We just model the battery as a device that creates a potential difference between its terminals.

In the circuit suppose that we let a little time dt go by. Then the amount of charge that moves from a to b is $i \cdot dt$

(from definition of current)

then what is the change in potential energy of charge dq for a fixed potential difference V?

$$dU = dq\, V = i\, dt\, V$$

Dividing both sides by dt gives

$$\frac{dU}{dt} = iV$$

rate of energy transfer is the power, so that means that

power $$P = iV$$

units are Volt · Amp $= \left[\frac{J}{C}\right]\left[\frac{C}{s}\right] = \left[\frac{J}{s}\right] = [W]$

If the device is a resistor, then the ~~energy is~~ electrical energy is converted to thermal energy. How much energy is dissipated? Just use $R = \frac{V}{i}$

to write $P = iV = \frac{V}{R} V = \frac{V^2}{R}$

or $P = iV = i\,iR = i^2 R$

Be careful when applying formulas

$P = iV$ applies for electrical energy transfer of all kind

whereas

$P = \frac{V^2}{R} = i^2 R$ applies only for conversion of electrical energy to thermal energy

QUESTION: given some length of uniform heating wire w/ resistance of $72\,\Omega$

a) If a potential difference of 120 V is applied across terminals, what is rate of energy dissipation?

$$P = \frac{V^2}{R} = \frac{(120\,V)^2}{72\,\Omega} = 200\,W$$

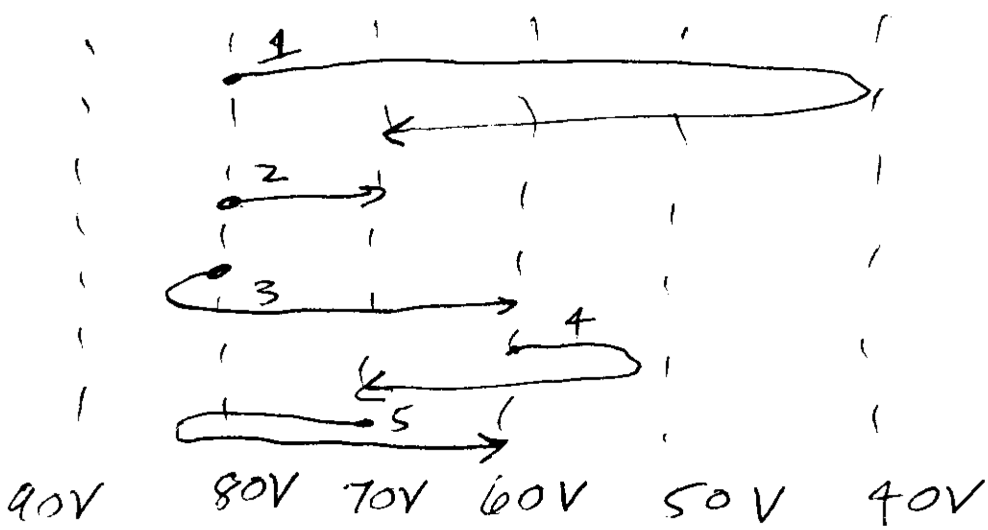
b) Now you cut the wire in half and apply a potential difference of 120 V across each half. What is rate of energy dissipation?

Each half has resistance $36\,\Omega$

so P for one half is $\frac{(120\,V)^2}{36\,\Omega} = 400\,W$

$\rightarrow$ total is 800W ?

Some review problems



a) What is direction of E-field?

$\longrightarrow$ (always from higher to lower potential)

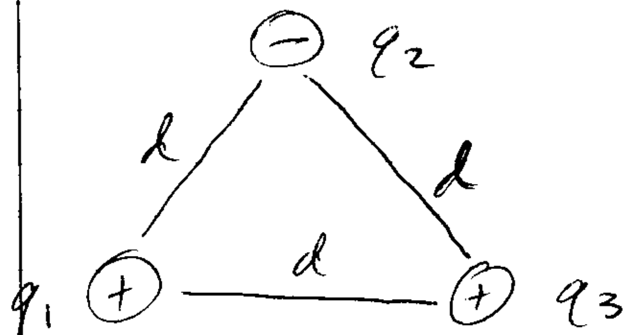
b) For each path, is the work invested by us +, −, or 0? (to move an electron)

1, 2, 3, 5 we have to go against field, so +

4, it is negative (remember electron!)

c) rank paths according to work done in moving an electron $3 > 1 = 2 = 5 > 4$

Potential energy of a system of three charged particles:



think about work needed to assemble system (when they all start @ infinity)

bring in charge q_2, no cost since no others present

bring in charge q_1,

cost is $\frac{k q_1 q_2}{d}$ (this was derived from Coulomb $\frac{1}{r^2}$ law & $\vec{F} \cdot \vec{d}$)

bring in charge q_3

q_1 + q_2 create a potential + total potential is sum of individual ones, so cost is $\frac{k q_1 q_3}{d} + \frac{k q_2 q_3}{d}$

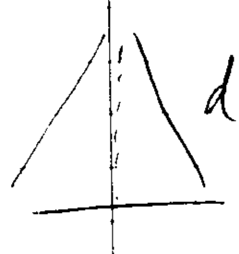
total is

$$k\left(\frac{q_1 q_2 + q_1 q_3 + q_2 q_3}{d}\right)$$

For any system it will be the sum of potential energy of all pairs of charges.

What is electric potential at

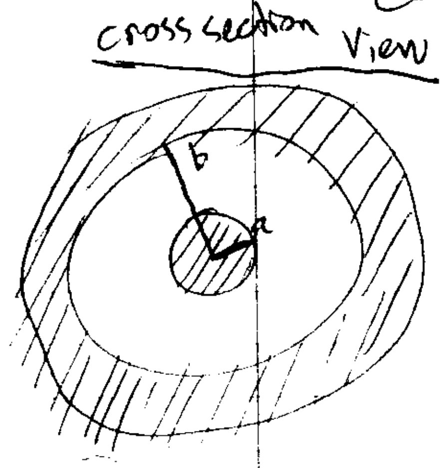
Sum all of the individual potentials



$$\frac{k q_2}{d'} + \frac{k q_1}{\frac{d}{2}} + \frac{k q_3}{\frac{d}{2}}$$

$$d^2 = \left(\frac{d}{2}\right)^2 + (d')^2 \Rightarrow d' = \frac{\sqrt{3}}{2} d$$

Example of Gauss' law for calculating capacitance of concentric spherical shells



Capacitance is defined as

$q = VC$

So imagine inner sphere has charge $+q$ & outer one (on inner surface) has charge $-q$

then we need to get potential difference but for this we need E-field.

We know from symmetry that E-field will go radially outward from inner to outer. So pick Gaussian surface to be a sphere of radius r & Gauss' law says

$$\text{flux} = \frac{\text{net charge enclosed}}{\epsilon}$$

flux is going to

$E\, 4\pi r^2$ here b/c it is the same magnitude at all locations on surface of Gaussian sphere & pointing radially outward, just as all of the normal vectors are

net enclosed charge is +q

so $E\, 4\pi r^2 = \frac{q}{\varepsilon_0}$

$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$ (notice consistency w/ shell theorem)

to get V (potential difference) we need to integrate along a path & it makes sense to take path going radially inward from outer to inner sphere.

So

$$V = -\int_b^a \frac{q}{4\pi\varepsilon_0 r^2}\,dr$$

$$= \frac{-q}{4\pi\varepsilon_0}\int_b^a \frac{1}{r^2}\,dr$$

$$= \frac{+q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

$$\Rightarrow \quad C = \frac{q}{V} = 4\pi\varepsilon_0 \frac{ab}{b-a}$$