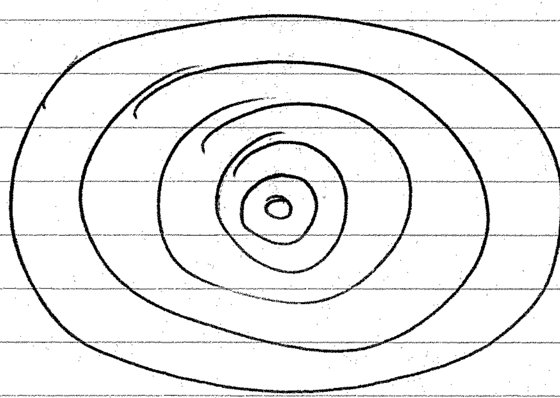
Lecture 2

27 AUG 2014

- pickup from pages 10–12 of Lecture 1

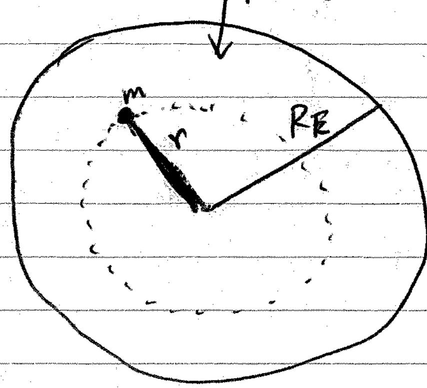
13-5: Gravitation inside the Earth

can think of the Earth as consisting of concentric spherical shells



So suppose that a particle is inside the earth at radius $r < R_E$

no net force on m



From the shell theorem we know that the shells outside radius r contribute no net force to the particle

The only force is due to concentric shells w/ radius less than r.

So suppose that the mass of the Earth is uniformly distributed in the full volume of the sphere

By Newton's law of universal gravitation, & the shell theorem, the magnitude of the force on the particle is

$$F = \frac{G \cdot M_{ins} \cdot m}{r^2}$$

where M_{ins} is the mass inside the sphere of radius r.

Let M be the total mass of earth then the mass density is $\rho = \frac{M}{V}$

where $V = \frac{4\pi R_E^3}{3}$ (volume of sphere)

we can get M_{ins} by $M_{ins} = \rho \cdot V_{ins}$

where $V_{ms} = \frac{4\pi}{3} r^3$

So $M_{ins} = M \cdot \frac{r^3}{R_E^3}$

& then finally

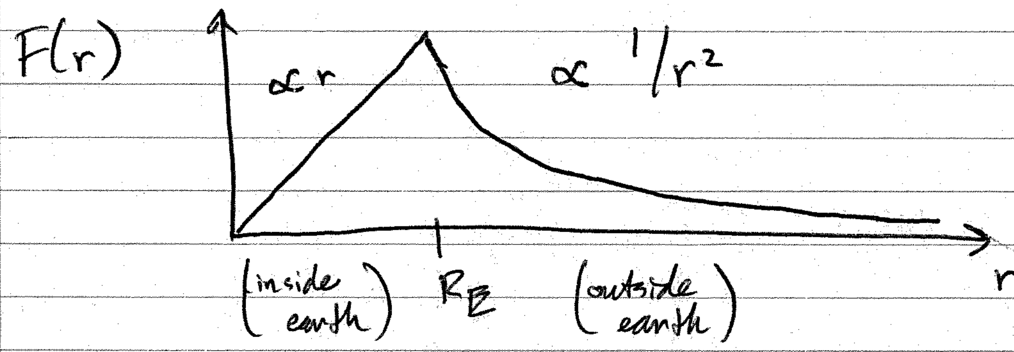
$$F = \frac{G \cdot \left(M \cdot \frac{r^3}{R_E^3}\right) \cdot m}{r^2} = \left(\frac{G \cdot M \cdot m}{R_E^3}\right) \cdot r$$

thinking of G, M, m & R_E as constants, this means that the force scales linearly with r when inside the earth

This ~~[illegible]~~ is analogous to Hooke's law for a mass on a spring

Question: From this analogy w/ Hooke's law, can ~~anyone~~ you conclude what would happen if we dropped a point mass through a tunnel that goes through the center of the earth?

so magnitude of force as a function of r looks like



Question: In the 2012 remake of "Total Recall", Colin Farrell rides a train that falls through the center of the Earth.

In the movie, he experiences normal gravity until hitting the core, a moment of weightlessness at the core & then gravity in the other direction as the train goes to the other side of the Earth.

Is this what would happen or just "Hollywood BS"?

B-6 Gravitational Potential Energy

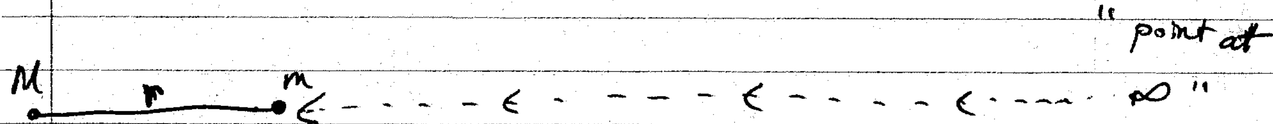
work is required to move objects against a gravitational force. The energy associated to an object after it "has been raised" is called potential energy

(Example: water behind a dam has potential energy that can be harnessed)

We need some reference point in order to establish ~~the magnitude of the~~ the value of the potential energy at various points in space. The usual convention is to take this reference point to be "the point at infinity"

~~The potential energy~~

Suppose there is a single point mass M then ~~we imagine~~ to calculate the potential energy between M & another point mass m at distance r from M, we calculate the work required to bring in m from ∞ to distance r. This

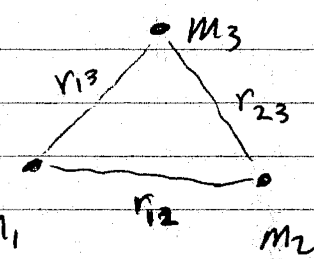


So the work is the force applied over this distance. Since the force varies w/ distance, we need to integrate.

$$U = \int_{\infty}^{r} F \cdot dx = \int_{\infty}^{r} \frac{G \cdot m \cdot M}{x^2}\, dx = \frac{-G M \cdot m}{r}$$

(By convention, the potential energy at point x is taken to be negative & increasing to zero as $x \to \infty$.)

For multiple particles,
we are interested in computing the total potential energy. To do so, we calculate for each pair, then sum the results. E.g., for a 3-particle system,



it would be

$$U = -\left(\frac{G m_1 m_2}{r_{12}} + \frac{G m_1 m_3}{r_{13}} + \frac{G m_2 m_3}{r_{23}} \right)$$

gravitational potential is just the gravitational potential energy of a point mass of 1 kg.

$$V = -\frac{GM}{r}$$

Given that the ~~[illegible]~~ magnitude of the gravitational field is $g = \frac{GM}{r^2}$

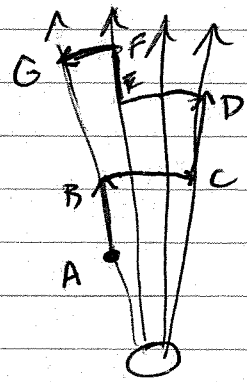
& that for the potential is $-\frac{GM}{r} = V$

we have that

$$g = -\frac{dV}{dr}$$

(also follows from definitions & fundamental theorem of calculus)

The potential energy calculation is independent of the path taken from ∞, b/c gravity is a conservative force



work done along circular arcs is zero b/c force is perpendicular to the direction of these arcs

so contributions only come from $A \to B$, $C \to D$, + $E \to F$

this shows path independence of work calculation