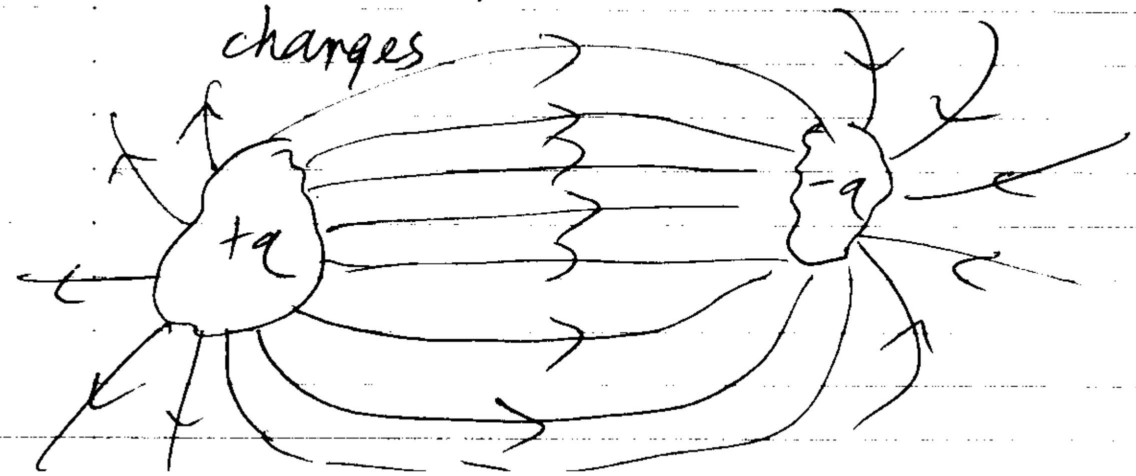
Lecture 15

29 SEP 2014

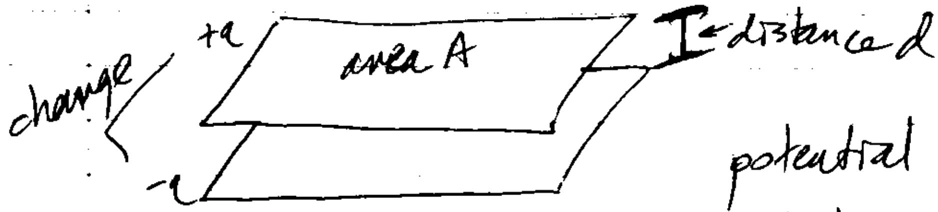
Ch. 25 - capacitance

- capacitor is a device that can store electrical energy. It can be used in applications that require a burst of electrical energy that batteries cannot provide (such as for the flash of a camera)
- First goal is to figure out how much charge a capacitor can store.

Capacitor consists of any two conducting materials w/ opposite charges

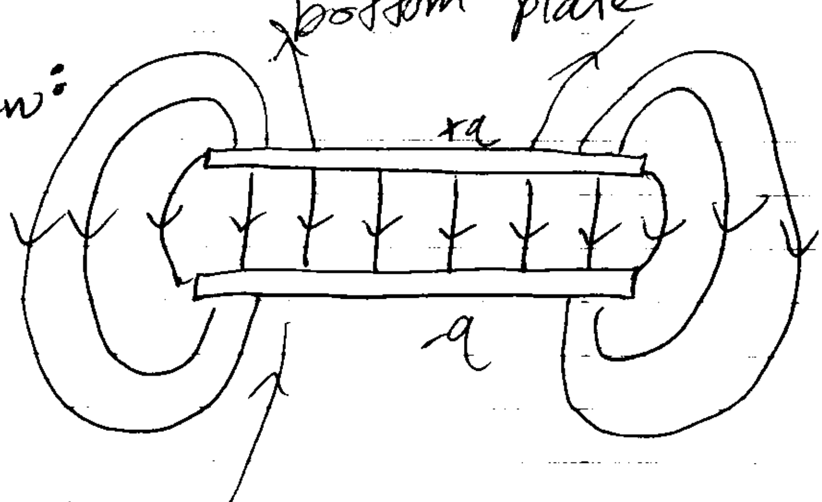


conventional setup is a "parallel plate" capacitor



potential difference V between top plate + bottom plate

other view:



Assume for now that there is no material between plates.

A capacitor is charged if plates have charges of equal magnitudes but opposite signs
Say that charge is q (even though actually +q for one + -q for the other)

Plates are conductors $\Rightarrow$
equipotential for all points on a plate.

represent potential difference between plates by V (for historical reasons)

<u>Capacitance</u> of a capacitor
is a measure of how much charge must be placed on the plates in order to create a certain potential difference across them.

$\boxed{\text{Its value depends only on the geometry of the plates}}$

$q = VC \qquad C = \dfrac{q}{V}$

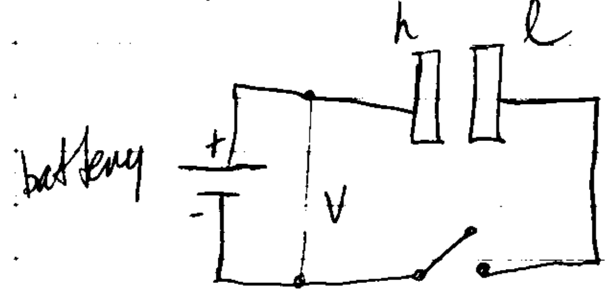
If it takes a charge of 1 C to have a potential difference of 1V, then capacitance is equal to

$$1 \ \frac{C}{V} = 1 \ F \ \text{(Farad)}$$

Charging a Capacitor w/ a battery

Just model a battery as a device which maintains a certain potential difference between its terminals

- can draw a circuit



1) After closing switch, charge can flow through conducting wires. (battery sets up an electric field in the wires)

2) field drives electrons from h to + terminal of battery, so h becomes + charged

3) field drives just as many electrons from - terminal of battery to l plate

Initially, potential is zero between plates

But then charge increases so that potential difference across capacitor is equal to that across battery.

Eventually capacitor becomes fully charged ~~until~~ so that there is no more flow of electrons & electric field between + of battery & r plate is zero (& same for − of battery & l plate)

QUESTION: Does capacitance increase, decrease, or remain the same when

a) charge is doubled

b) potential difference across it is tripled ?

remains the same

Calculating Capacitance

Recipe:

1) Assume a charge q on the plates

2) Calculate E-field between plates using Gauss' law

3) w/ $\vec{E}$ calculated, get potential difference V between the plates

get C from $q = VC$

So for 2) we get E-field from Gauss' law:

$$\frac{q_{enc}}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A}$$

To simplify things, we will always pick a Gaussian surface such that $\vec{E}$ is uniform + in the same direction as $d\vec{A}$ so that $\frac{q_{enc}}{\varepsilon_0} = E \cdot A$

for potential difference

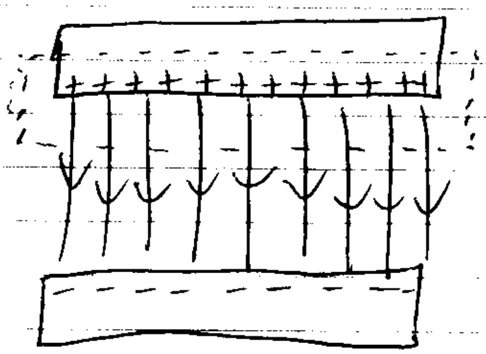
$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

To simplify, choose a path such that path follows electric field line from negative to positive plate, so that

$$\vec{E} \cdot d\vec{s} = -E\,ds \quad \&$$

$$V_f - V_i = \int_i^f E\,ds = \int_-^+ E\,ds$$

Examples: Parallel plate



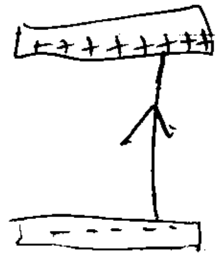
Gaussian surface

$\boxed{\text{QUESTION:}}$ Gauss' law

$\Rightarrow \quad q = \varepsilon_0 E A$

(neglecting fringing effects at edges)

take path to be



so that

$$V = \int_{-}^{+} E\,ds = E\int_{0}^{d} ds = Ed$$

E-field constant

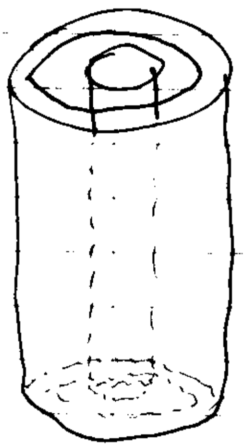
$\Rightarrow$ $q = VC$

$$q = \varepsilon_0 E A = E d C$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d}$$

so capacitance depends only on geometry

Another example: Cylindrical capacitor



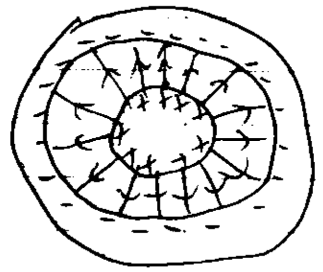
take Gaussian surface to be a cylinder ~~to~~ enclosing inner cylinder.

Then

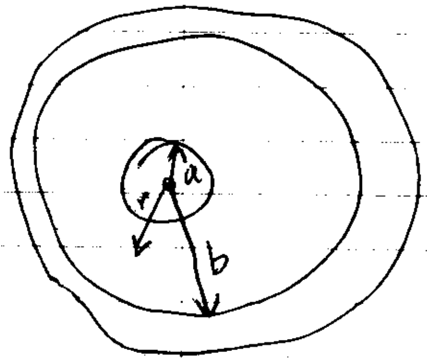
$$\frac{q}{\varepsilon_0} = EA = E(2\pi r \cdot L)$$

where r is radius of Gaussian surface cylinder

Another view:



So $E = \frac{q}{2\pi\varepsilon_0 L} \cdot \frac{1}{r}$



Then

$$V = \int_{-}^{+} E\,ds$$

$$= \frac{-q}{2\pi\varepsilon_0 L} \int_b^a \frac{dr}{r}$$

b/c $dr = -ds$ (going in opposite

$$= \frac{q}{2\pi\varepsilon_0 L} \log\left(\frac{b}{a}\right)$$

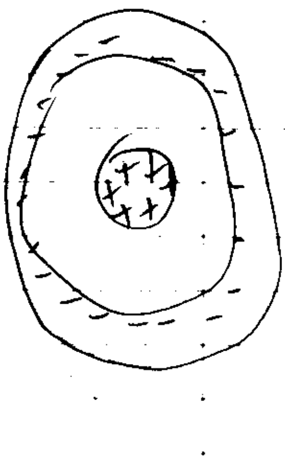
$\Rightarrow$ from $q = VC$

that

$$C = 2\pi\varepsilon_0 \frac{L}{\log(b/a)}$$

Another example: Spherical capacitor

two concentric spherical shells



Pick Gaussian surface to be a sphere enclosing inner sphere shell

Gauss' law $\Rightarrow \frac{q}{\varepsilon_0} = EA = E\,4\pi r^2$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow V = \int_-^+ E\,ds = \frac{-q}{4\pi\varepsilon_0} \int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$