Optical Cluster State Generation without Number-Resolving Photon Detectors

Mark M. Wilde\(^1\), Federico Spedalieri\(^2\), Jonathan P. Dowling\(^3\), Hwang Lee\(^3\)

\(^1\) Department of Electrical Engineering, University of Southern California

\(^2\) Department of Electrical Engineering, University of California, Los Angeles

\(^3\) Hearne Institute for Theoretical Physics, Department of Physics and Astronomy, Louisiana State University
1 Introduction

- Knill, Laflamme, and Milburn (KLM) proposed a scheme for quantum computation with linear-optical elements and post selection.
- KLM showed how to implement a two-qubit controlled-phase gate that succeeds with probability arbitrarily close to one.
- **Our scheme does not require number-resolving detectors.** We modify the KLM scheme significantly by using a simple four-qubit ancilla state and a method for generating this state offline that does not require number-resolving detectors.
- Can implement our scheme with currently available detectors that have low dark count.
- Success probability of $1/4$ for our scheme.
2 Scheme

- Use a polarization-encoding scheme for reliable optical cluster-state generation.
- Polarization-encoding immune to photon loss errors because the number of photons is fixed before the computation proceeds.
- Outline our method for a controlled-phase gate below.

![Controlled-Phase Gate Diagram]
3 Legend

Qubits — thin lines, Classical bits — thick lines.
Classical processor performs post processing on computational modes.
Trash symbol denotes tracing out or discarding a mode.
Polarization-independent detector (PID) detects photons independent of their polarization.
Polarization rotator (PR) rotates the basis of polarization.
Phase shifter (PS) rotates the global phase.
Polarizing beam splitter (PBS) transmits H photons and reflects V photons.
Polarization-dependent phase shifter (PDPS) shifts the phase of V photons only.
4 Polarization-Independent Detectors

- Polarization-independent detectors (PID) detect the number of photons in a spatial mode independent of their polarization.
- Its operation on the computational basis \( \{|H\rangle, |V\rangle\} \) is the following:

\[
|H\rangle \xrightarrow{\text{PID}} (|H0\rangle + |0V\rangle)/\sqrt{2} \tag{1}
\]

\[
|V\rangle \xrightarrow{\text{PID}} (-|H0\rangle + |0V\rangle)/\sqrt{2} \tag{2}
\]

- Detectors cannot determine the polarization of the incoming photon.
5 B2G (Bell-to-GHZ converter)

- The B2G operation (Bell-to-GHZ converter) converts Bell states to GHZ states with a success probability of $\frac{1}{2}$.
- Suppose we have a source of Bell states $|\Phi^+\rangle \equiv (|HH\rangle + |VV\rangle ) / \sqrt{2}$.
- $|\text{GHZ}^+\rangle \equiv (|HHH\rangle + |VVV\rangle ) / \sqrt{2}$, $|\text{GHZ}^-\rangle \equiv (|HHH\rangle - |VVV\rangle ) / \sqrt{2}$.
- B2G converts these pure Bell states to a mixture of $|\text{GHZ}^+\rangle$ and $|V0H\rangle$. 
6 Operation of the B2G

- Feed in two Bell states at the four input ports of the B2G.
- Initial state $|\Phi^+\rangle |\Phi^+\rangle$ propagates as follows:

$$
|\Phi^+\rangle |\Phi^+\rangle \xrightarrow{\text{PBS}} \frac{1}{\sqrt{2}} \left( |HHH\rangle |H\rangle + |VVV\rangle |V\rangle \right)
\left( + \frac{1}{2} |V0H\rangle |(H, V)\rangle + \frac{1}{2} |H (H, V) V\rangle |0\rangle \right)
$$

- Perform a PID operation on the last mode. The quantum state becomes the following just before the detectors in the PID:

$$
\text{PID} \quad \frac{1}{\sqrt{2}} \left( |GHZ^-\rangle |H0\rangle + |GHZ^+\rangle |0V\rangle \right)
\left( - \frac{1}{2\sqrt{2}} \left( |V0H\rangle \left( |H^20\rangle - |0V^2\rangle \right) \right) + \frac{1}{2} |H (H, V) V\rangle |00\rangle ,
$$

$|H^2\rangle$ denotes two horizontally polarized photons in a given path.
7 Mixed State after the B2G

- Detectors in the PID measure the last two modes and cannot distinguish between $|H0\rangle$ and $|H^20\rangle$ or $|0V\rangle$ and $|0V^2\rangle$ because they cannot resolve photon number.

- Perform the following post-processing operations: discard the operation and start over if we measure $|00\rangle$, perform a PDPS of $\pi$ on the first mode if we measure $|H^n0\rangle$, or do nothing if we measure $|0V^n\rangle$.

- The state becomes the mixture $\rho_{\text{PGHZ}}$ (the partial GHZ mixture) after performing the above conditional operations.

$$\rho_{\text{PGHZ}} = \frac{2}{3} |\text{GHZ}^+\rangle \langle \text{GHZ}^+ | + \frac{1}{3} |V0H\rangle \langle V0H |$$  \hspace{1cm} (5)

We obtain a pure GHZ state with probability $1/2$ after performing the B2G operation on two pure Bell states.
8 G2A (GHZ-to-four-qubit-Ancilla converter)

- G2A has a mechanism to correct for possible errors introduced in the B2G operation.
- G2A converts the mixture above to the four-qubit ancilla state $|t_1'\rangle$ [? , ?],

$$|t_1'\rangle \equiv \frac{1}{2} (|HV VH\rangle + |V HV H\rangle + |V HHV\rangle - |HV HV\rangle).$$

- Probability of this conversion, given two pure GHZ states, is $1/2$. 
9 Intrinsic Error Correction Circuit

- 1st part of G2A corrects for the error that B2G introduces.
- ECC only has a non-unit probability of success, but we know whether the correction fails or succeeds.
10 Operation of ECC

- Second mode of $\rho_{\text{PGHZ}}$ may have an error.
- ECC first detects whether two pure GHZ states are actually at the input of the G2A operation.
- ECC then produces a state which we can convert deterministically to the four-qubit ancilla state $|t'_1\rangle$.
- ECC only produces the convertible state if two pure GHZ states are at the input of the G2A operation.

Suppose state $|V0H\rangle |V0H\rangle$ input to G2A.
- Can detect this state uniquely because no one of the four detectors in the two PIDs fire.
- Discard the operation and start over if we detect zero photons.
11 Operation of ECC (ctd.)

Suppose either state $|\text{GHZ}^+\rangle |V0H\rangle$ or $|V0H\rangle |\text{GHZ}^+\rangle$ input to G2A.

- Left column of Table gives possible states of the two middle modes in either of the two superpositions: $|\text{GHZ}^+\rangle |V0H\rangle$ or $|V0H\rangle |\text{GHZ}^+\rangle$.

- Discard the computation and start over if we detect zero photons in exactly three modes.

| Init. $\left| \begin{array}{l} 0H \rangle \\ 0V \rangle \\ H0 \rangle \\ V0 \rangle \end{array} \right.$ | Resulting States $\left( \begin{array}{l} \left| 0H00 \right\rangle - e^{i\pi/4} \left| H000 \right\rangle + \left| 000V \right\rangle + e^{i\pi/4} \left| 00V0 \right\rangle \\ \left| H000 \right\rangle - e^{i\pi/4} \left| 0H00 \right\rangle + \left| 00V0 \right\rangle + e^{i\pi/4} \left| 000V \right\rangle \\ \left| 0H00 \right\rangle - e^{i\pi/4} \left| 0H00 \right\rangle - \left| 00V0 \right\rangle + e^{i\pi/4} \left| 000V \right\rangle \\ \left| H000 \right\rangle - e^{i\pi/4} \left| 0H00 \right\rangle - \left| 00V0 \right\rangle + e^{i\pi/4} \left| 000V \right\rangle \end{array} \right)/2$ |
12 Operation of ECC (ctd.)

Suppose two pure GHZ states $|\text{GHZ}^+\rangle |\text{GHZ}^+\rangle$ are input to ECC.

- Discard the computation if we measure zero photons in exactly three modes.
- Keep the state if we measure zero photons in only two modes.
- Determine with certainty whether two pure GHZ states are input to G2A.
- Employ the following shorthand notation: $|0H0V\rangle \equiv |1\rangle$, $|H0V0\rangle \equiv |2\rangle$, $|H00V\rangle \equiv |3\rangle$, $|0HV0\rangle \equiv |4\rangle$, $|HH00\rangle \equiv |5\rangle$, $|00VV\rangle \equiv |6\rangle$, $|H^2000\rangle \equiv |7\rangle$, $|0H^200\rangle \equiv |8\rangle$, $|00V^20\rangle \equiv |9\rangle$, $|000V^2\rangle \equiv |10\rangle$.

<table>
<thead>
<tr>
<th>Init.</th>
<th>Resulting States</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>HH\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>HV\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>VH\rangle$</td>
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<tr>
<td>$</td>
<td>VV\rangle$</td>
</tr>
</tbody>
</table>
13 Operation of ECC (ctd.)

Suppose two pure GHZ states $|\text{GHZ}^+\rangle |\text{GHZ}^+\rangle$ are input to G2A.

- Resulting state after the ECC operation.
- Can convert this state deterministically to the four-qubit ancilla $|t_1\rangle$.

<table>
<thead>
<tr>
<th>Meas.</th>
<th>Resulting States</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>5\rangle,</td>
</tr>
<tr>
<td>$</td>
<td>3\rangle,</td>
</tr>
</tbody>
</table>
14 **A2C (Four-qubit Ancilla to Controlled-Phase)**

- A2C uses the four-qubit ancilla $|t'_1\rangle$ to perform a controlled-phase operation with a success probability of $1/4$.
- Know for certain whether the gate succeeds using photon detectors that do not resolve photon number.

The A2C operation consists of a 50:50 beam splitter followed by two PIDs.
15 **Operation of A2C**

- Controlled-phase gate performs the following action: $|HH\rangle \rightarrow |HH\rangle$, $|HV\rangle \rightarrow |HV\rangle$, $|VV\rangle \rightarrow -|VV\rangle$.

- Analyze the computational basis elements as inputs assuming we have four-qubit ancilla state $|t_1\rangle$.

- Determine the propagation of the following four states through the latter half of the controlled-phase gate

$$|H\rangle_1 |t'_1\rangle_{2345} |H\rangle_6, \quad |H\rangle_1 |t'_1\rangle_{2345} |V\rangle_6, \quad |V\rangle_1 |t'_1\rangle_{2345} |H\rangle_6, \quad |V\rangle_1 |t'_1\rangle_{2345} |V\rangle_6 \quad (7)$$

- Determine the state of modes three and four after the two A2C operations by first analyzing the A2C operation acting on the four basis states $|HH\rangle$, $|HV\rangle$, $|VH\rangle$, and $|VV\rangle$ (all four combinations occur).
16 Operation of A2C (ctd.)

- Discard the operation of the controlled-phase gate and start over if the result of the measurement gives zero photons in exactly three modes.
- Consider the operation a success if we measure zero photons in exactly two modes.


17 Conclusion

• Perform a successful operation of the controlled-phase gate with probability $1/4$ given the four-qubit ancilla state $|t'_1\rangle$.

• B2G has success probability $1/2$. (need 2 B2Gs)

• G2A has success probability $1/2$.

• Generate the four-qubit ancilla state $|t'_1\rangle$ offline with success probability $1/8$.

• Success probability of the controlled-phase gate is $1/4$ given the four-qubit ancilla state $|t'_1\rangle$.

• It is possible to remove the need for number-resolving detectors in linear optical quantum computation with cluster states.
18 Acknowledgements

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