Entanglement generation with a quantum channel and a shared state

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2010 IEEE International Symposium on Information Theory
Austin, Texas, USA
Friday, June 18, 2010

Overview

Quickly overview some protocols from **quantum Shannon theory**

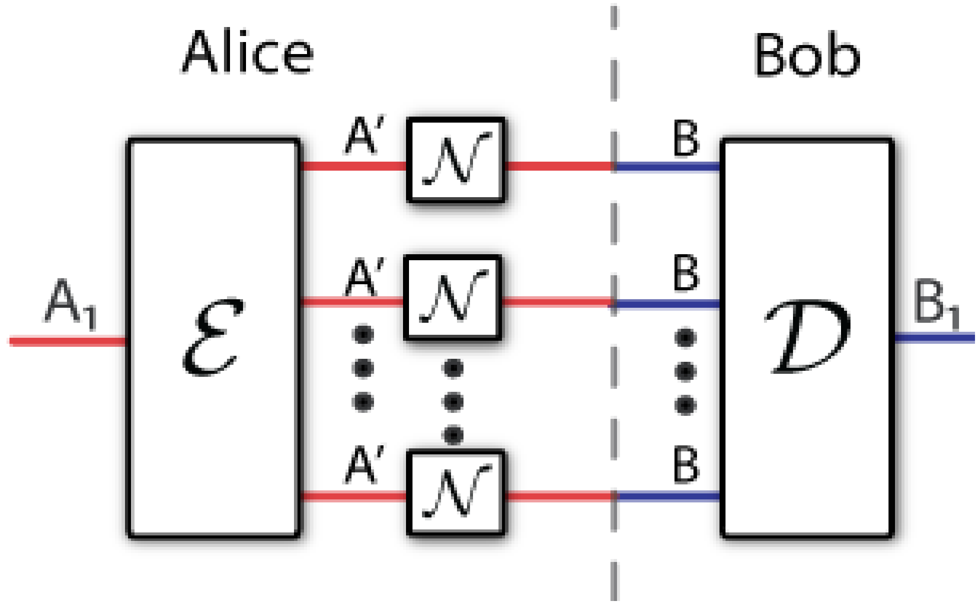
Discuss the **practical motivation** for our work and potential strategies that are suboptimal

Discuss our main result: **capacity theorem** with achievability proof and converse proof

Give an example of **superactivation** and discuss future questions

Quantum Communication

One important quantum information processing task is to transmit quantum information reliably



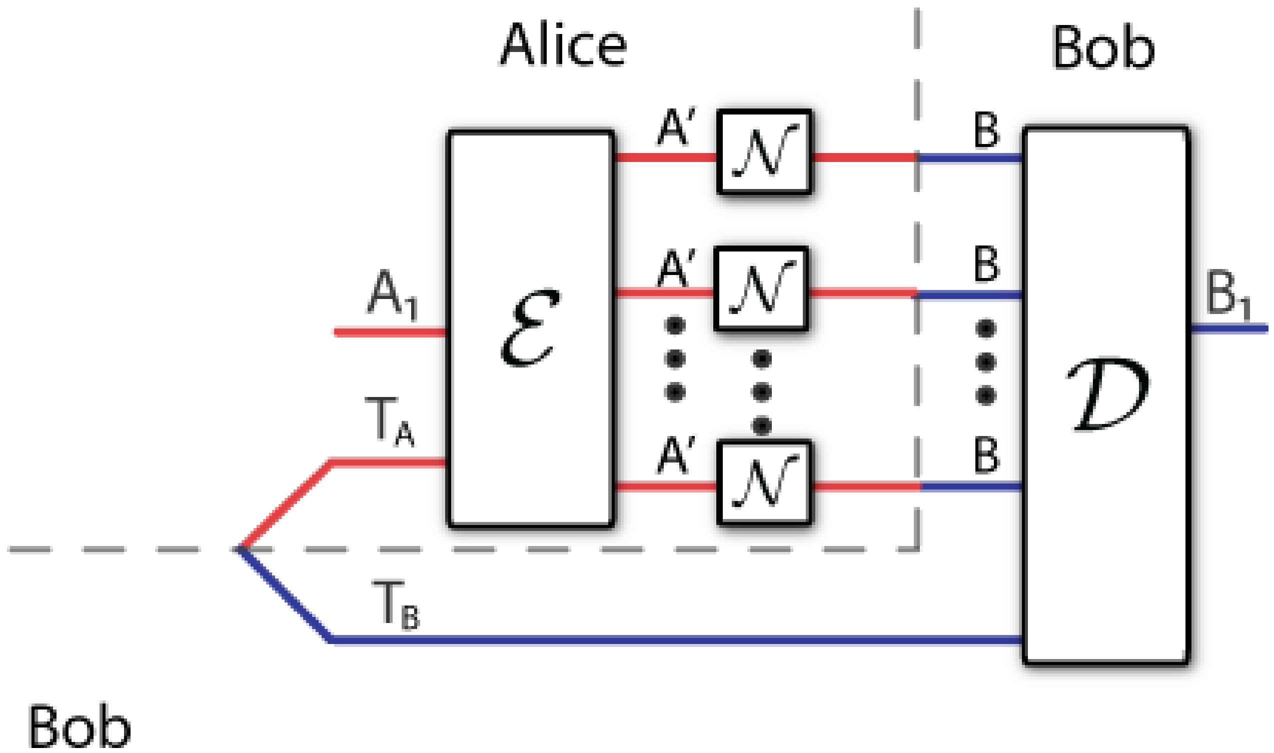
Regularized channel coherent information
is an **achievable rate**

$$Q(\mathcal{N}) \equiv \max_{\phi} I(A\rangle B)$$

$I(A\rangle B) \equiv H(B) - H(AB)$ and $\mathcal{N}^{A'\to B}(\phi^{AA'})$

Lloyd 1997, Shor 2002, Devetak 2005

Father Protocol

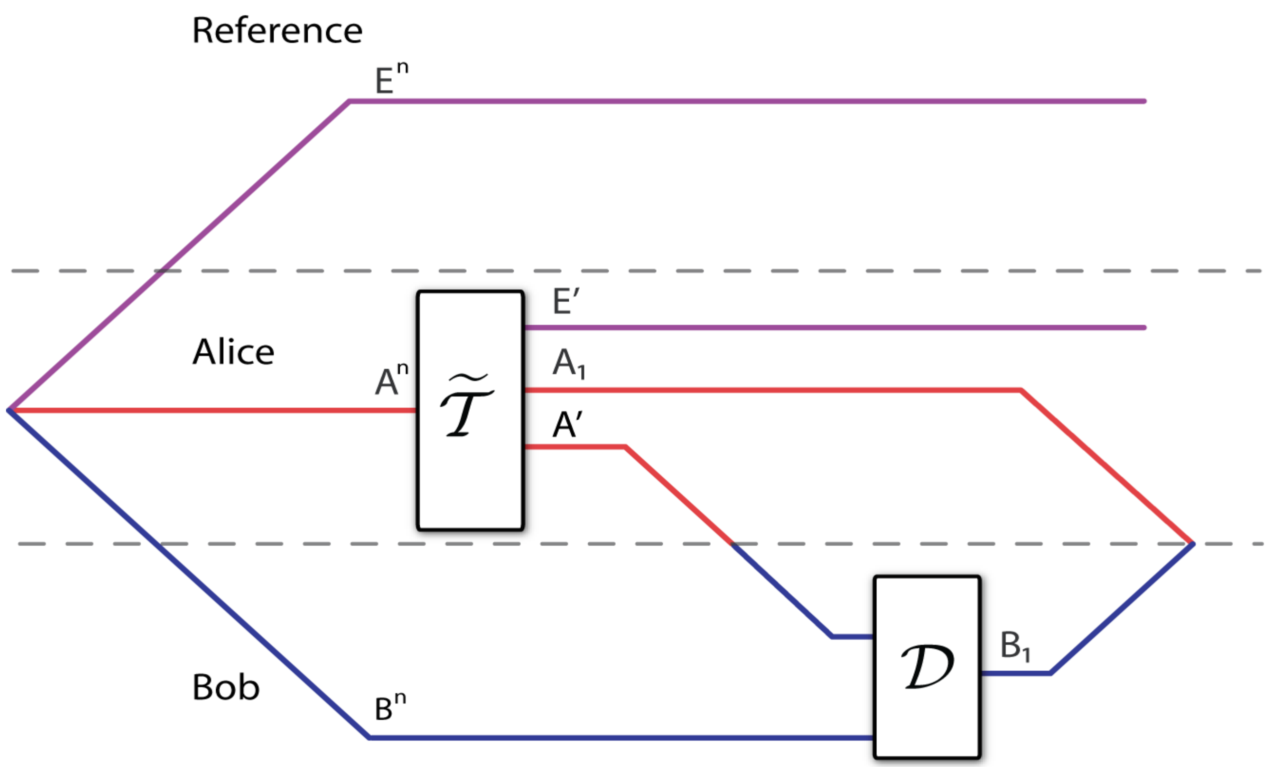


$$\langle \mathcal{N} \rangle + \frac{1}{2} I(A;E)[qq] \geq \frac{1}{2} I(A;B)[q \to q]$$

Trade-off between **entanglement consumption** and **quantum transmission**

Devetak, Harrow, Winter. IEEE Trans Inf. Theory 2008

Mother Protocol



$$\langle \rho^{AB} \rangle + \frac{1}{2} I(A;E)[q \rightarrow q] \geq \frac{1}{2} I(A;B)[qq]$$

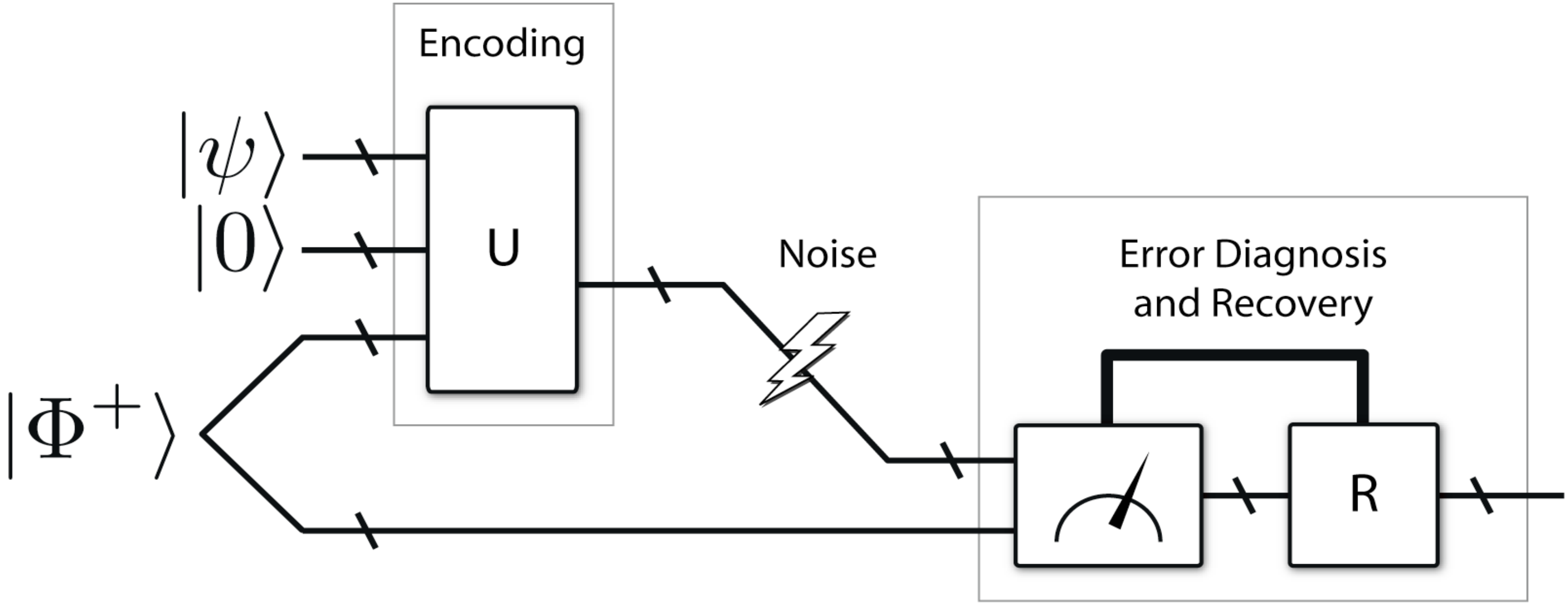
Trade-off between **quantum communication consumption** and **entanglement generation**

Devetak, Harrow, Winter. IEEE Trans Inf. Theory 2008

Motivation for Present Work

What if **both** the quantum channel and the shared state are noisy?

Practical Application:
Entanglement-assisted quantum codes where shared entanglement is **noisy**



Brun, Devetak, Hsieh. Science 2006

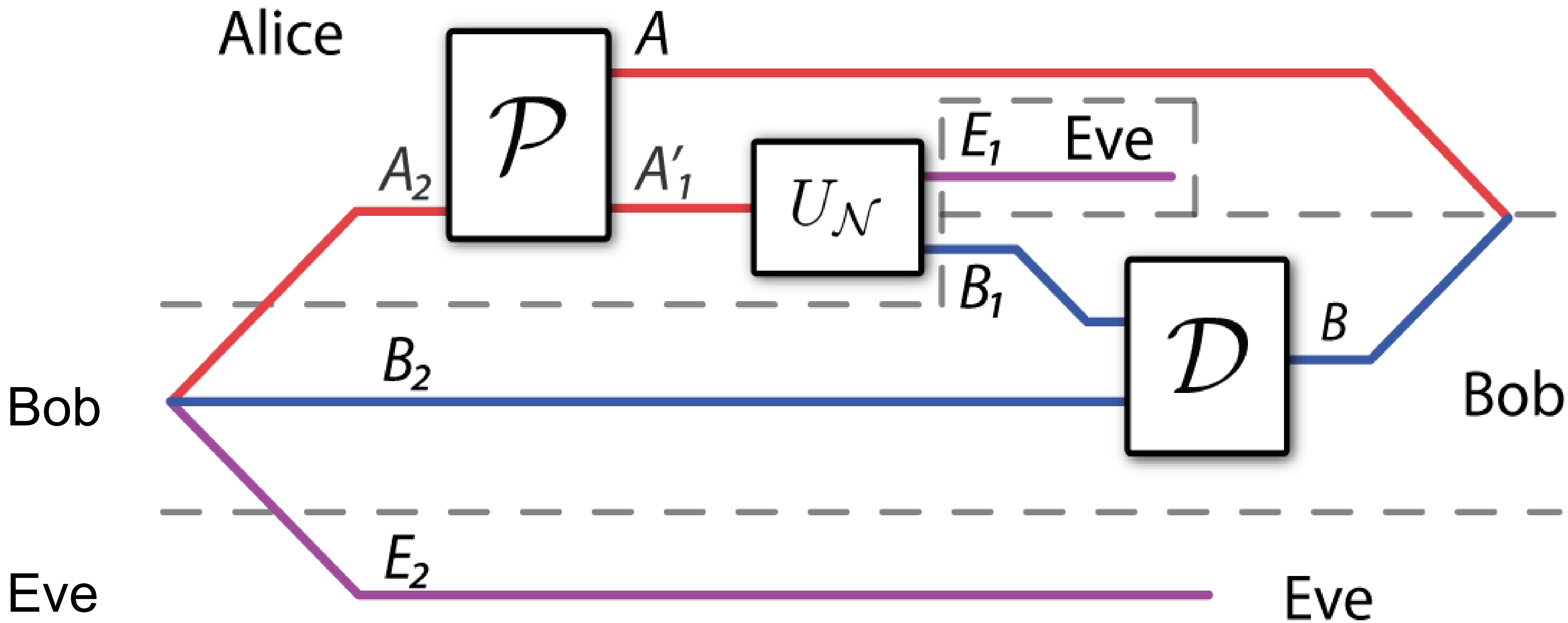
Potential Yet Suboptimal Strategies

Use an **LSD random quantum code** for the channel
and **independently distill entanglement** from the state

Distill the entanglement and **execute the father protocol**
if there is enough entanglement available

Use the channel to generate quantum communication
and **execute the mother protocol**
if enough quantum communication is available

Information Processing Task



Initial: Alice and Bob share a noisy state

Preparation: Alice performs some preparation map

Transmission: Alice transmits encoded state over channel
(*allow classical communication*)

Decoding: Bob decodes

Channel-State Capacity Theorem

The entanglement generation capacity $E(\mathcal{N} \otimes \rho)$ of a quantum channel $\mathcal{N}$ and a bipartite state ρ is

$$E(\mathcal{N} \otimes \rho) = \lim_{l \to \infty} \frac{1}{l} E^{(1)}(\mathcal{N}^{\otimes l} \otimes \rho^{\otimes l}), \tag{1}$$

where the "one-shot" capacity $E^{(1)}(\mathcal{N} \otimes \rho)$ is

$$E^{(1)}(\mathcal{N} \otimes \rho) = \max_{\mathcal{P}} I\left(A_1 A_2 \rangle B_1 B_2\right)_{\omega}. \tag{2}$$

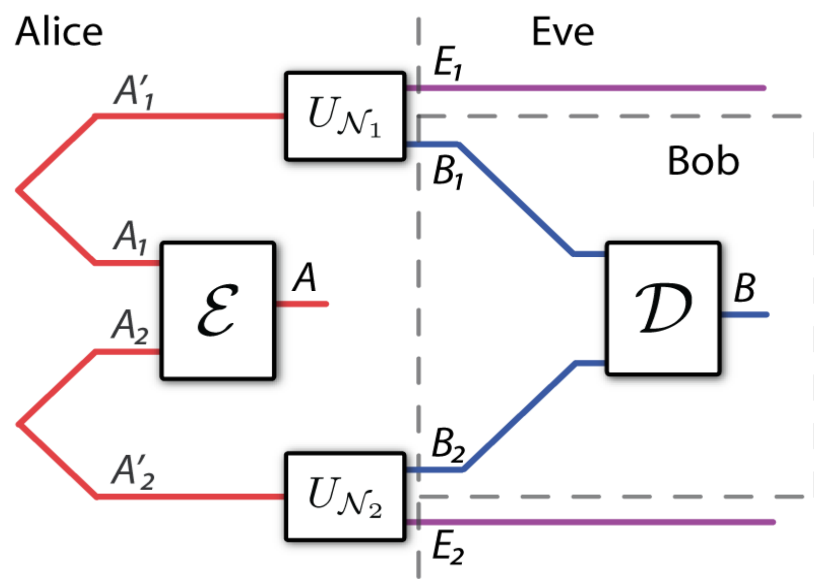
The maximization is over all preparations $\mathcal{P}^{A_2 \to A_1 A_2 A_1'}$ and the coherent information $I\left(A_1 A_2 \rangle B_1 B_2\right)_{\omega}$ is with respect to the following state:

$$\mathcal{N}^{A_1' \to B_1}(\mathcal{P}^{A_2 \to A_1 A_2 A_1'}(\rho^{A_2 B_2})). \tag{3}$$

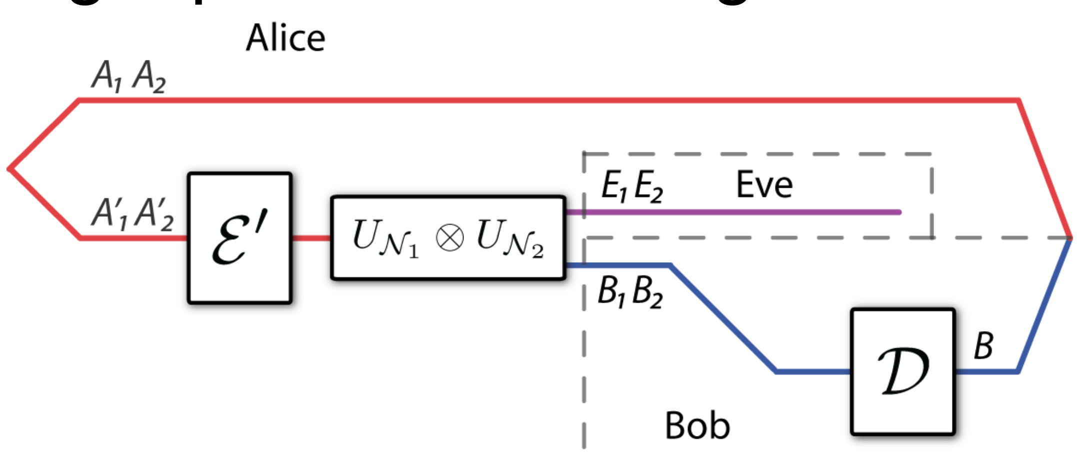
Converse Proof

$$\begin{aligned} nR &= I(A\rangle B)_{\Phi} && \text{Evaluate coherent information of Bell state} \\ &= I(A_1^n A_2^n \rangle B)_{\Phi} && \text{Isometry relates Alice's systems} \\ &\leq I(A_1^n A_2^n \rangle B)_{\omega'} + \epsilon' && \text{Fannes' inequality} \\ &\leq I(A_1^n A_2^n \rangle B_1^n B_2^n)_{\omega} + \epsilon' && \text{Quantum data processing} \end{aligned}$$

Achievability Proof



Can think of noisy state as arising
from sending a pure state through a second channel



Project onto a type subspace and use standard techniques
for entanglement generation over a quantum channel

Smith-Yard Like Superactivation

Example in which there is a **dramatic benefit** to channel-state coding

Alice and Bob share a state
with **no distillable entanglement**,
but some secret key (a Horodecki state)

The channel connecting them is a
zero-capacity 50% erasure channel

Using an argument similar to Smith and Yard,
can show that there is a
non-zero entanglement generation rate

This would be **impossible** using
independent strategies outlined earlier!

Conclusion and Open Questions

Open question: How to achieve a protocol that generates quantum communication without classical communication?

Open question: How does a noisy channel and noisy state perform in a trade-off scenario with classical communication, quantum communication and entanglement?

Open question: Examples of channels and states for which we can evaluate the formula? (*degradability is a start*)

Open question: What about varying the proportions of channels and states? For example, 2 states for every 1 channel use?