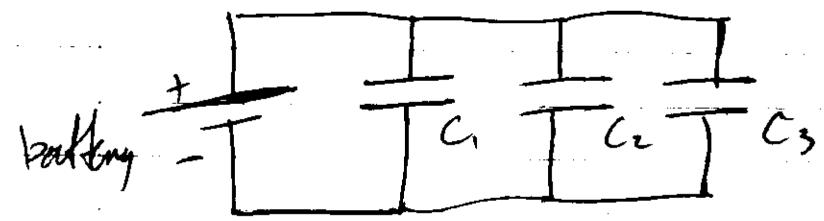
Lecture 16

1 OCT 2014

continuing w/ Chapter 25 – Capacitors

Capacitors in a Parallel Configuration

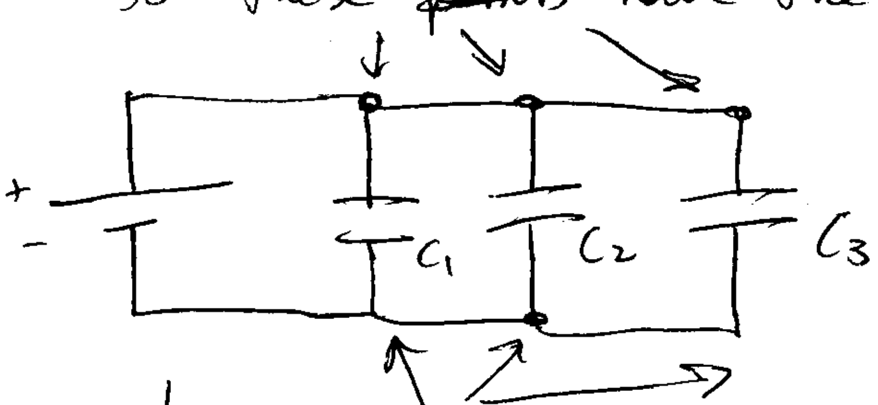


want to replace them w/
an "equivalent capacitor" that
has the same capacitance as
the combination of capacitors.
can be helpful in the design of electrical
circuits.

Important thing to remember about conductors:

charges arrange themselves in such a way
that E-field inside conductor is zero
⇒ all points inside conductor have the
same potential.

So these locations have the same potential



& so do these

⇒ potential difference across each capacitor is the same

So if it is V across battery, then it is V across each capacitor.

geometries of each capacitor could be different

⇒ in general, $C_1 \neq C_2 \neq C_3$

but $q_1 = VC_1$, $q_2 = VC_2$ & $q_3 = VC_3$

& total charge on + side is

$$q = q_1 + q_2 + q_3$$

So the equivalent capacitor has charge q + potential difference V

So its capacitance is given by

$$q = V\,C$$

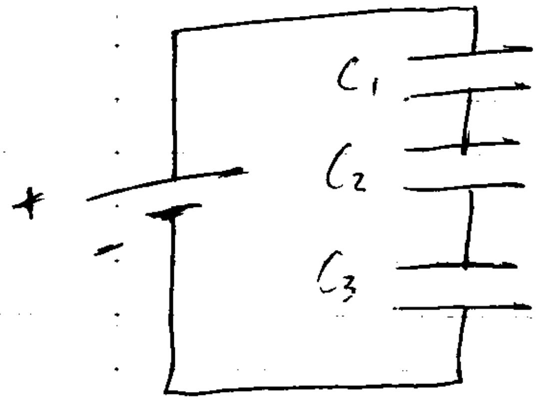
$$\Leftrightarrow \quad q_1 + q_2 + q_3 = VC$$

$$\Leftrightarrow \quad VC_1 + V\,C_2 + VC_3 = VC$$

$$\Leftrightarrow \quad C = C_1 + C_2 + C_3$$

So for capacitors in a **parallel** configuration, equivalent capacitor has capacitance equal to the **sum** of the individual ones.

Capacitors in a series configuration



When battery applies a potential difference, each capacitor ends up w/ the same charge.

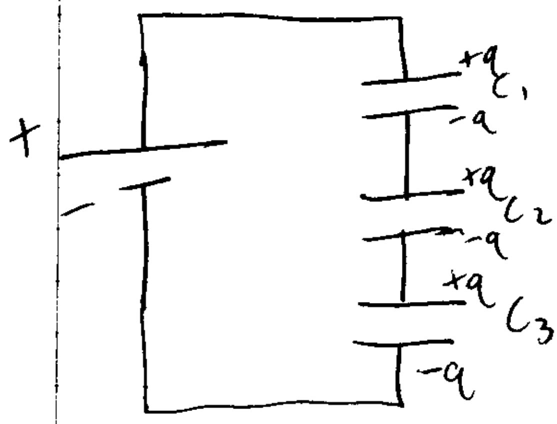
Why is this?

a "chain reaction" occurs in which each capacitor charges the next.

1. When battery is connected, electrons from its negative terminal are repelled & go to bottom plate of C_3. So this acquires a charge of $-q$.
2. Negative charges on top plate are repelled & go to bottom plate of C_2, leaving a charge of $+q$ on top plate of C_3 & creating

$-q$ charge on bottom plate of C_2.

- this continues for C_2 to C_1 & we're left w/



Call potential difference across C_1 as V_1 & same for V_2 & V_3

If potential difference across battery is V, then $V = V_1 + V_2 + V_3$

using that $V_1 = \frac{q}{C_1}$, $V_2 = \frac{q}{C_2}$, $V_3 = \frac{q}{C_3}$

$$\Rightarrow V = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

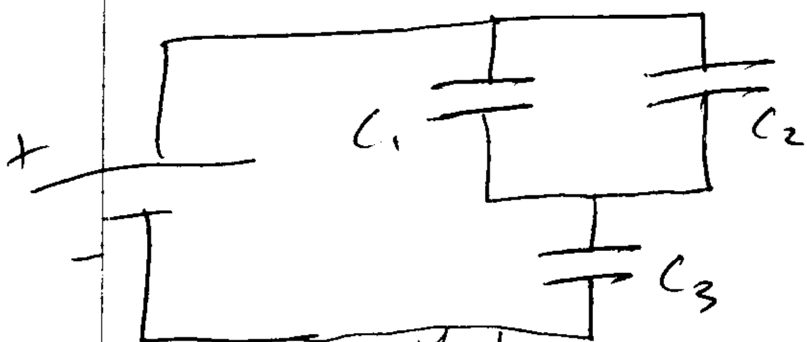
but charge for equivalent capacitor is q & using $C = \frac{q}{V}$, we get

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

or

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

<u>QUESTION</u>: How to find potential & charge on each capacitor in the following circuit?



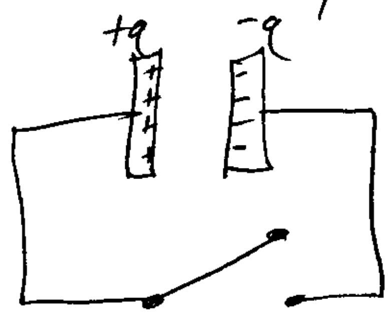
given that battery creates a potential difference of V across its terminals. Idea: find equivalent capacitor, mark

Energy stored in a capacitor

If we charge up a capacitor, it stores electrical potential energy that can be used to light a light bulb temporarily (for example)

How to quantify the potential energy in a capacitor?

Suppose that we charge a capacitor



so when it is charged it has potential energy

if we close the circuit, then charge will flow from right plate around the circuit to left plate

Recall that potential is defined in such a way that potential energy is equal to charge q times potential difference V

$$U = qV$$

So one is tempted to say that this is the potential energy of the capacitor ...

The problem w/ this is that as the charge flows from one plate to the other, the potential difference between the plates decreases. So we need to account for this.

We can do so by using calculus

Suppose that the change on capacitor at some instant is q'. Then the potential difference across the plates at that time instant is

$$V' = \frac{q'}{C}$$

If we then transfer a differential amount of change dq' the little bit of work needed to do so is

$$dW = \frac{q'}{C}\, dq'$$

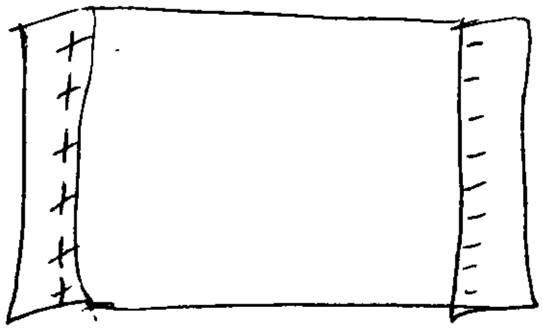
To get the total work, we then sum all of these small amounts of work to get

$$\int dW = \int_0^q \frac{q'}{C}\, dq' = \frac{1}{C}\left[\frac{q'^2}{2}\Big|_0^q\right]$$
$$= \frac{q^2}{2C}$$

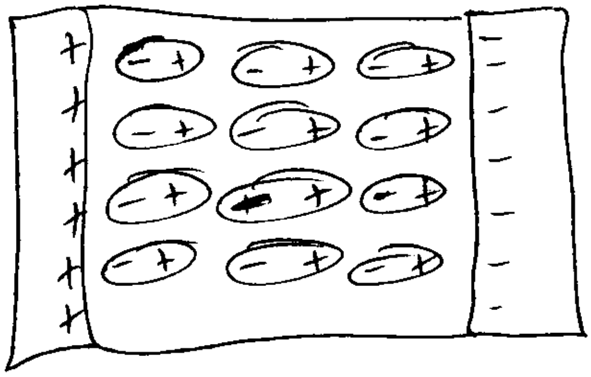
So potential energy stored in a capacitor is $U = \frac{q^2}{2C} = \frac{1}{2}CV^2$

↑ using $q = VC$

Dielectric Material – insert a nonconducting material between plates of a capacitor to increase the capacitance by a factor $K > 1$



atomic view of what's going on ...



molecules in dielectric material will polarize all in one direction

there is the E-field set up by the plates ⟶

but then the dielectric material sets up an opposing E-field ⟵

this has the effect of lowering the potential difference V of the plates which in turn raises capacitance b/c

$C = \frac{Q}{V}$ can quantify this increase w/ K