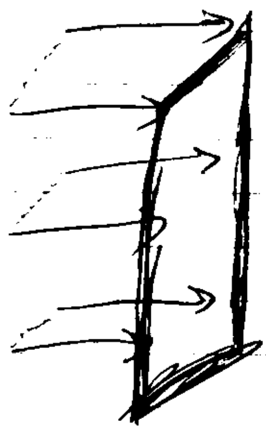
Lecture 10

17 SEP 2014

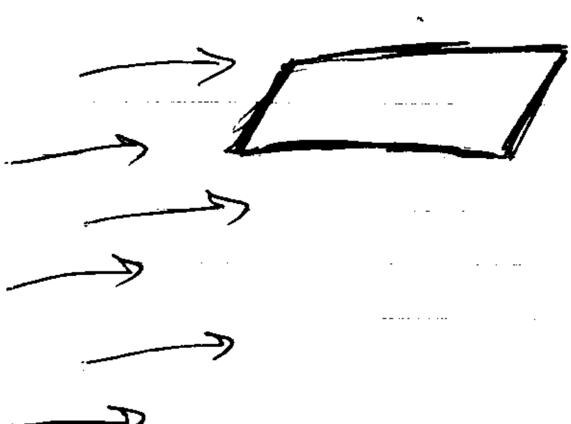
Continuing w/ Gauss' law ...

Recall the notion of electric flux:

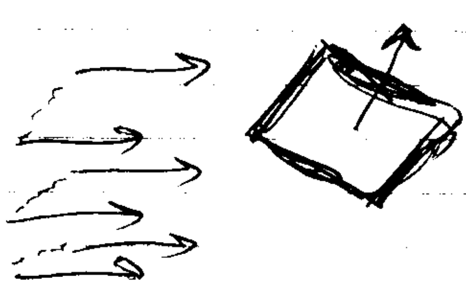
analogy w/ air & a window



volume flow rate is $v \cdot A$ where v is velocity of air & A is area of window



in this situation zero



in general, it is

$v \cos\theta \, A$

or $\vec{V} \cdot \vec{A}$

For E-field, electric flux is $\vec{E} \cdot \vec{A} = \Phi$ ($\vec{E}$ is like air & $\vec{A}$ normal to window)

For a general Gaussian surface,

Divide it into little windows, compute the flux for each window & add them all up.

$$\Phi = \sum_i \vec{E}_i \cdot \Delta\vec{A}$$

Take the limit as the little windows become infinitesmally small, & we get a surface integral

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

refers to E-field vectors on the surface.

Statement of Gauss' Law

The flux through a Gaussian surface is proportional to the net change enclosed by that surface

$$\Phi = \frac{q_{enc}}{\varepsilon_0}$$

q_{enc} is net charge enclosed by surface

ε_0 is from

$$k = \frac{1}{4\pi\varepsilon_0}$$

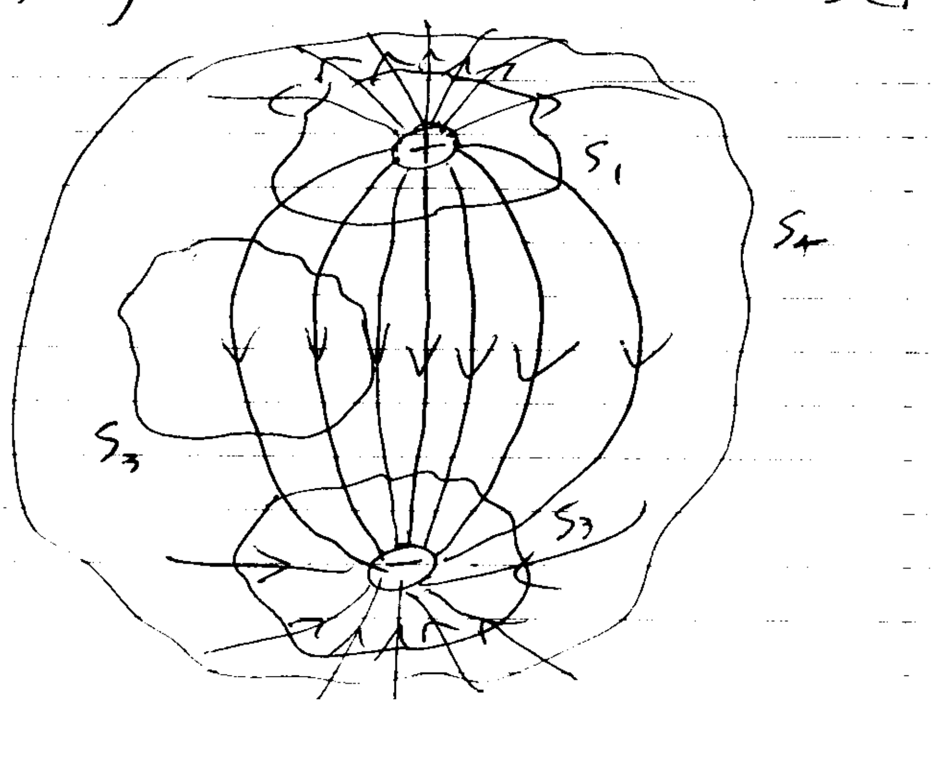
so we can write as

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

impressive aspect of this theorem:

from a parameter related to the surface, we can figure out what's going on inside & vice versa.

QUESTION:



What is flux through surface S_1, S_2, S_3, S_4?

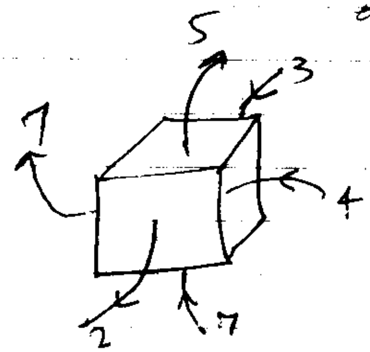
1: $\frac{+q}{\varepsilon_0}$ – consistent w/all field lines going out

2: $\frac{-q}{\varepsilon_0}$ – consistent w/all field lines going in

3: 0 – no change & all field lines going in are exiting

4: 0 – net charge is zero – same # of field lines entering & exiting

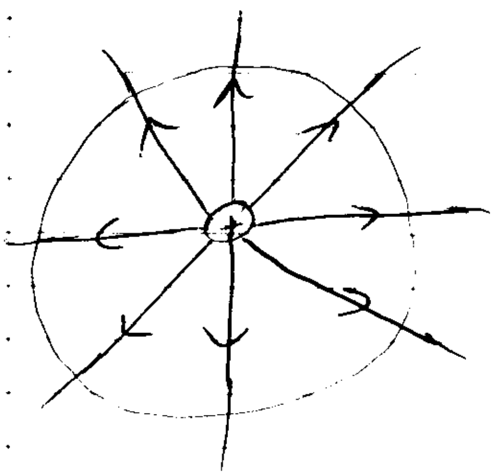
QUESTION:



E-field directions & magnitude of flux given. What is the net charge inside?

Deriving Gauss' Law from Coulomb's Law for E-field

Suppose spherical Gaussian surface w/ radius r enclosing a point charge +q



magnitude of E-field at any point on surface is

$$E = \frac{kq}{r^2} = \frac{q}{4\pi\varepsilon_0 r^2}$$

from before,

flux on surface $\Phi = \oint \vec{E} \cdot d\vec{A}$

in this scenario, E-field always points radially outward + thus is always aligned w/ normal vectors of "little windows" that we put on the surface of the sphere. So, $\theta = 0$ + $\cos\theta = 1$ $\Rightarrow \vec{E} \cdot d\vec{A} = E\,dA$

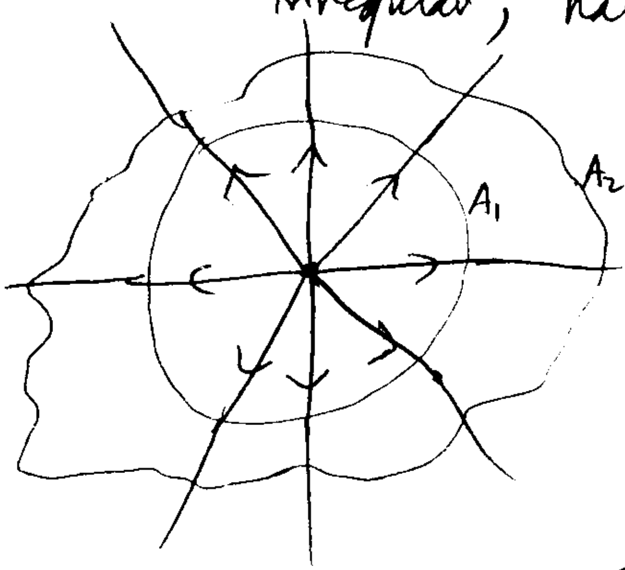
So then $\Phi = \oint \underbrace{\frac{q}{4\pi\varepsilon_0 r^2}}_{\text{all constants wrt integral, so}} dA$

$\Phi = \frac{q}{4\pi\varepsilon_0 r^2} \underbrace{\oint dA}$

this is an integral over the surface of the sphere, which gives its surface area $4\pi r^2$

$\Rightarrow \Phi = \frac{q}{4\pi\varepsilon_0 r^2} \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$

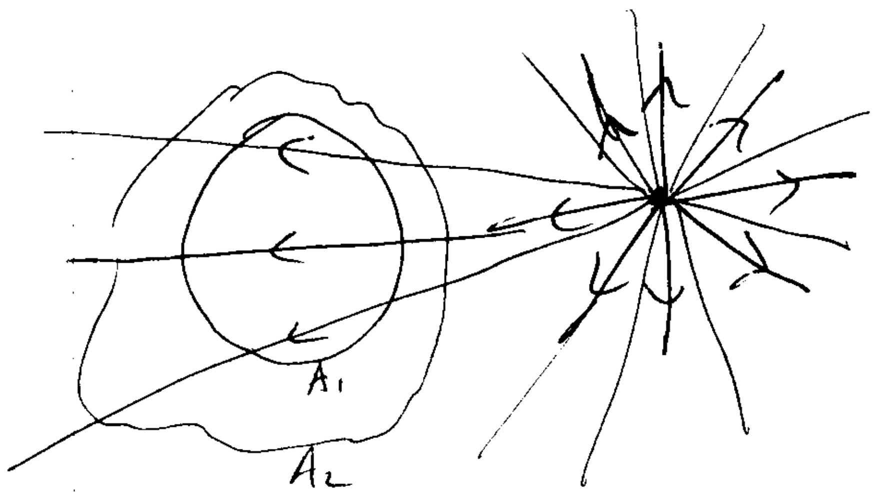
Now what if we make the surface irregular, having an arbitrary shape



the same # of field lines go thru A_2 as there are going thru A_1. So the flux through A_2 & A_1 should be the same:

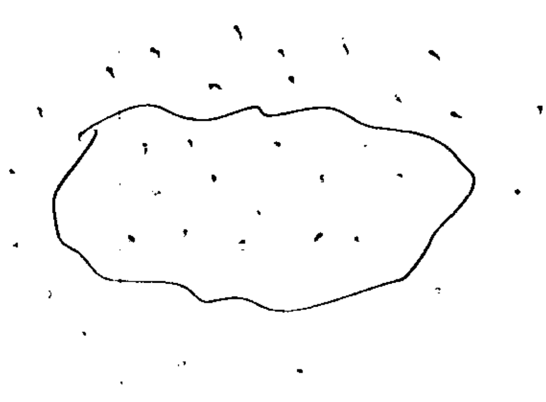
$\Phi = \oint_{A_2} \vec{E} \cdot d\vec{A} = \oint_{A_1} \vec{E} \cdot d\vec{A}$

What if the point change is outside the surface?



field lines all enter + exit the area, so flux due to these is <u>zero</u>.

<u>Arbitrary situation</u>:



<u>QUESTION</u>: What principle to use to finish off the proof?

<u>superposition principle</u>

label each charge as q_i + E-field at various locations on surface as $\vec{E}_i$.

Then net field at some location on surface is

$$\vec{E} = \sum_i \vec{E}_i \Rightarrow$$

net flux through surface

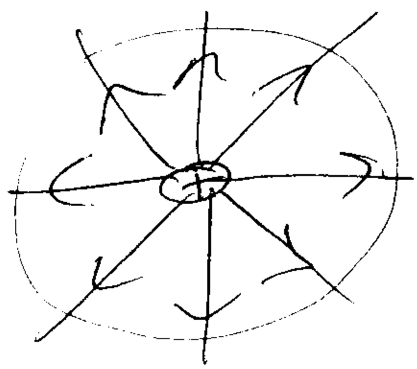
$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \oint \left(\sum_i \vec{E}_i\right) \cdot d\vec{A}$$

$$= \sum_i \oint \vec{E}_i \cdot d\vec{A}$$

$$= \sum_{\substack{i \\ (\text{those inside} \\ \text{surface})}} \frac{q_i}{\varepsilon_0} = \frac{q_{enc}}{\varepsilon_0} \qquad \Box$$

Deriving Coulomb Law from Gauss' Law (almost)



take Gaussian surface to be sphere

then $\Phi = \frac{q}{\varepsilon_0}$

$\Phi = \oint \vec{E} \cdot d\vec{A}$

need to assume that E-field is pointing radially outward & has same magnitude at every point of sphere.

then

$$\Phi = \oint E\,dA = E \oint dA$$

$$= E \cdot 4\pi r^2$$

$$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0 r^2}$$