Explicit receivers for pure-interference bosonic multiple access channels

Mark M. Wilde

School of Computer Science,
McGill University

Joint work with **Saikat Guha**

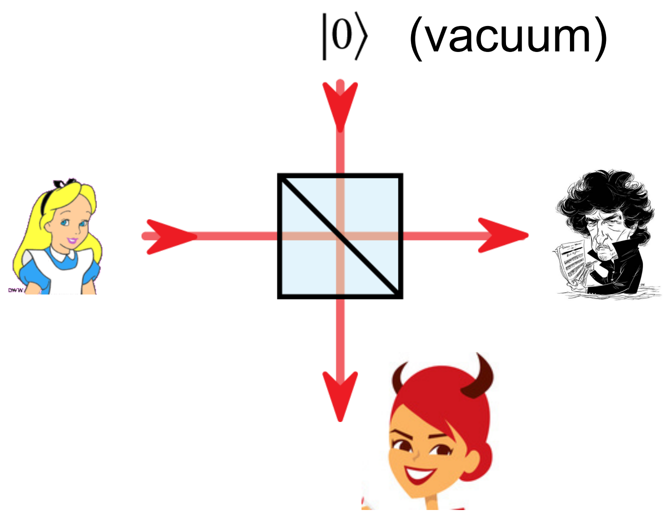
arXiv:1204.0521

International Symposium on Information Theory and its Applications,
Honolulu, Hawaii, October 29, 2012

Pure-Loss Bosonic Channels

Pure-Loss Bosonic Channel

(*models fiber optic or free space transmission*)



Heisenberg input-output relation for channel:

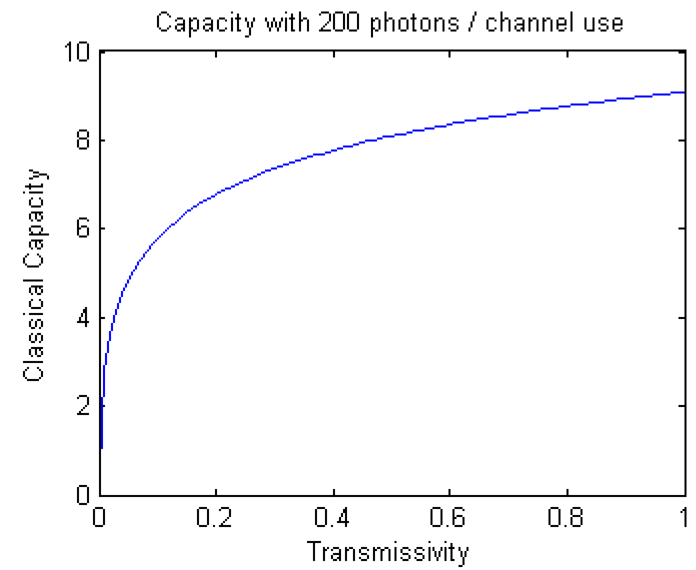
$$\hat{b} = \sqrt{\eta}\hat{a} + \sqrt{1-\eta}\hat{e}$$

Weedbrook *et al*., Gaussian Quantum Information, *Reviews of Modern Physics* (2011).

Sending Classical Data over Bosonic Channels

Classical capacity of **lossy bosonic channel** is exactly

$$g(\eta N_S)$$

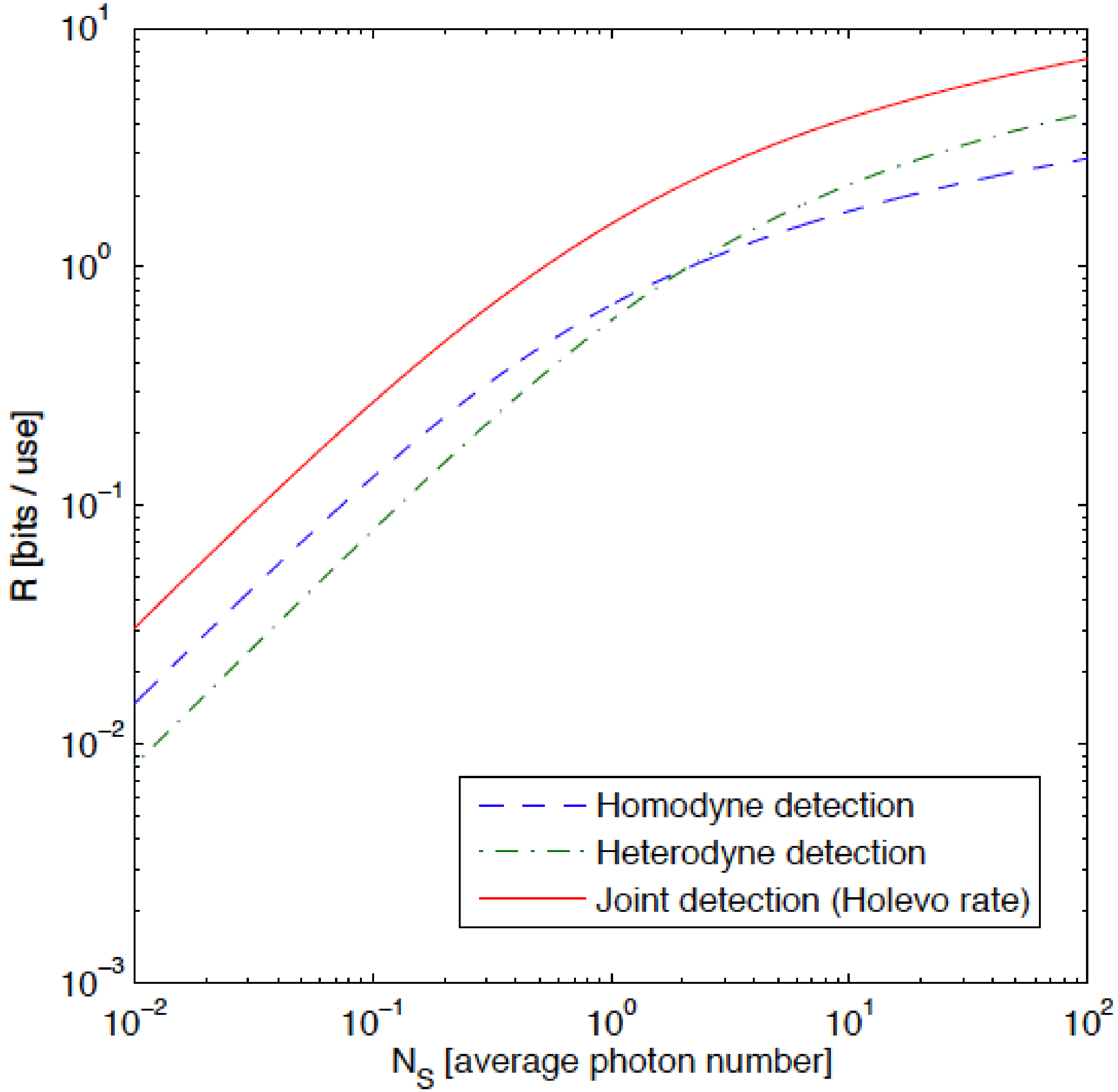


where η is **transmissivity** of channel,
N_S is the **mean input photon number**,
and $g(x) = (x+1)\log(x+1) - x \log x$
is the **entropy** of a **thermal state**
with photon number x

Can **achieve** this capacity by selecting **coherent states** randomly according to a complex, isotropic Gaussian prior with variance N_S

Giovannetti *et al*., *Physical Review Letters* 92, 027902 (2004)

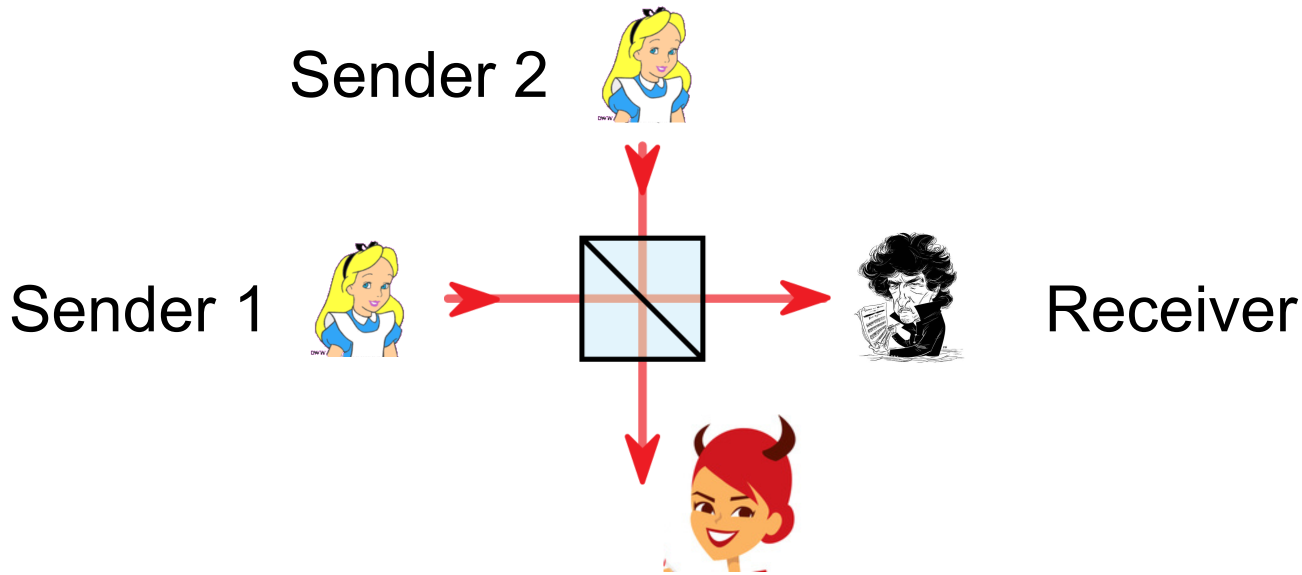
Quantum strategies are better



Giovannetti et al., PRL 92, 027902 (2004).

This work: Bosonic Multiple Access Channel

Simple model of a "pure-interference" bosonic multiple access channel:



Coherent-state inputs $|\alpha\rangle$ and $|\beta\rangle$ lead to output

$$|\sqrt{\eta}\alpha + \sqrt{1-\eta}\beta\rangle$$

Yen, Shapiro. PRA (2005)

Codebook for bosonic MAC

First sender chooses codewords **randomly** according to

$$p_{N_S}(\alpha) \equiv (1/\pi N_S) \exp\left\{-|\alpha|^2/N_S\right\}$$

First sender's codebook has the form: $\{\,|\alpha^n(l)\rangle\,\}_l$

where $|\alpha^n(l)\rangle \equiv |\alpha_1(l)\rangle \otimes \cdots \otimes |\alpha_n(l)\rangle$

$$|\alpha\rangle \equiv D(\alpha)|0\rangle \equiv \exp\left\{\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right\}|0\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

2nd sender's codebook is similar: $\{\,|\beta^n(m)\rangle\,\}_m$

Output Codewords

If first sender chooses message *l* and second sender chooses message *m*, then input to channel is

$$|\alpha^n(l)\rangle \otimes |\beta^n(m)\rangle$$

The channel output is as follows:

$$|\gamma^n(l,m)\rangle \equiv |\gamma_1(l,m)\rangle \otimes |\gamma_2(l,m)\rangle \otimes \cdots \otimes |\gamma_n(l,m)\rangle$$

where $\gamma_i(l,m) \equiv \sqrt{\eta}\,\alpha_i(l) + \sqrt{1-\eta}\,\beta_i(m)$

Sequential Decoding for bosonic MAC

Sequential decoding measurements are

$$\left\{ \left|\gamma^{n}\left(l,m\right)\right\rangle \left\langle \gamma^{n}\left(l,m\right)\right|, I^{\otimes n} - \left|\gamma^{n}\left(l,m\right)\right\rangle \left\langle \gamma^{n}\left(l,m\right)\right| \right\}$$

Observing that

$$\left|\gamma^{n}\left(l,m\right)\right\rangle = D\left(\gamma_{1}\left(l,m\right)\right) \otimes \cdots \otimes D\left(\gamma_{n}\left(l,m\right)\right) \left|0\right\rangle^{\otimes n}$$

1) Displace the *n*-mode codeword state by

$$D\left(-\gamma_{1}\left(l,m\right)\right) \otimes \cdots \otimes D\left(-\gamma_{n}\left(l,m\right)\right)$$

2) Perform a “vacuum-or-not” measurement:

$$\left\{ \left|0\right\rangle \left\langle 0\right|^{\otimes n}, \quad I^{\otimes n} - \left|0\right\rangle \left\langle 0\right|^{\otimes n} \right\}$$

3) If “NOT VAC,” displace back:

$$D\left(\gamma_{1}\left(l,m\right)\right) \otimes \cdots \otimes D\left(\gamma_{n}\left(l,m\right)\right)$$

Key Tool: Noncommutative Union Bound

Holds for a subnormalized state ρ and projectors Π_1, …, Π_N:

$$1 - \text{Tr}\{\Pi_N \cdots \Pi_1 \rho \Pi_1 \cdots \Pi_N\} \leq 2\sqrt{\sum_{i=1}^{N} \text{Tr}\{(I - \Pi_i)\rho\}}$$

Consider similarity with union bound:

$$\Pr\{(A_1 \cap \cdots \cap A_N)^c\} = \Pr\{A_1^c \cup \cdots \cup A_N^c\} \leq \sum_{i=1}^{N} \Pr\{A_i^c\}$$

P. Sen, "Achieving the Han-Kobayashi inner bound ...", arXiv:1109.0802

Technical Difficulty

There is a technical issue with the current analysis

We can only show that the following rate region is achievable with this decoding strategy:

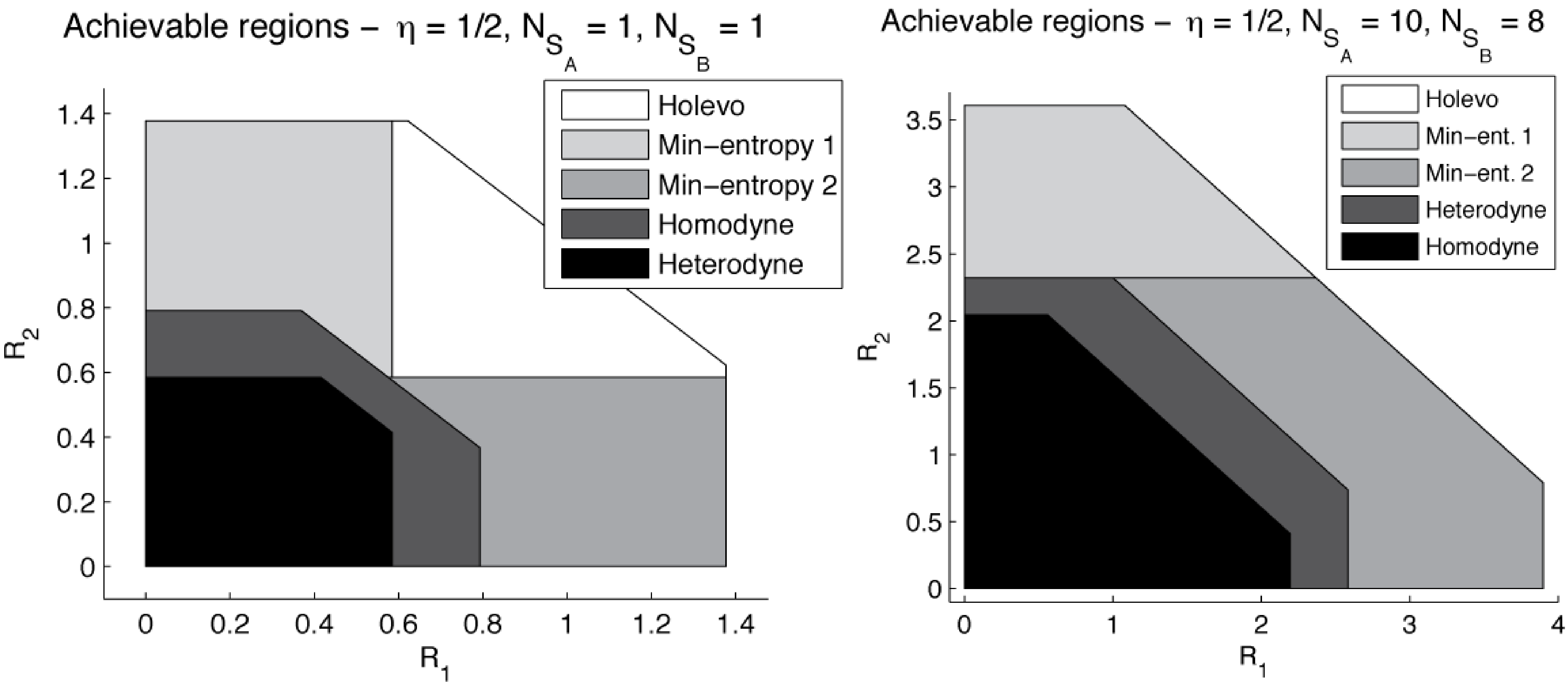
$$R_1 \leq \log_2(\eta N_{S_A} + 1)$$
$$R_2 \leq g((1-\eta) N_{S_B})$$
$$R_1 + R_2 \leq g(\eta N_{S_A} + (1-\eta) N_{S_B})$$

or the following symmetric one:

$$R_1 \leq g(\eta N_{S_A})$$
$$R_2 \leq \log_2((1-\eta) N_{S_B} + 1)$$
$$R_1 + R_2 \leq g(\eta N_{S_A} + (1-\eta) N_{S_B})$$

But not the Holevo rates found by Yen and Shapiro!

Examples



(a) (b)

In some cases, the decoding strategy achieves the Yen-Shapiro rates for the bosonic MAC (right figure),

while in others, it does not (left figure)

Conclusion and Current Work

Quantum sequential decoding leads to a “practical” receiver
(“practical” in the sense that we can implement)

It is **impractical** because it requires
an exponential number of measurements

Open question: How to reduce the number of measurements?

Polar codes might be helpful here (arXiv:1202.0533)

We conjecture that the decoding strategy given here
can achieve the Yen-Shapiro rates,
and it is just the error analysis that needs improvement