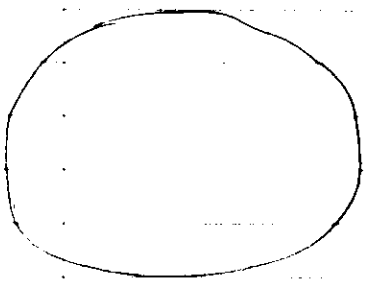
Lecture 11

19 SEP 2014

Gauss' Law + conductors

- recall that a conductor is a material through which electrons can flow freely (metals, ...)
  - think of a conducting sphere. (filled in)



Suppose an excess charge is placed on the sphere, so that it becomes charged.

Then we can show w/ Gauss' law + another assumption that all of the excess charge will be on the surface of the conductor.

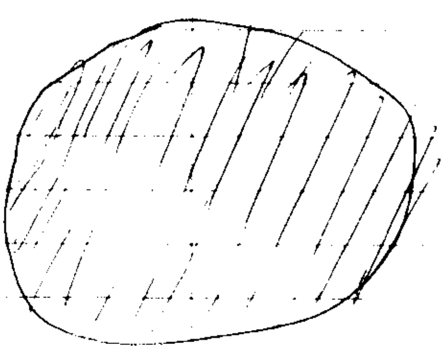
The other assumption needed is that the electric field on the inside of the conductor is $\boxed{\text{zero}}$ after some time...

If it were not zero, then there would be charges incessantly flowing through the conductor, but this is not observed (i.e., the system settles into an equilibrium)

so the charges redistribute themselves in such a way that the E-field inside the conductor is zero.

We can prove this w/ Gauss' law.

$$\Phi = \frac{q_{enc}}{\varepsilon_0}$$

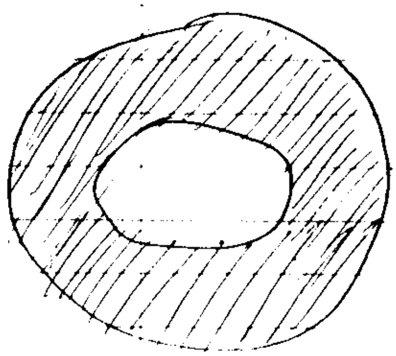


Take Gaussian surface to be any ~~spherical~~ spherical shell strictly inside the conducting sphere.

Then since E-field is zero, $\Phi = 0$ & total enclosed charge is zero.

So if the sphere is charged,
then all of this excess charge
must be @ the surface of
the sphere.

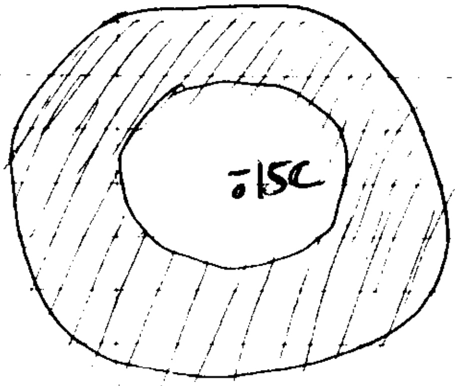
Now suppose that we drill a
cavity through the center of
the conducting sphere, but the
material still has an excess charge.



<u>QUESTION:</u> Where is all
of the excess charge
located?

still on the out side,
b/c no net charge in
cavity

Now suppose that there is a ~~[illegible]~~ point charge placed at the center of the cavity:



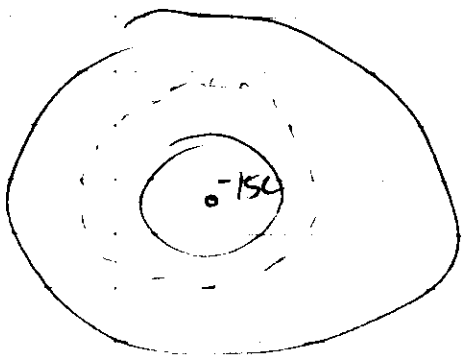
& the conducting material has an excess charge of +10 C

Then what is the charge on the inner & outer surface of the conducting material?

<u>Remember</u>: in conducting material, charge distributes in such a way that E-field in it is zero.

So we apply Gauss' law ...

~~[illegible]~~ to determine charge on inner surface, take the Gaussian surface in the middle of the conducting material



Since E-field is zero, flux on this Gaussian surface is zero &

$$\Phi = \frac{q_{enc}}{\varepsilon_0} \quad \& \quad \Phi = 0$$

$$\Rightarrow \quad q_{\text{point charge}} + q_{\text{inner surface}} = 0$$

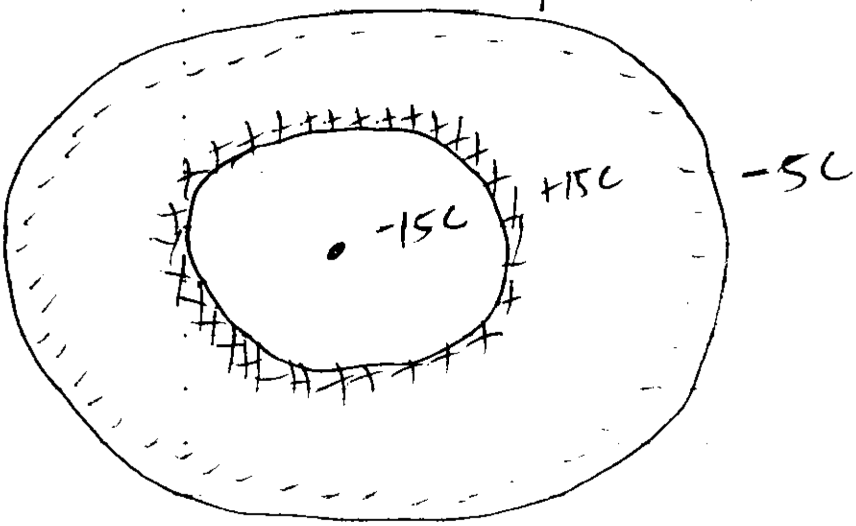
$$\Rightarrow \quad q_{\text{inner surface}} = 15\,C$$

to get charge on outer surface, we use that excess charge for conducting material is +10C, conservation of charge, & that the charge cannot be in the conducting material to conclude that

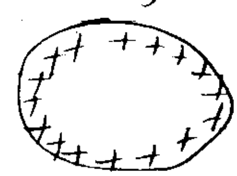
$q_{outer} + q_{inner} = 10\,C$, so that

q_{outer} is $-5\,C$

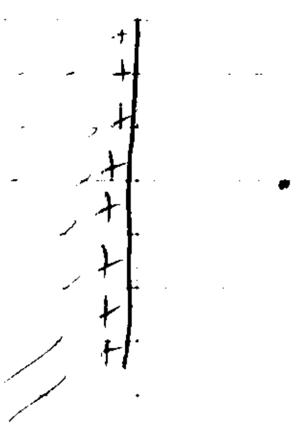
So the picture is



calculating the E-field just outside a conductor



can zoom in to



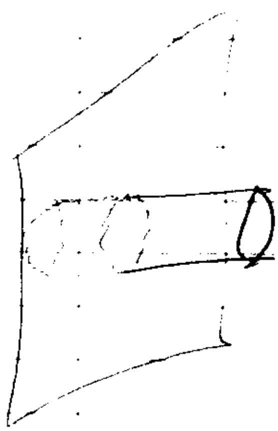
what is E-field here?

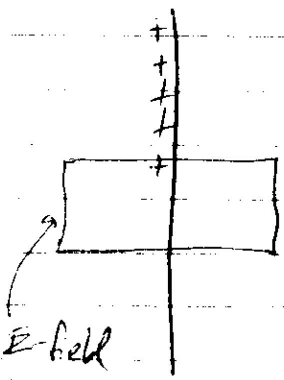
1st, the direction of E-field is perpendicular to surface

$\vec{E} \longrightarrow$

this is from symmetry or the fact that any other direction would cause charges to move which would violate the equilibrium condition

So then pick the Gaussian surface to be a cylinder going through the conductor





E-field here is zero b/c no field inside conductor
$\Rightarrow$ flux here is zero

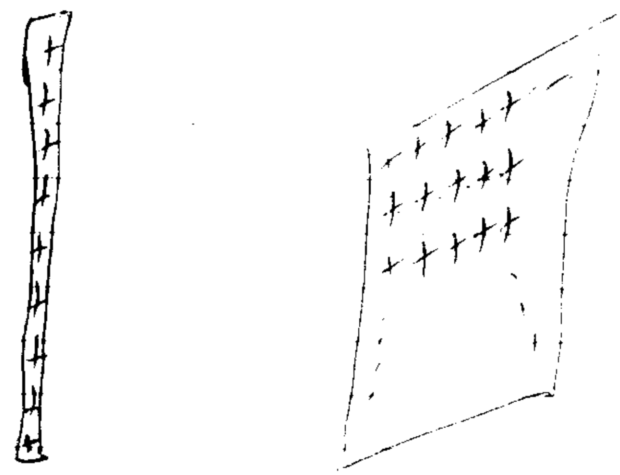
flux around cylinder is zero (inside it is zero & outside E-field skims by this part)

flux through the outer cap is $E \cdot A$
where A is area of cap

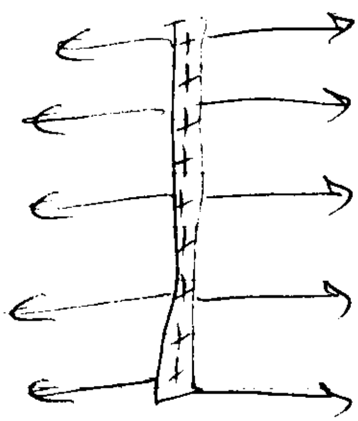
the total charge enclosed by cylinder is
$\sigma \cdot A$ where σ is surface charge density

Gauss' Law $\Rightarrow$ $E \cdot A = \frac{\sigma A}{\epsilon_0} \Rightarrow E = \frac{\sigma}{\epsilon_0}$

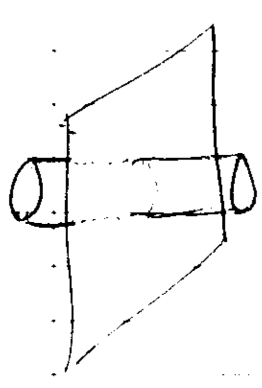
What about a charged plate?



E-field is like (from symmetry argument)



again pick a cylinder going thru sheet & perpendicular to it



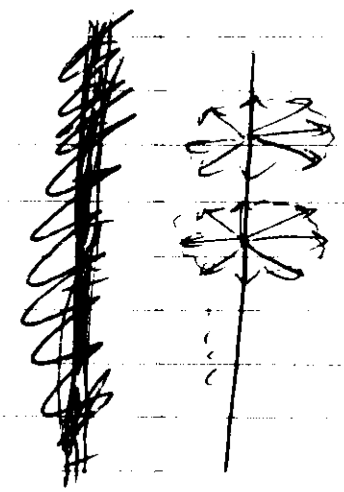
<u>flux</u> through curved part is zero & thru end caps is

$$EA + EA = 2EA$$

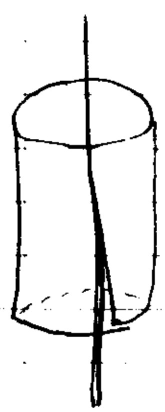
Gauss' law $\Rightarrow$ $2EA = \frac{\sigma A}{\varepsilon_0}$ $\Rightarrow$ $E = \frac{\sigma}{2\varepsilon_0}$

What about E-field from a line of charge?

from symmetry, E-field points radially outward



so best Gaussian surface to take is a cylinder enclosing the line



Since E points outward radially, flux through the end caps is zero (it's just "skimming" by these)

E-field is aligned w/ normal vectors for "little windows" around the cylinder, so

$\Phi = E \cdot$ (area of curved part of cylinder)

$= E \cdot 2\pi r \cdot h$

where r is radius & h is height

Gauss' law $\Rightarrow$

$$\Phi = \frac{q_{enc}}{\varepsilon_0}$$

In this case, $q_{enc} = \lambda \cdot h$ where λ is linear charge density

$$\Rightarrow \quad E \cdot 2\pi r \cdot h = \frac{\lambda \cdot h}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\lambda}{2\pi \varepsilon_0 r}$$

these were all calculations that we did last week that were much more tedious ...