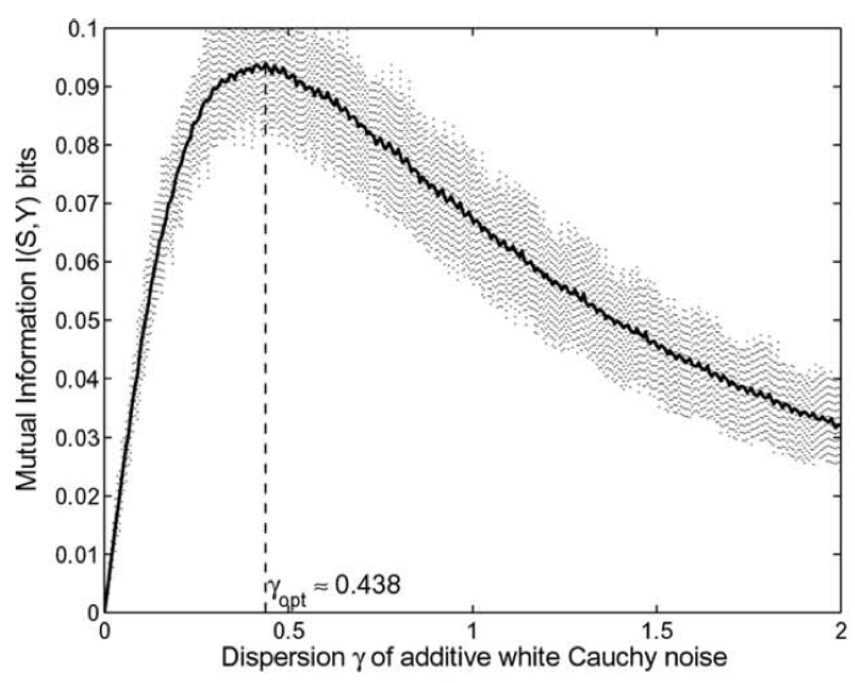
Quantum Forbidden-Interval Theorem for Stochastic Resonance with Squeezed Light

Mark M. Wilde and Bart Kosko

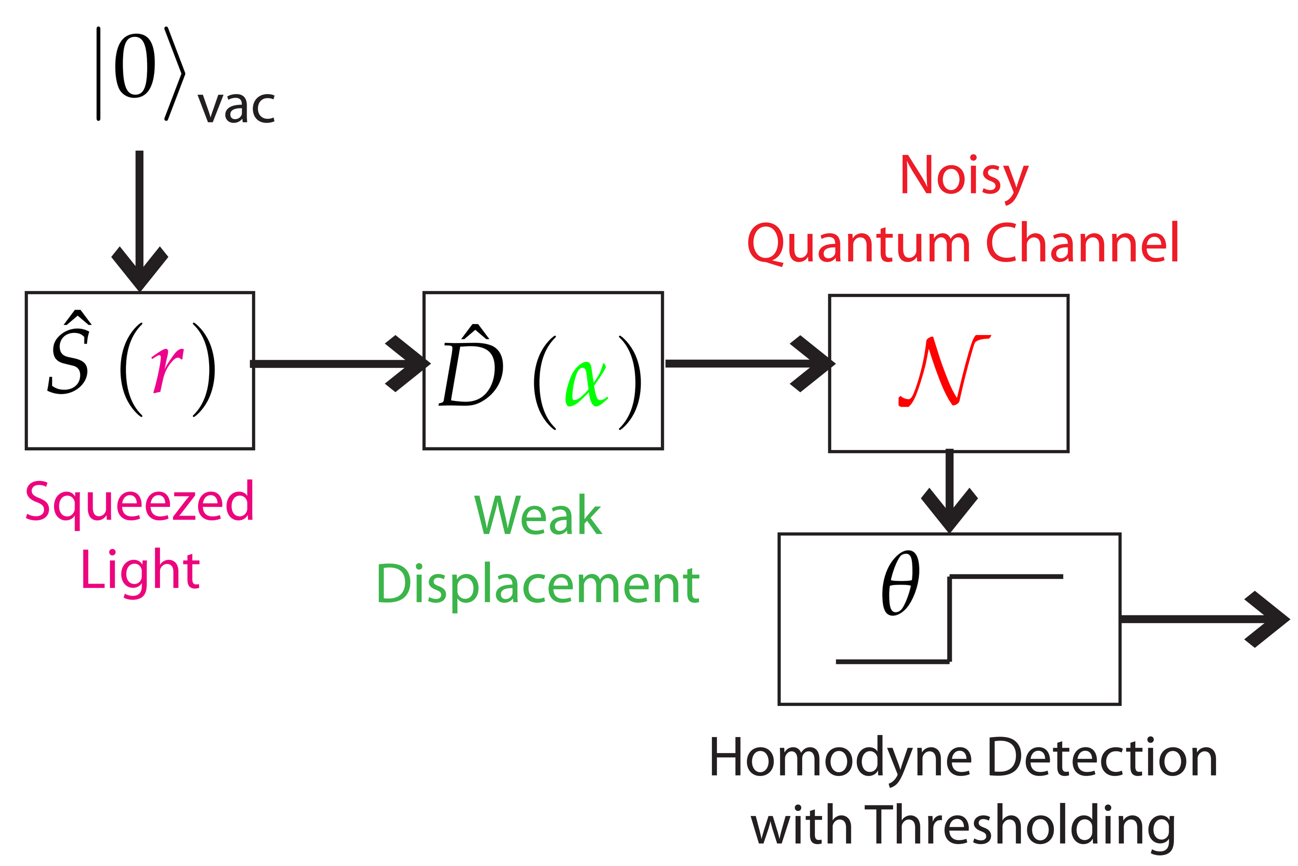
Signal and Image Processing Institute, Department of Electrical Engineering
University of Southern California, Los Angeles, California 90007 USA

What is Stochastic Resonance?

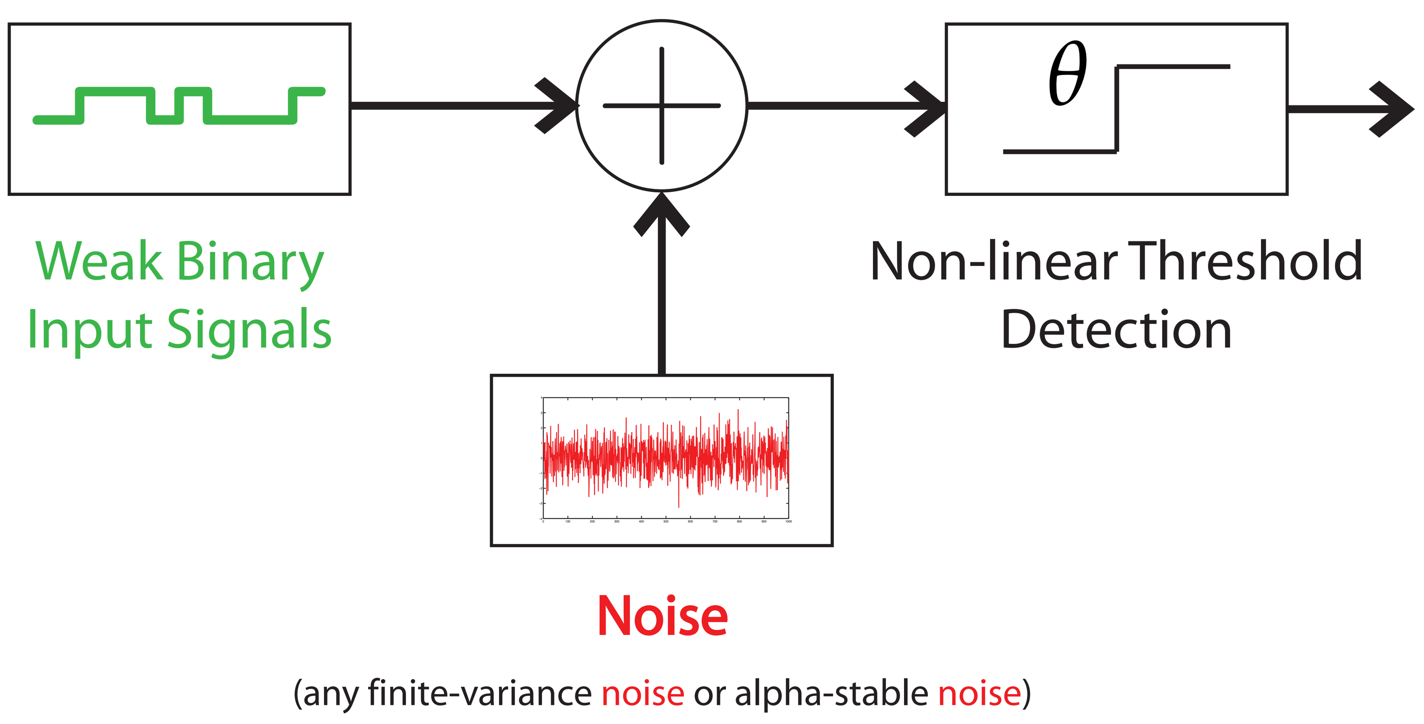


Typical Inverted-U
Stochastic Resonance (SR) curve

Quantum Model: Squeezed Light



Classical Model: Binary Threshold Neuron



Quantum Proof Strategy

MI vanishes if noise vanishes and squeezing becomes large

$$\sigma \rightarrow 0 \text{ and } r \rightarrow \infty \quad \Rightarrow \quad I(A;B) \rightarrow 0$$

MI increases as channel noise increases and squeezing decreases

SR effect iff quantum noise mean not in forbidden interval:

$$\mu \notin [\theta - \alpha_x, \theta + \alpha_x]$$

Proof Strategy: What Goes Down Must Go Up

Mutual Information (MI) vanishes if noise variance vanishes

$$\sigma \rightarrow 0 \quad \Rightarrow \quad I(A;B) \rightarrow 0$$

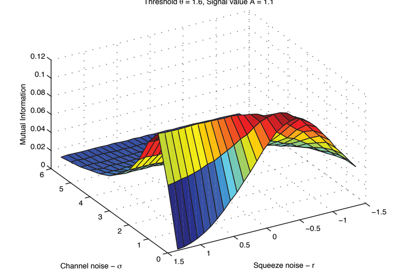
MI goes up from zero as channel noise increases

SR effect iff noise mean not in forbidden interval:

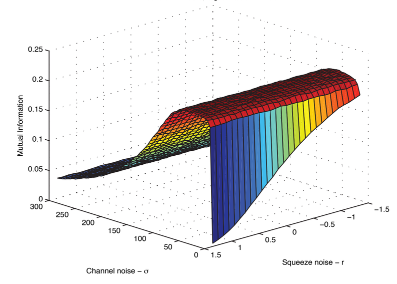
$$\mu \notin [\theta - A, \theta + A]$$

Simulation Results

Gaussian Noise Case

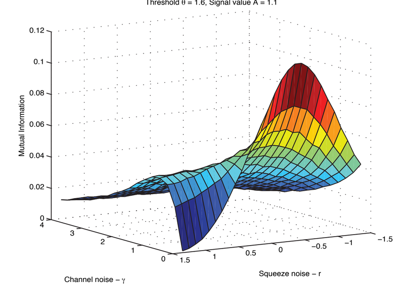


Low Photon Number

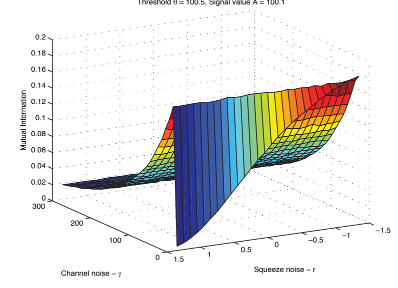


High Photon Number

Cauchy Noise Case



Low Photon Number



High Photon Number

References

[1] Bart Kosko and Sanya Mitaim. Stochastic resonance in noisy threshold neurons. Neural Networks, 16:755–761, 2003.
[2] Bart Kosko and Sanya Mitaim. Robust stochastic resonance for simple threshold neurons. Phys. Rev. E, 70:031911/1–031911/1, 2004.