Optical Cluster State Generation without Number-Resolving Photon Detectors

Mark M. Wilde[1], Federico Spedalieri[2], Jonathan P. Dowling[3], Hwang Lee[3]

[1] Department of Electrical Engineering, University of Southern California

[2] Department of Electrical Engineering, University of California, Los Angeles

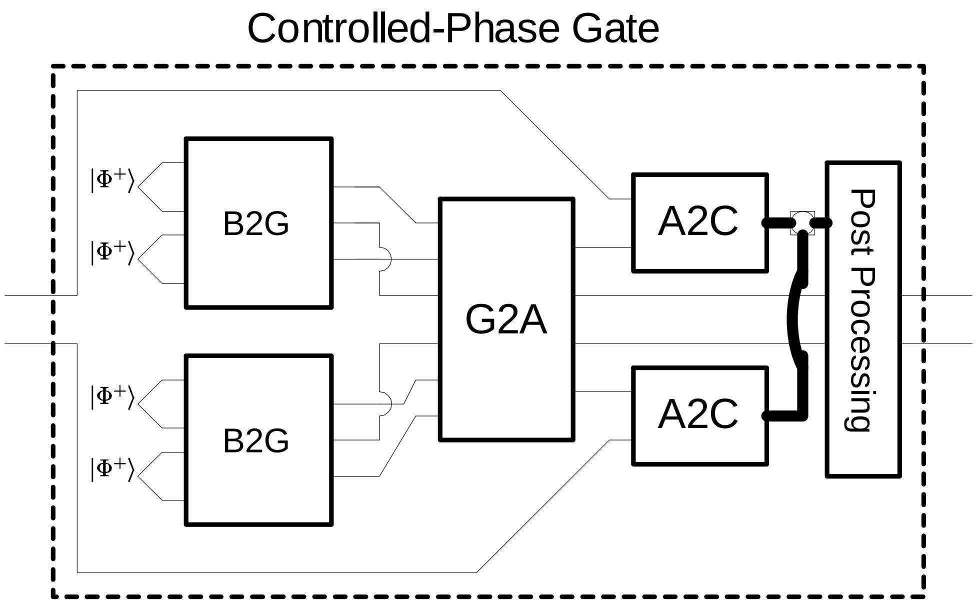
[3] Hearne Institute for Theoretical Physics, Department of Physics and Astronomy, Louisiana State University

1 Introduction

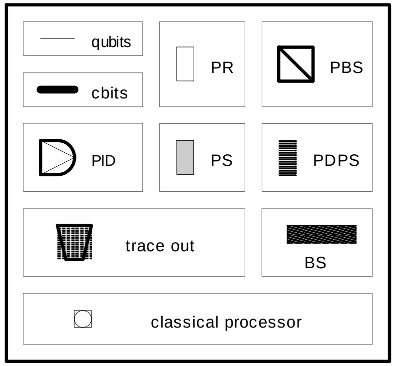
- Knill, Laflamme, and Milburn (KLM) proposed a scheme for quantum computation with linear-optical elements and post selection.
- KLM showed how to implement a two-qubit controlled-phase gate that succeeds with probability arbitrarily close to one.
- **Our scheme does not require number-resolving detectors**. We modify the KLM scheme significantly by using a simple four-qubit ancilla state and a method for generating this state offline **that does not require number-resolving detectors**.
- Can implement our scheme with currently available detectors that have low dark count.
- Success probability of $1/4$ for our scheme.

2 Scheme

- Use a polarization-encoding scheme for reliable optical cluster-state generation.
- Polarization-encoding immune to photon loss errors because the number of photons is fixed before the computation proceeds.
- Outline our method for a controlled-phase gate below.

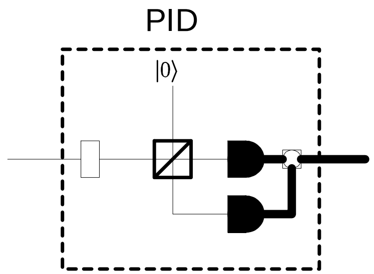


3 Legend



Qubits — thin lines, Classical bits — thick lines.
Classical processor performs post processing on computational modes.
Trash symbol denotes tracing out or discarding a mode.
Polarization-independent detector (PID) detects photons independent of their polarization.
Polarization rotator (PR) rotates the basis of polarization.
Phase shifter (PS) rotates the global phase.
Polarizing beam splitter (PBS) transmits H photons and reflects V photons.
Polarization-dependent phase shifter (PDPS) shifts the phase of V photons only.

4 Polarization-Independent Detectors



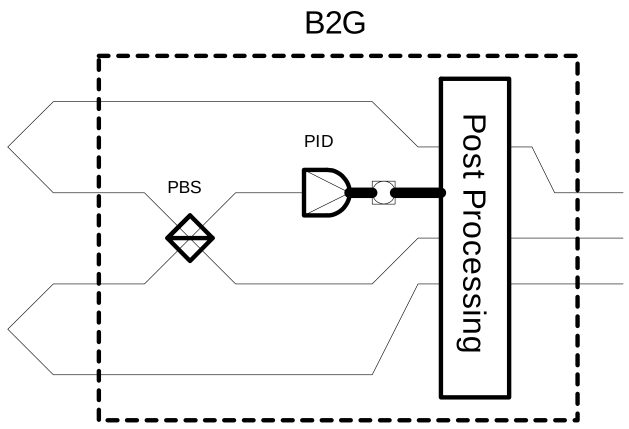
- Polarization-independent detectors (PIDs) detect the number of photons in a spatial mode independent of their polarization.
- Its operation on the computational basis $\{|H\rangle, |V\rangle\}$ is the following:

$$|H\rangle \quad \underrightarrow{\text{PID}} \quad (|H0\rangle + |0V\rangle)/\sqrt{2} \tag{1}$$

$$|V\rangle \quad \underrightarrow{\text{PID}} \quad (-|H0\rangle + |0V\rangle)/\sqrt{2} \tag{2}$$

- Detectors cannot determine the polarization of the incoming photon.

5 B2G (Bell-to-GHZ converter)



- The B2G operation (Bell-to-GHZ converter) converts Bell states to GHZ states with a success probability of $1/2$.
- Suppose we have a source of Bell states $|\Phi^+\rangle \equiv (|HH\rangle + |VV\rangle)/\sqrt{2}$.
- $|\text{GHZ}^+\rangle \equiv (|HHH\rangle + |VVV\rangle)/\sqrt{2}$, $|\text{GHZ}^-\rangle \equiv (|HHH\rangle - |VVV\rangle)/\sqrt{2}$.
- B2G converts these pure Bell states to a mixture of $|\text{GHZ}^+\rangle$ and $|V0H\rangle$.

6 Operation of the B2G

- Feed in two Bell states at the four input ports of the B2G.
- Initial state $|\Phi^+\rangle\,|\Phi^+\rangle$ propagates as follows:

$$|\Phi^+\rangle\,|\Phi^+\rangle \quad \xrightarrow{\text{PBS}} \quad \frac{1}{\sqrt{2}}\left(\frac{|HHH\rangle\,|H\rangle + |VVV\rangle\,|V\rangle}{\sqrt{2}}\right) + \frac{1}{2}\,|V0H\rangle\,|(H,V)\rangle + \frac{1}{2}\,|H\,(H,V)\,V\rangle\,|0\rangle \qquad (3)$$

- Perform a PID operation on the last mode. The quantum state becomes the following just before the detectors in the PID:

$$\xrightarrow{\text{PID}} \frac{1}{\sqrt{2}}\left(\left|\text{GHZ}^-\right\rangle|H0\rangle + \left|\text{GHZ}^+\right\rangle|0V\rangle\right) - \frac{1}{2\sqrt{2}}\left(|V0H\rangle\left(\left|H^2 0\right\rangle - \left|0V^2\right\rangle\right)\right) + \frac{1}{2}\,|H\,(H,V)\,V\rangle\,|00\rangle\,, \qquad (4)$$

$\left|H^2\right\rangle$ denotes two horizontally polarized photons in a given path.

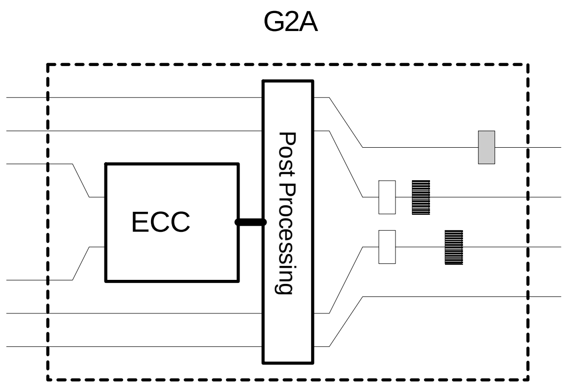
7 Mixed State after the B2G

- Detectors in the PID measure the last two modes and cannot distinguish between $|H0\rangle$ and $\left|H^2 0\right\rangle$ or $|0V\rangle$ and $\left|0V^2\right\rangle$ **because they cannot resolve photon number**.
- Perform the following post-processing operations: discard the operation and start over if we measure $|00\rangle$, perform a PDPS of π on the first mode if we measure $|H^n 0\rangle$, or do nothing if we measure $|0V^n\rangle$.
- The state becomes the **mixture** ρ_{PGHZ} (the partial GHZ mixture) after performing the above conditional operations.

$$\rho_{\text{PGHZ}} = \frac{2}{3}\left|\text{GHZ}^+\right\rangle\left\langle\text{GHZ}^+\right| + \frac{1}{3}|V0H\rangle\langle V0H| \quad (5)$$

We obtain a pure GHZ state with probability $1/2$ after performing the B2G operation on two pure Bell states.

8 G2A (GHZ-to-four-qubit-Ancilla converter)

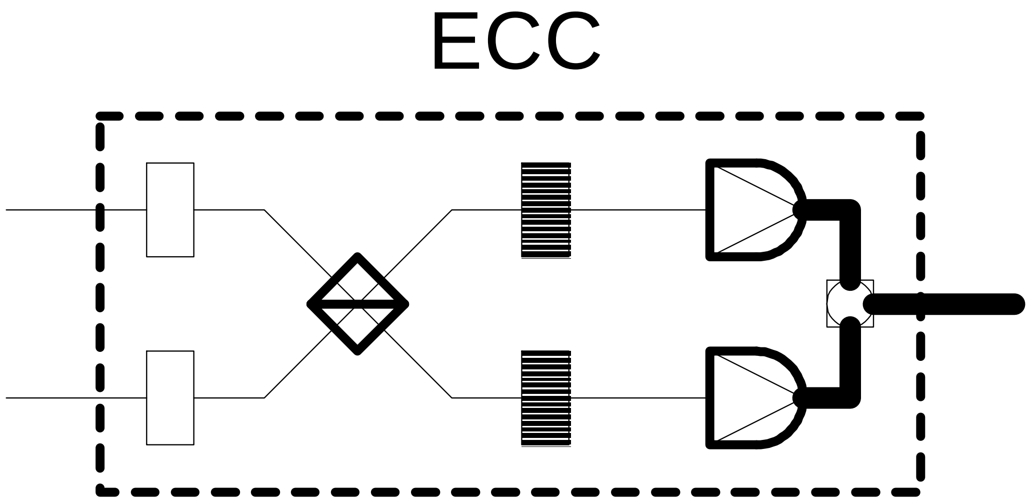


- G2A has a mechanism to correct for possible errors introduced in the B2G operation.
- G2A converts the mixture above to the four-qubit ancilla state $|t_1'\rangle$ [?, ?],

$$|t_1'\rangle \equiv \frac{1}{2}\left(|HVVH\rangle + |VHVH\rangle + |VHHV\rangle - |HVHV\rangle\right). \quad (6)$$

- Probability of this conversion, given two pure GHZ states, is $1/2$.

9 Intrinsic Error Correction Circuit



- 1st part of G2A corrects for the error that B2G introduces.
- ECC only has a non-unit probability of success, but **we know whether the correction fails or succeeds**.

10 Operation of ECC

- Second mode of ρ_{PGHZ} may have an error.
- ECC first detects whether two pure GHZ states are actually at the input of the G2A operation.
- ECC then produces a state which we can convert deterministically to the four-qubit ancilla state $|t_1'\rangle$.
- ECC only produces the convertible state if two pure GHZ states are at the input of the G2A operation.

Suppose state $|V0H\rangle\,|V0H\rangle$ input to G2A.

- Can detect this state uniquely because no one of the four detectors in the two PIDs fire.
- Discard the operation and start over if we detect zero photons.

11 Operation of ECC (ctd.)

Suppose either state $\left|\text{GHZ}^+\right\rangle |V0H\rangle$ or $|V0H\rangle \left|\text{GHZ}^+\right\rangle$ input to G2A.

- Left column of Table gives possible states of the two middle modes in either of the two superpositions: $\left|\text{GHZ}^+\right\rangle |V0H\rangle$ or $|V0H\rangle \left|\text{GHZ}^+\right\rangle$.
- Discard the computation and start over if we detect zero photons in exactly three modes.

Init.	Resulting States
$\|0H\rangle$	$\left(\|0H00\rangle - e^{i\pi/4}\|H000\rangle + \|000V\rangle + e^{i\pi/4}\|00V0\rangle\right)/2$
$\|0V\rangle$	$\left(-\|0H00\rangle - e^{i\pi/4}\|H000\rangle - \|000V\rangle + e^{i\pi/4}\|00V0\rangle\right)/2$
$\|H0\rangle$	$\left(\|H000\rangle - e^{i\pi/4}\|0H00\rangle + \|00V0\rangle + e^{i\pi/4}\|000V\rangle\right)/2$
$\|V0\rangle$	$\left(-\|H000\rangle - e^{i\pi/4}\|0H00\rangle - \|00V0\rangle + e^{i\pi/4}\|000V\rangle\right)/2$

12 Operation of ECC (ctd.)

Suppose two pure GHZ states $\left|\text{GHZ}^{+}\right\rangle \left|\text{GHZ}^{+}\right\rangle$ are input to ECC.

- Discard the computation if we measure zero photons in exactly three modes.
- Keep the state if we measure zero photons in only two modes.
- Determine with certainty whether two pure GHZ states are input to G2A.
- Employ the following shorthand notation: $|0H0V\rangle \equiv |1\rangle$, $|H0V0\rangle \equiv |2\rangle$, $|H00V\rangle \equiv |3\rangle$, $|0HV0\rangle \equiv |4\rangle$, $|HH00\rangle \equiv |5\rangle$, $|00VV\rangle \equiv |6\rangle$, $\left|H^2000\right\rangle \equiv |7\rangle$, $\left|0H^200\right\rangle \equiv |8\rangle$, $\left|00V^20\right\rangle \equiv |9\rangle$, $\left|000V^2\right\rangle \equiv |10\rangle$.

Init.	Resulting States
$\|HH\rangle$	$e^{\frac{i\pi}{4}}(\|5\rangle - i\|3\rangle - i\|4\rangle + \|6\rangle - \|7\rangle + \|9\rangle - \|8\rangle + \|10\rangle)$
$\|HV\rangle$	$ie^{\frac{i3\pi}{4}}(-i\|5\rangle - \|3\rangle - \|4\rangle - i\|6\rangle + \|7\rangle - \|9\rangle - \|8\rangle + \|10\rangle)$
$\|VH\rangle$	$ie^{\frac{i3\pi}{4}}(-i\|5\rangle - \|3\rangle - \|4\rangle - i\|6\rangle - \|7\rangle + \|9\rangle + \|8\rangle - \|10\rangle)$
$\|VV\rangle$	$e^{\frac{i\pi}{4}}(\|5\rangle - i\|3\rangle - i\|4\rangle + \|6\rangle + \|7\rangle + \|9\rangle - \|8\rangle - \|10\rangle)$

13 Operation of ECC (ctd.)

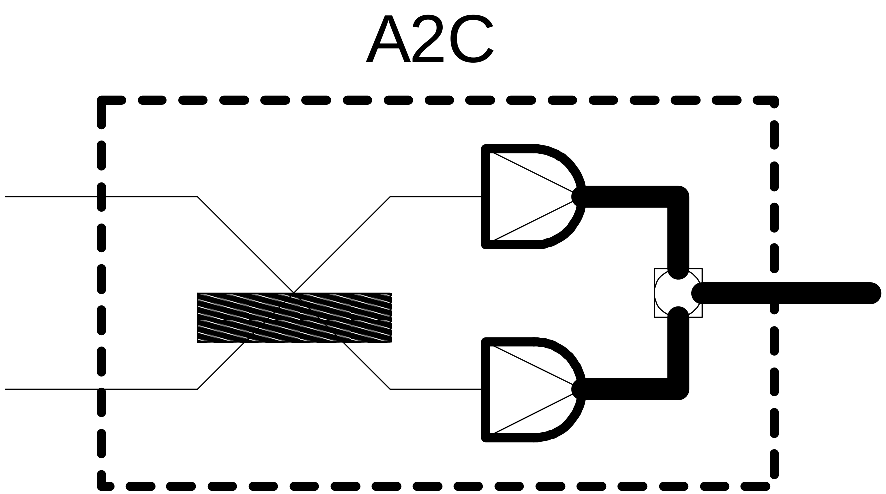
Suppose two pure GHZ states $\left|\text{GHZ}^+\right\rangle \left|\text{GHZ}^+\right\rangle$ are input to G2A.

- Resulting state after the ECC operation.
- Can convert this state deterministically to the four-qubit ancilla $|t'_1\rangle$.

Meas.	Resulting States
$\|5\rangle, \|6\rangle$	$e^{\frac{i\pi}{4}} \left(\|HHHH\rangle + i\|HHVV\rangle + i\|VVHH\rangle + \|VVVV\rangle\right)/2$
$\|3\rangle, \|4\rangle$	$e^{-\frac{i\pi}{4}} \left(\|HHHH\rangle - i\|HHVV\rangle - i\|VVHH\rangle + \|VVVV\rangle\right)/2$

14 A2C (Four-qubit Ancilla to Controlled-Phase)

- A2C uses the four-qubit ancilla $|t_1'\rangle$ to perform a controlled-phase operation with a success probability of $1/4$.
- Know for certain whether the gate succeeds **using photon detectors that do not resolve photon number**.



The A2C operation consists of a 50:50 beam splitter followed by two PIDs.

15 Operation of A2C

- Controlled-phase gate performs the following action: $|HH\rangle \rightarrow |HH\rangle$, $|HV\rangle \rightarrow |HV\rangle$, $|VH\rangle \rightarrow |VH\rangle$, $|VV\rangle \rightarrow -|VV\rangle$.
- Analyze the computational basis elements as inputs assuming we have four-qubit ancilla state $|t_1'\rangle$.
- Determine the propagation of the following four states through the latter half of the controlled-phase gate

$$|H\rangle_1 \, |t_1'\rangle_{2345} \, |H\rangle_6 \, , \quad |H\rangle_1 \, |t_1'\rangle_{2345} \, |V\rangle_6 \, , \quad |V\rangle_1 \, |t_1'\rangle_{2345} \, |H\rangle_6 \, , \quad |V\rangle_1 \, |t_1'\rangle_{2345} \, |V\rangle_6 \tag{7}$$

- Determine the state of modes three and four after the two A2C operations by first analyzing the A2C operation acting on the four basis states $|HH\rangle$, $|HV\rangle$, $|VH\rangle$, and $|VV\rangle$ (all four combinations occur).

16 Operation of A2C (ctd.)

- Discard the operation of the controlled-phase gate and start over if the result of the measurement gives zero photons in exactly three modes.
- Consider the operation a success if we measure zero photons in exactly two modes.

17 Conclusion

- Perform a successful operation of the controlled-phase gate with probability $1/4$ given the four-qubit ancilla state $|t_1'\rangle$.
- B2G has success probability $1/2$. (need 2 B2Gs)
- G2A has success probability $1/2$.
- Generate the four-qubit ancilla state $|t_1'\rangle$ offline with success probability $1/8$.
- Success probability of the controlled-phase gate is $1/4$ given the four-qubit ancilla state $|t_1'\rangle$.
- **It is possible to remove the need for number-resolving detectors** in linear optical quantum computation with cluster states.

18 Acknowledgements

M.M.W. would like to thank Austin Lund for stimulating discussions. We acknowledge the support of the Army Research Office, the Hearne Foundation, and the Disruptive Technologies Office.