Entanglement generation with a quantum channel and a shared state

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Overview

Quickly overview some protocols from quantum Shannon theory

Discuss the practical motivation for our work and potential strategies that are suboptimal

Discuss our main result: capacity theorem with achievability proof and converse proof

Give an example of superactivation and discuss future questions
Quantum Communication

One important quantum information processing task is to transmit quantum information reliably.

Regularized channel coherent information is an achievable rate

\[ Q(\mathcal{N}) \equiv \max_{\phi} I(A\rangle B) \]

\[ I(A\rangle B) \equiv H(B) - H(AB) \quad \text{and} \quad \mathcal{N}_{A'\rightarrow B}^{A'\rightarrow B}(\phi^{AA'}) \]

Father Protocol

Trade-off between entanglement consumption and quantum transmission

Trade-off between quantum communication consumption and entanglement generation

\[ \langle \rho^{AB} \rangle + \frac{1}{2} I(A; E)[q \to q] \geq \frac{1}{2} I(A; B)[qq] \]

Motivation for Present Work

What if both the quantum channel and the shared state are noisy?

Practical Application:
Entanglement-assisted quantum codes where shared entanglement is noisy

Brun, Devetak, Hsieh. Science 2006
Potential Yet Suboptimal Strategies

Use an **LSD random quantum code** for the channel and **independently distill entanglement** from the state.

Distill the entanglement and **execute the father protocol** if there is enough entanglement available.

Use the channel to generate quantum communication and **execute the mother protocol** if enough quantum communication is available.
Initial: Alice and Bob share a noisy state

Preparation: Alice performs some preparation map

Transmission: Alice transmits encoded state over channel
(allow classical communication)

Decoding: Bob decodes
Channel-State Capacity Theorem

The entanglement generation capacity $E(\mathcal{N} \otimes \rho)$ of a quantum channel $\mathcal{N}$ and a bipartite state $\rho$ is

$$E(\mathcal{N} \otimes \rho) = \lim_{l \to \infty} \frac{1}{l} E^{(1)}(\mathcal{N}^\otimes l \otimes \rho^\otimes l), \quad (1)$$

where the “one-shot” capacity $E^{(1)}(\mathcal{N} \otimes \rho)$ is

$$E^{(1)}(\mathcal{N} \otimes \rho) = \max_{\mathcal{P}} I(A_1 A_2 B_1 B_2)_\omega. \quad (2)$$

The maximization is over all preparations $\mathcal{P}^{A_2 \rightarrow A_1 A_2 A'_1}$ and the coherent information $I(A_1 A_2 B_1 B_2)_\omega$ is with respect to the following state:

$$\mathcal{N}^{A'_1 \rightarrow B_1} (\mathcal{P}^{A_2 \rightarrow A_1 A_2 A'_1} (\rho^{A_2 B_2})). \quad (3)$$
Converse Proof

\[ nR = I(A \rangle B)_{\Phi} \]  
Evaluate coherent information of Bell state

\[ = I(A_{1}^{n} A_{2}^{n} \rangle B)_{\Phi} \]  
Isometry relates Alice's systems

\[ \leq I(A_{1}^{n} A_{2}^{n} \rangle B)_{\omega'} + \epsilon' \]  
Fannes' inequality

\[ \leq I(A_{1}^{n} A_{2}^{n} \rangle B_{1}^{n} B_{2}^{n})_{\omega} + \epsilon' \]  
Quantum data processing
Achievability Proof

Can think of noisy state as arising from sending a pure state through a second channel.

Project onto a type subspace and use standard techniques for entanglement generation over a quantum channel.
Example in which there is a **dramatic benefit** to channel-state coding

Alice and Bob share a state with **no distillable entanglement**, but some secret key (a Horodecki state)

The channel connecting them is a **zero-capacity 50% erasure channel**

Using an argument similar to Smith and Yard, can show that there is a **non-zero entanglement generation rate**

This would be **impossible** using independent strategies outlined earlier!
Open question: How to achieve a protocol that generates quantum communication without classical communication?

Open question: How does a noisy channel and noisy state perform in a trade-off scenario with classical communication, quantum communication and entanglement?

Open question: Examples of channels and states for which we can evaluate the formula? (degradability is a start)

Open question: What about varying the proportions of channels and states? For example, 2 states for every 1 channel use?