Lecture 17

6 Oct 2014

Ch. 26

Electric current - concept is applicable in many fields

- Biologists study electric current flowing in the nervous system
- of course EE
- space engineers study the flow of charged particles from the sun

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Electric current is a characteristic of flowing charge.

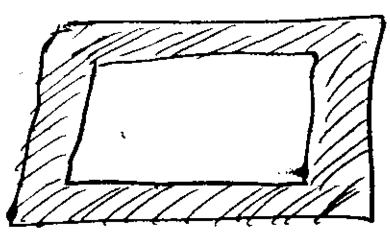
It is **not**

1. due to random motion of charges in a conductor. In this case, charges may go back and forth through a plane of a copper wire but there is not a net transport
2. due to flow of water, e.g.,

We could say that flow of water transports positive charge but it transports just as much negative charge, so there is no net charge flow.

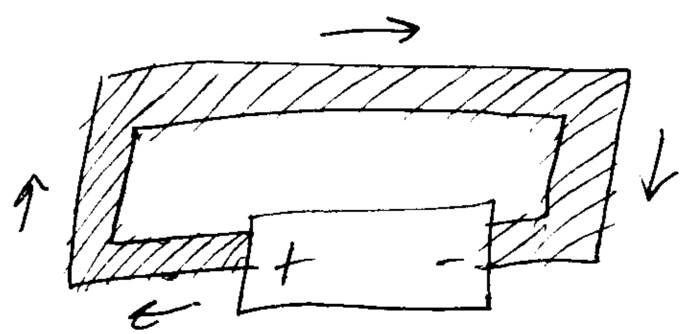
So to have an electric current, there must be a net flow of charge through a surface.

Picture an isolated conducting loop:



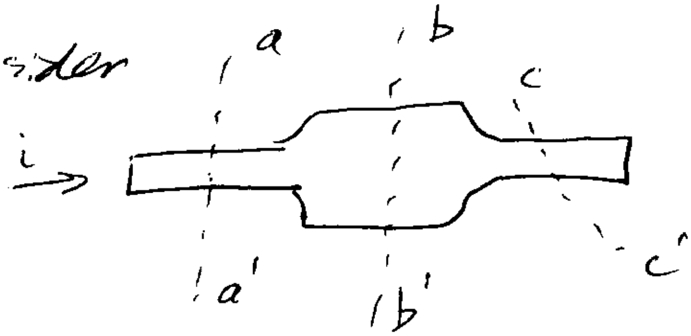
even if there is an excess charge, No **E**-field exists within it or on the surface, so that all points are @ the same potential

Now if we place a battery in the loop, we can no longer say that all points are @ the same potential:



electron flow will reach a steady state and there is a current.

consider



current is equal to rate of change of charge w/ time

$$i = \frac{dq}{dt}$$

i.e., if a small charge dq passes through plane aa' in time dt, then

$$i = \frac{dq}{dt}$$

due to conservation of charge & if the current is constant, then it is the same for planes aa', bb', & cc'

to get the total charge flowing through plane aa' for a time t we integrate

$$\int dq = \int_0^t i \, dt = q$$

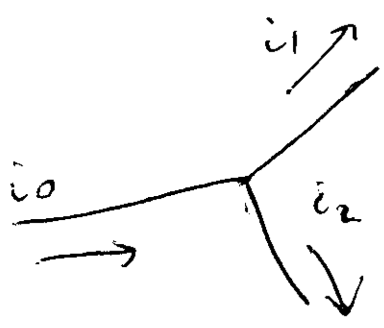
units of current are

Amperes (amps)

$1\ A = 1\ C/s$

current is a scalar quantity but we often use arrows to indicate its direction.

at a junction in a wire



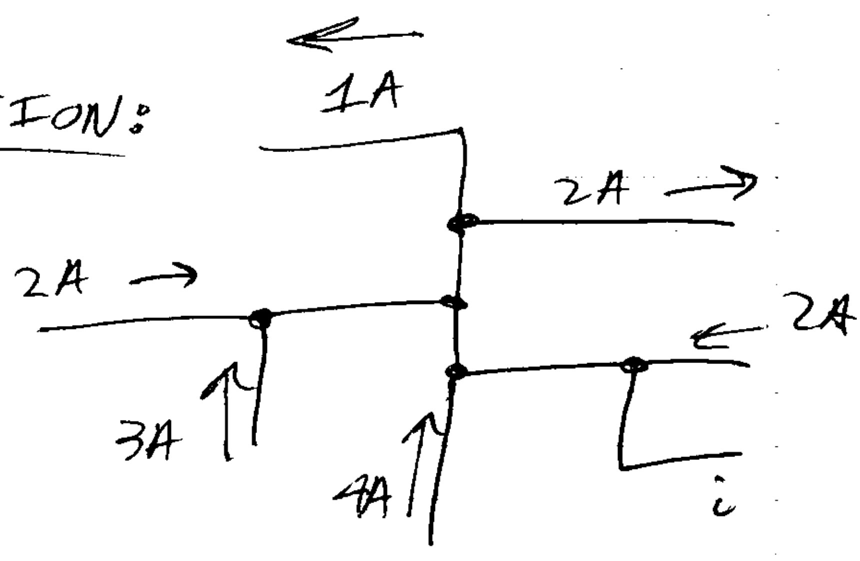
current in = current out due to conservation of charge:

$$i_0 = i_1 + i_2$$

(6)

arrows for current (by convention) go in the direction that positive charge carriers would go, even if charge carriers are actually negative electrons & go in the opposite direction

QUESTION:



What is the current & its direction at location i?

We defined current as the flow of charge through a given area per unit time, so it is a kind of flux. What is it a flux of?

<u>Current density</u> $\vec{J}$ (vector quantity)

has same direction as <u>velocity</u> of moving positive charges

(but opposite direction for moving negative charges)

given a small window w/ area dA & normal $d\vec{A}$

current flowing through it is

$$\vec{J} \cdot d\vec{A}$$

, so the total current through a surface is

$$i = \int \vec{J} \cdot d\vec{A}$$

If current is uniform & parallel to a surface w/ area A, then so is current density $\vec{J}$, so in this case

$$i = \int J \, dA = J \int dA$$

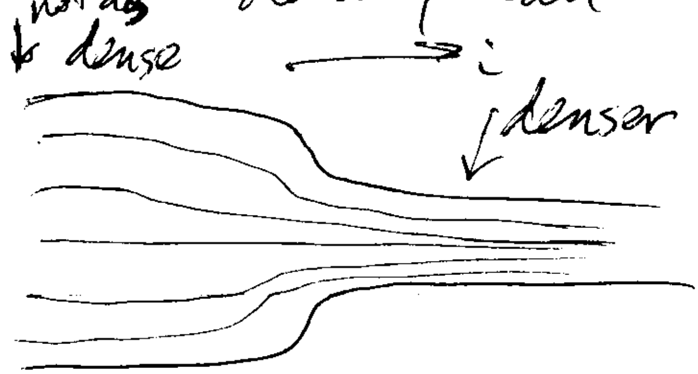
$$= JA$$

$$\Rightarrow \quad J = \frac{i}{A}$$

& units of current density are

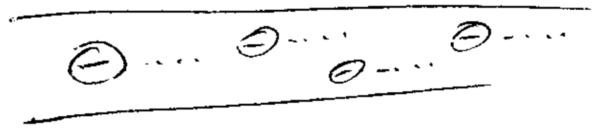
Amps / m^2

can represent current density w/ streamlines. Even though steady current flowing through wire doesn't change, the current density can



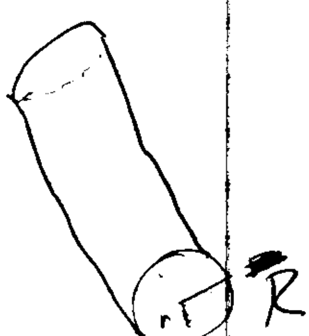
QUESTION:

electrons are moving leftward



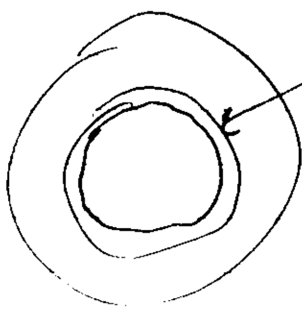
Which way is current flowing?
" " " current density pointing?
" " " E-field " ?

Suppose that the current density through a cylindrical wire varies as $J = a r^2$ where r is radius & $a = 3 \times 10^{11}$ A/m^4.



What is the current through outer portion of wire between $R/2$ & R?
need to use calculus...

Assume that current density through a small ring is uniform



Since $\vec{J}$ is parallel to $d\vec{A}$, we get

$$\vec{J} \cdot d\vec{A} = J\,dA$$

area dA is equal to

$2\pi r\,dr$

so then

$$i = \int \vec{J} \cdot d\vec{A} = \int_{R/2}^{R} a r^2\, 2\pi r\, dr$$

$$= 2\pi a \int_{R/2}^{R} r^3\, dr = 2\pi a \left[\frac{r^4}{4} \Big|_{R/2}^{R} \right]$$

$$= \frac{\pi a}{2} \left\{ R^4 - \frac{R^4}{16} \right\} = \frac{15}{32} \pi a R^4$$