Lecture 13

24 SEP 2014

brief review -

change in potential energy is equal to negative of work done by field on particle

$$\Delta U = U_f - U_i = -W$$

Define zero of potential energy to be when particles have ∞ separation between them

so $$U = -W_\infty$$

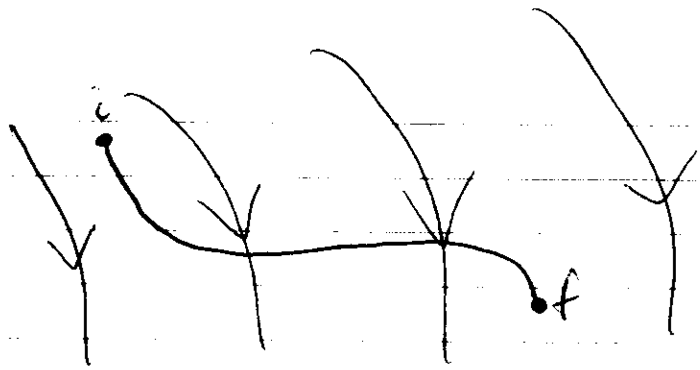
work done by field to bring in from ∞.

Define electric potential to be quantity related to potential energy but independent of charge, i.e.,

$$V = \frac{U}{q}$$

electric potential difference is then

2



$\Delta U = -W$ where

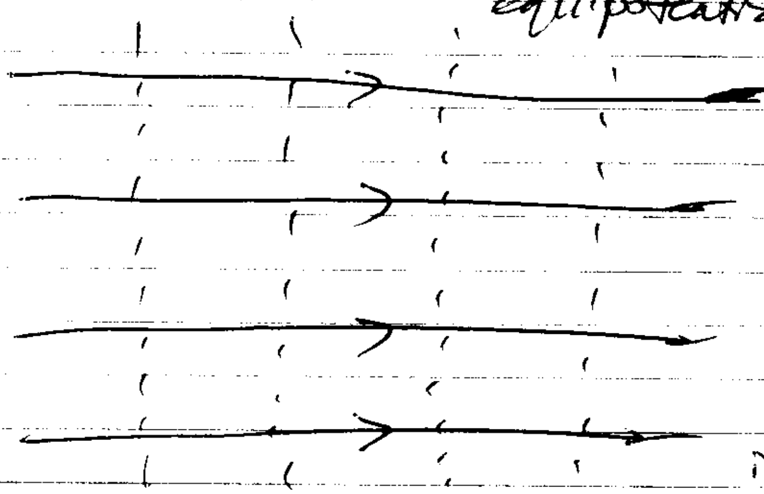
$$W = \int_i^f dW = \int_i^f \vec{F} \cdot ds$$

$$\Rightarrow \Delta V = V_f - V_i$$

$$= \frac{U_f - U_i}{q_0} = -\frac{W}{q_0} = -\int_i^f \vec{E} \cdot d\vec{s}$$

since $\vec{F} = q_0 \vec{E}$

for uniform electric field, lines of equipotential



which lines have higher potential? (think about placing a positive charge in there + figuring out ...

Potential Energy $= U =$ [J]

Electric Potential $= V = \frac{U}{q} = \left[\frac{J}{C}\right] = [V]$ Volts

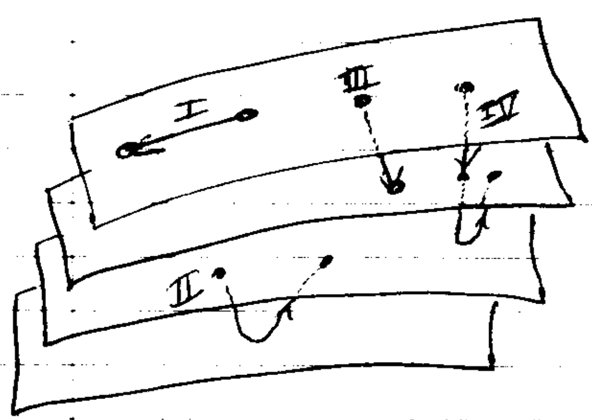
Electric Field $= E = \left[\frac{N}{C}\right]$

$F = qE$

$U = qV$

Electron-Volt $= 1eV =$ Work needed to move an electron thru a potential difference of 1V

Question: Equipotential surfaces

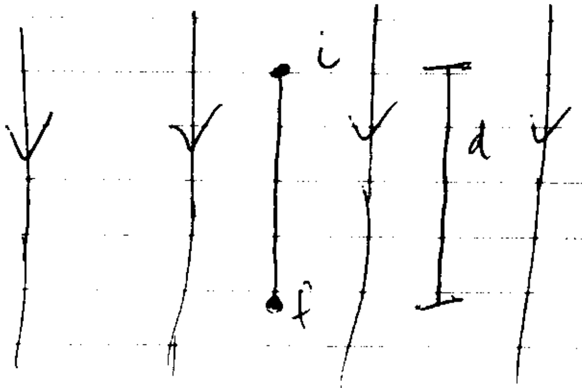


For which paths is work $= 0$? I, II

For which paths is work $\neq 0$? III IV

For which paths is the work the same? I II, III IV

Example: uniform E-field, find potential difference between i + f



use formula

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

Since E-field is aligned w/ direction $\vec{E} \cdot d\vec{s} = E\,ds$

$$\& \quad \int_i^f \vec{E} \cdot d\vec{s} = \int_i^f E\,ds$$

$$= E \int_i^f ds$$

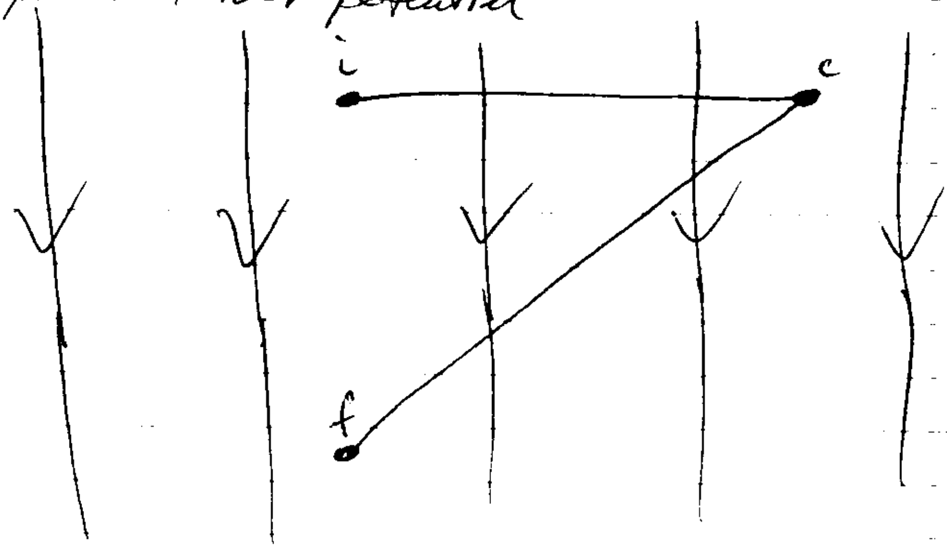
$$= Ed$$

So

$$V_f - V_i = -Ed$$

Suppose we ~~take~~ a different path?
potential difference should be the same b/c electric force is conservative

direction of E-field is from higher to lower potential



So $V_f - V_i = -\left[\int_i^c \vec{E} \cdot d\vec{s} + \int_c^f \vec{E} \cdot d\vec{s} \right]$

for the first path from i to c
E-field is perpendicular to direction of path so
$\vec{E} \cdot d\vec{s} = 0$

For second part from c to f

direction of path is @ angle
45° wrt field so

$$\vec{E} \cdot d\vec{s} = E(ds)\cos(45°)$$

So

$$\int_c^f \vec{E} \cdot d\vec{s} = \int_c^f E(ds)\cos(45°)$$

$$= E\cos(45°)\int_c^f ds$$

How long is path?
equal to $d/\cos(45°)$

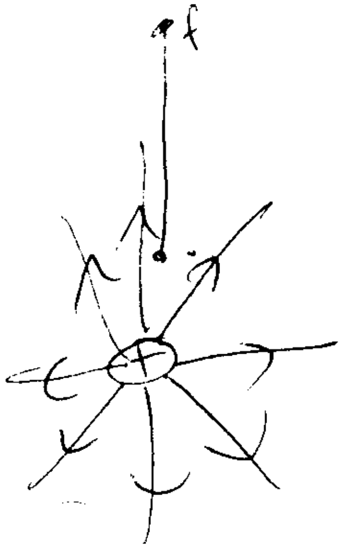
so $\int_c^f ds = d\cos(45°)$

& $V_f - V_i = -\left(E\cos(45)\dfrac{d}{\cos 45}\right)$

$= -Ed$

so consistent w/ other path

Finding electric potential around a point charge



Let the initial ~~state~~ potential be at a distance R from a point charge & final potential be @ ∞

$$V_f - V_i = -W$$

$$= -\int_i^f \vec{E} \cdot d\vec{s}$$

by convention, potential @ ∞ $= 0$ So $V_f = 0$

call $V_i = V$

So $$-V = -\int_i^f \vec{E} \cdot d\vec{s}$$

E-field is aligned w/ path

so $\vec{E} \cdot d\vec{s} = E\,ds = \frac{kq}{s^2} ds$

$$\Rightarrow -V = -\int_R^\infty \frac{kq}{s^2} ds = -kq\left[-s^{-1}\Big|_R^\infty\right] =$$

$$= -\frac{kq}{R} \Rightarrow \boxed{V = \frac{kq}{R}}$$

equation for potential at a distance R from changed particle w/ change q.

What about for a group of changes?

can apply the <u>principle of superposition</u>

$$V_f - V_i = -W = -\int_i^f \vec{E} \cdot d\vec{s}$$

but E-field is vector sum of fields due to individual changes, so

$$\vec{E} = \sum_j \vec{E}_j$$

where $\vec{E}_j$ is E-field from jth particle

$$\Rightarrow V_f - V_i = -\int_i^f \left(\sum_j \vec{E}_j\right)\cdot d\vec{s}$$

$$= -\int_i^f \sum_j \left(\vec{E}_j \cdot d\vec{s}\right)$$

$$= \sum_j -\int_i^f \vec{E}_j \cdot d\vec{s}$$

$$= -\sum_j \underline{V_j}$$

where V_j is potential due to j^{th} particle

w/ $V_f = 0$ + $V_i = V$

$$\Rightarrow V = \sum_j V_j = k \sum_j \frac{q_j}{r_j}$$

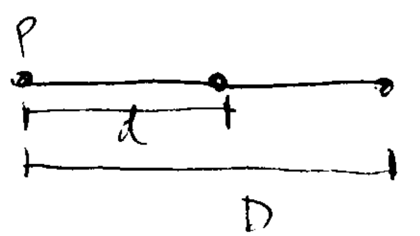
q_j is j^{th} charge + r_j is distance from it

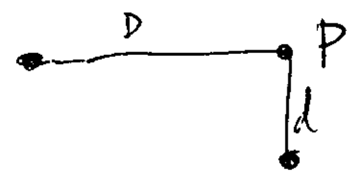
so potentials are easier to calculate

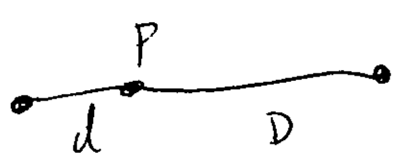
QUESTION: Remember that potential is a scalar

Given are 2 protons

Rank arrangements according to the net potential at point P

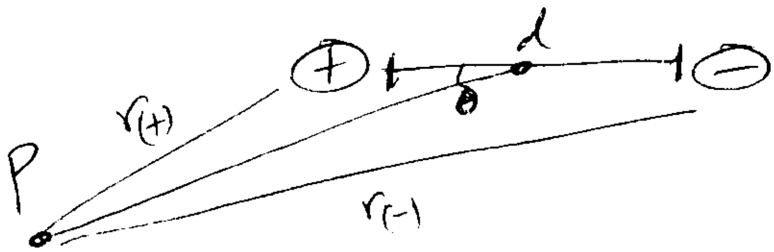






all the same...

Potential due to a dipole



Net potential at point P is

$$V = V_{(+)} + V_{(-)} = \frac{kq}{r_{(+)}} - \frac{kq}{r_{(-)}}$$

$$= kq\left(\frac{1}{r_{(+)}} - \frac{1}{r_{(-)}}\right)$$

$$= kq\left(\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}\right)$$

for P far away,

can approximate $r_{(-)} - r_{(+)} = d\cos\theta$

$r_{(-)}r_{(+)} = r^2$

$$\Rightarrow V = \frac{kq\,d\cos\theta}{r^2} = \frac{kp\cos\theta}{r^2}$$