Convolutional Entanglement Distillation

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Joint work with
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Overview

Review entanglement distillation and quantum convolutional codes

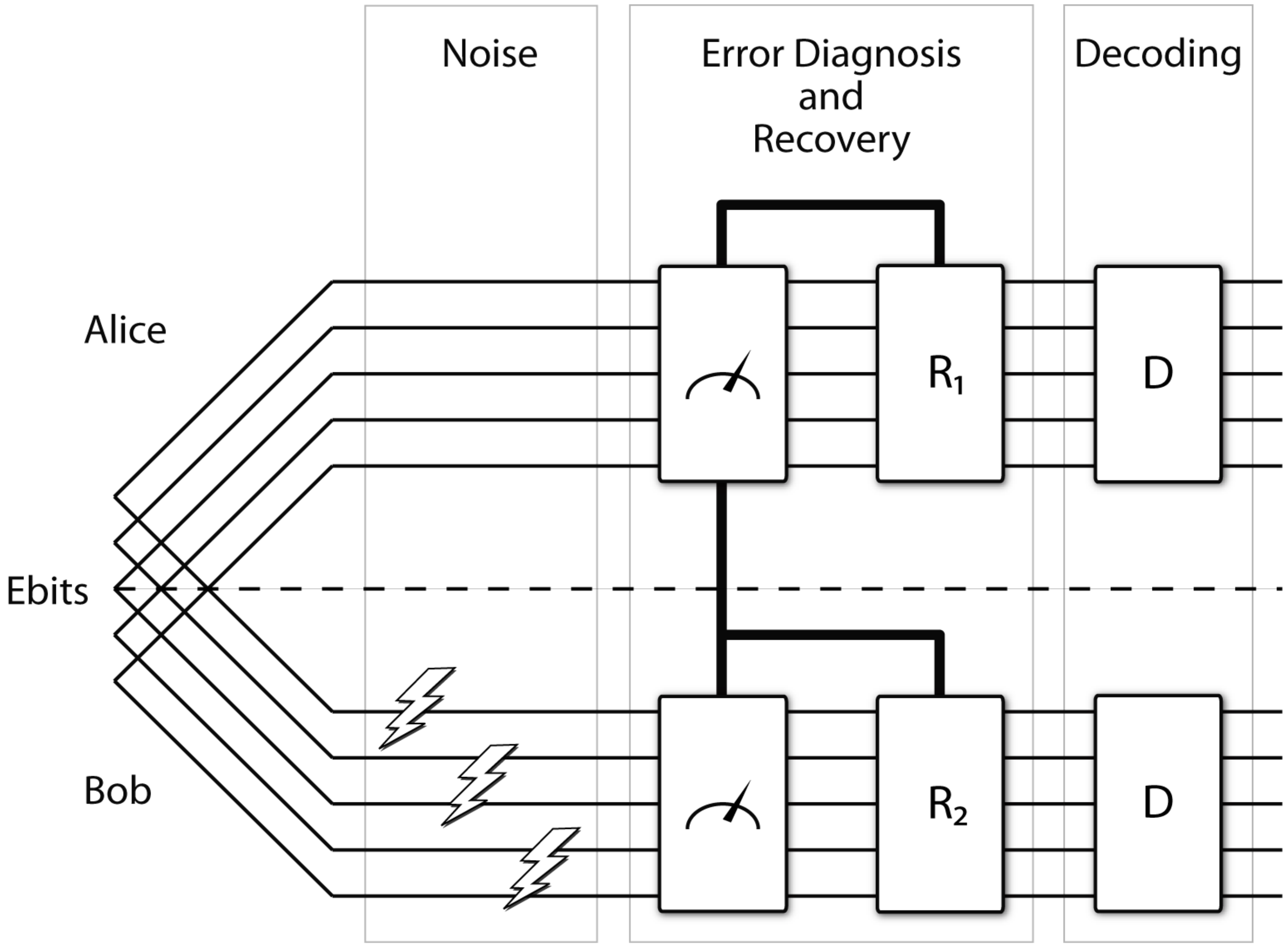
Shifted Symplectic Product

Introduce convolutional entanglement distillation

Protocol Construction

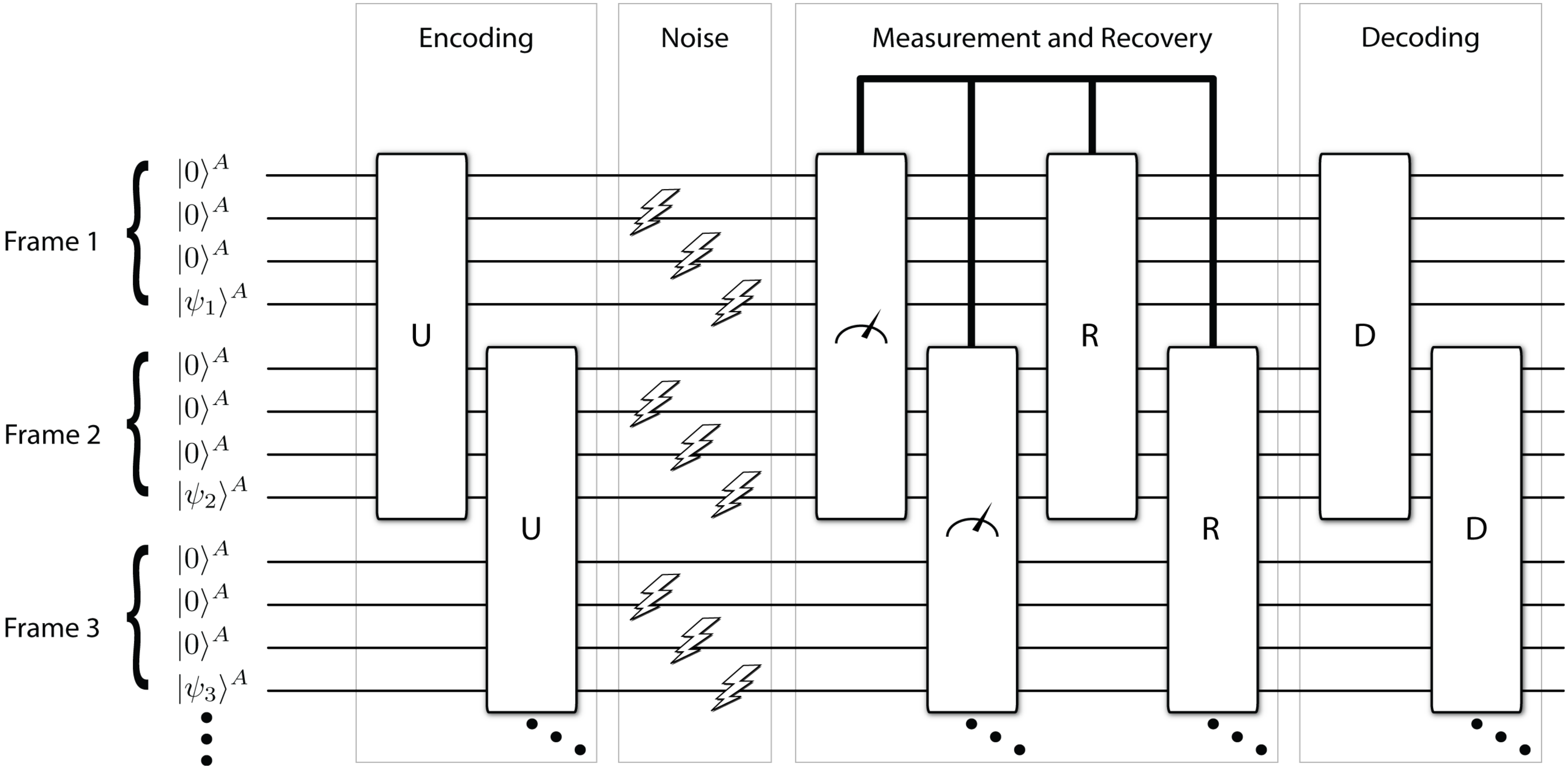
Conclude with open questions

Entanglement Distillation



C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, W. K. Wootters, Mixed State Entanglement and Quantum Error Correction. *Physical Review A* **54**, 3824 (1996).

Quantum Convolutional Coding



Ollivier, Tillich, Physical Review Letters ***91,*** *177902 (2003).*
Forney, Grassl, Guha, IEEE Trans. Inf. Theory ***53****, 865-880 (2007).*
Grassl, Rötteler, In proceedings of ISIT (2005, 2006, 2007).

Example Stabilizer for a QCC

$$\cdots \left| \begin{array}{c} III \\ III \\ III \\ III \end{array} \right| \begin{array}{c} ZII \\ IZI \\ III \\ III \end{array} \left| \begin{array}{c} III \\ III \\ ZII \\ IZI \end{array} \right| \begin{array}{c} III \\ III \\ III \\ III \end{array} \left| \begin{array}{c} III \\ III \\ III \\ III \end{array} \right| \cdots$$

Unencoded Stabilizer

$$\cdots |0\rangle \, |0\rangle \, |\psi_1\rangle \, |0\rangle \, |0\rangle \, |\psi_2\rangle \cdots$$

$$\cdots \left| \begin{array}{c} III \\ III \\ III \\ III \end{array} \right| \begin{array}{c} XXX \\ ZZZ \\ III \\ III \end{array} \left| \begin{array}{c} XZY \\ ZYX \\ XXX \\ ZZZ \end{array} \right| \begin{array}{c} III \\ III \\ XZY \\ ZYX \end{array} \left| \begin{array}{c} III \\ III \\ III \\ III \end{array} \right| \cdots$$

Encoded Stabilizer

*Forney, Grassl, Guha, IEEE Trans. Inf. Theory **53**, 865-880 (2007).*

Shifted Symplectic Product

Simple way to specify the **commutativity constraint** on the generators of a quantum convolutional code.

Given a **binary polynomial check matrix**

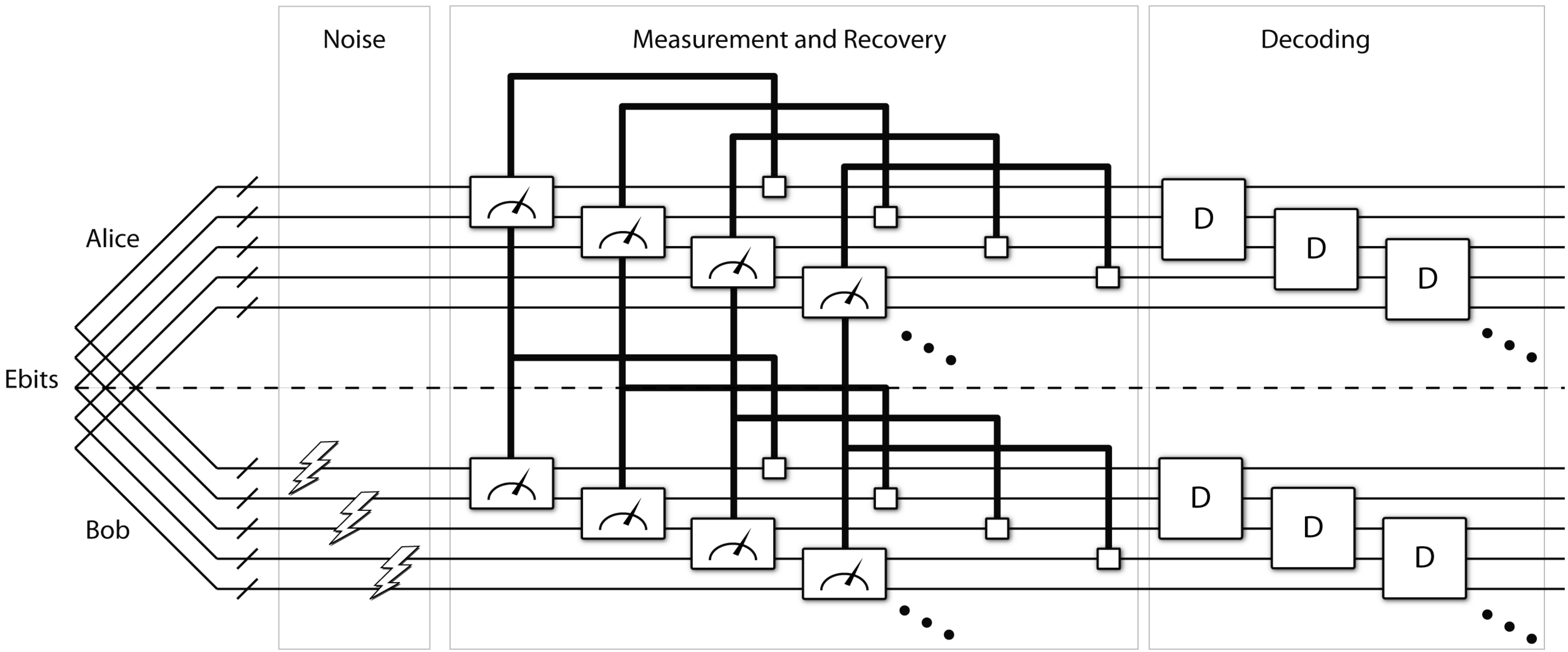
$$H(D) = [Z(D)|X(D)]$$

The **shifted symplectic product matrix** gives the full commutativity relations

$$\Omega(D) = Z(D)X^T(D^{-1}) + X(D)Z^T(D^{-1})$$

This matrix should vanish for a **valid quantum convolutional code**, but does not need to if we have **extra entanglement**

Convolutional Entanglement Distillation



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Convolutional Protocol Construction

How to construct a protocol
when generators **do not commute**?

$$\begin{bmatrix} \mathbf{u}_1(D) \\ \mathbf{u}_2(D) \\ \vdots \\ \mathbf{u}_m(D) \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{z}_1(D) & \mathbf{x}_1(D) \\ \mathbf{z}_2(D) & \mathbf{x}_2(D) \\ \vdots & \vdots \\ \mathbf{z}_m(D) & \mathbf{x}_m(D) \end{array}\right]$$

Simply **augment** the check matrix
and assume **extra noiseless entanglement** available

$$\mathbf{u}_i^+ \equiv (\mathbf{u}_i \odot \mathbf{u}_i)(D)^+, \qquad \mathbf{u}_{i,j} \equiv (\mathbf{u}_i \odot \mathbf{u}_j)(D)$$

$$\left[\begin{array}{ccccc|cc} \mathbf{z}_1(D) & \mathbf{u}_1^+ & \mathbf{u}_{2,1} & \cdots & \mathbf{u}_{m,1} & \mathbf{x}_1(D) & \\ \mathbf{z}_2(D) & 0 & \mathbf{u}_2^+ & \cdots & \mathbf{u}_{m,2} & \mathbf{x}_2(D) & \\ \vdots & \vdots & & \ddots & \vdots & \vdots & \mathbf{I}_{m\times m} \\ \mathbf{z}_m(D) & 0 & \cdots & 0 & \mathbf{u}_m^+ & \mathbf{x}_m(D) & \end{array}\right]$$

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Performance

Can import arbitrary classical convolutional codes for use as a **convolutional entanglement distillation protocol** and rates and distance translates

Requires $2^{C(n+m)}$ **catalyst ebits** to get the protocol going
(where C is some constant depending on decoding)

Ebit yield is equal to $(n-m)/n$
(assuming that generated ebits are not corrupt)

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Example

Classical convolutional code over GF(4) with distance 3:

$$(\cdots|0000|1\bar{\omega}10|1101|0000|\cdots)$$

Consider the mapping between **GF(4)** and **Paulis**:

Π	I	X	Y	Z
$\mathbb{F}_4$	0	ω	1	$\bar{\omega}$

Convert to **quantum generators** with the above mapping:

$$(\cdots|IIII|ZXZI|ZZIZ|IIII|\cdots),$$
$$(\cdots|IIII|XYXI|XXIX|IIII|\cdots)$$

Produce **valid quantum convolutional code** by augmenting:

$$(\cdots|IIIIII|ZXZIXI|ZZIZZI|IIIIII|\cdots),$$
$$(\cdots|IIIIII|XYXIIX|XXIXZZ|IIIIII|\cdots)$$

Ent. Dist. Protocol has **yield ½** and **distance 3**

Algorithm for CSS-like Protocols

Applies to generators with **CSS-like structure**

Use a **symplectic Gram-Schmidt orthogonalization** to process generators

Does not change code properties

It **minimizes** the initial number of ebits required for catalysis

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Conclusion and Open Questions

Constructed a
convolutional entanglement distillation protocol
from arbitrary generators

The **ideal yield** of such a protocol is $(n\text{-}m)/n$
where m is number of generators and
n is the number of qubits in each frame

Open question: How does such a protocol behave under realistic conditions such as a depolarizing channel? Is there a threshold at which performance rapidly deteriorates?

THANK YOU!